A Statistical Evaluation of Cephalometric Prediction*

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INTRODUCTION

In recent years a variety of methods have been proposed for the cephalometric prediction of craniofacial growth. Certain of these have gained a considerable measure of clinical acceptance, apparently based on little else but testimonials from "satisfied users." Popularity, however, is no substitute for objective proof. At present such proof is almost completely lacking.

It will be the purpose of this paper to examine statistically the predictive significance of a wide variety of measures obtained from the lateral cephalogram and to explore, at some length, any apparent potentials and limitations which may be found.

REVIEW OF THE LITERATURE

According to Kendall and Buckland,[32] ". . . prediction is the process of forecasting the magnitude of statistical variates at some future point of time." Implicit in this definition is the supposition that the magnitude of the characteristic being forecast is initially unknown. Obviously, if the individual pattern of craniofacial growth ". . . is established at a very early age and that once attained it does not change,"[10] the concept of growth prediction is trivial. There is, however, abundant evidence that in many instances the individual pattern does indeed change.[3-9,14,42,45,46,48] If this is true, an investigation of prediction is, at least in potential, a meaningful exercise.

Many approaches to growth prediction are based on the presumed significance of certain characteristics of the craniofacial skeleton. Frequent reference is made to the mandibular plane angle,[16,50,53,54,59,60] the occlusal plane angle,[16,28,29,51,54] the Y axis,[17,51,54] the X-Y axis,[51,54] as well as a plethora of mandibular anatomical characteristics. Although these measures provide an abstraction of the growth which has already taken place, there is little substantive evidence which would indicate that they provide significant information concerning that growth which has yet to occur. Moreover, it is very difficult to imagine a method by which the efficacy of these intuitive "predictors" could be verified, as long as changes due to growth and changes due to treatment are conveniently confounded. As Burstone[12] has stated:

> "The starting place, therefore, for a growth prediction is an estimation of the changes which might occur without orthodontic intervention."

Studies by Björk,[5] Björk and Palling,[8] Harvold,[20] Lande,[34] Maj *et al.*,[35,36] and by Meredith and co-workers[13,38,40,41,43,44,47] have provided simple size-size, size-gain, and gain-gain coefficients of linear correlation for a number of craniofacial dimensions. Although a few statistically significant relationships have been uncovered, their clinical value is generally so questionable as to prompt the following comment from Horowitz and Hixon:[24]

> "The evidence is quite conclusive that no morphologic trait has yet been found that enhances the orthodontist's ability to predict future growth changes."

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Considering the complexity of craniofacial growth, it would seem highly improbable that any *one* measure could ever contain enough information to be of predictive significance. This limitation may, at least in theory, be circumvented by the use of more than one variable. Such an approach has been advocated repeatedly by Ricketts.[50,51,53,54] Statistical verification of this highly subjective method is, unfortunately, almost completely lacking. There are, however, statistical tools by which the predictive significance of multiple variables may be evaluated. Accordingly, several studies have recently been completed in which stepwise multiple regression and correlation have been applied to the problem.[30,2,15] The first of these, a more general treatment of the "multivariate" approach to cephalometric prediction, provides the basis for the present communication.

Materials and Methods

The cephalograms analyzed in this investigation were obtained from the University of Michigan Elementary School Growth Study. The sample consisted of ninety subjects (fifty males and forty females), each represented by two lateral cephalograms separated by a five-year interval. Selection was in no way influenced by the presence or absence of malocclusion, or upon any characteristic of the variables to be measured. None, of course, had undergone orthodontic therapy. Specifically, there were ten males and eight females in each of five age-pairs: 4 and 9, 5 and 10, 6 and 11, 7 and 12, and 8 and 13 (Figure 1).

From the first cephalograms of each pair were obtained a set of 42 linear, angular, area, and proportional measurements (independent variables) chosen to provide a thorough description of the size, shape, and position of a number of craniofacial structures. Seven measurements of maxillomandibular relation and maxillary prognathism were obtained from the second cephalograms. These seven measurements, along with their five-year changes, constituted the 14 variables to be predicted (dependent variables). The various measures are described in Tables 1 and 2 and depicted in Figures 2-5.[33] All measurements were executed in a conventional manner with the exception of areas (in artificial "vernier units") which were obtained with a compensating polar planimeter.

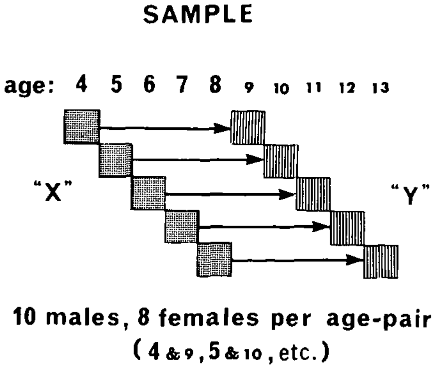

Figure 1 Distribution of sample by age and sex.

To test the extent of linear relationship between *individual* independent and dependent variables, use was made of the Pearson product-moment coefficient of linear correlation. This statistic may be represented by the notation, r_{xy}, where x represents the values for any of the 42 measures taken from the initial cephalogram, and y represents the values for a given measure obtained from the subsequent cephalogram. In this study coefficients of linear correlation were considered to be statistically significant if the odds against their occurring by chance were greater than 19:1 (5% level).[1]

When relatively large samples are employed, a given coefficient of correlation may indicate a relatively slight linear relationship between two variables and still be statistically significant.

TABLE 1

1. *Linear Measures* (see Figure 2)
   a. Size
      (1) Palatal length (Pal.): A to PNS. measured parallel to palatal plane.
      (2) Effective mandibular length (M): Cp to Me, measured parallel to palatal plane.
      (3) Anterior cranial base length: S to Na.
      (4) Posterior cranial base length: S to Ba.
      (5) Upper anterior face height (Ha): Na to palatal plane, measured perpendicular to palatal plane.
      (6) Upper posterior face height (Hp): S to palatal plane, measured perpendicular to palatal plane.
      (7) Lower face height (Hi): Me to palatal plane, measured perpendicular to palatal plane.
      (8) Sagittal dental arch length—maxilla: $\underline{e}$ to A, measured parallel to natural occlusal plane.
      (9) Sagittal dental arch length—mandible: $\bar{e}$ to A, measured parallel to natural occlusal plane.

   b. Position
      (1) Anteroposterior position of the condyle relative to S: Cp to S, measured parallel to palatal plane.
      (2) Anteroposterior position of posterior nasal spine relative to S: S to PNS, measured parallel to palatal plane.
      (3) Anteroposterior position of the condyle relative to posterior nasal spine: Cp to PNS, measured parallel to palatal plane.
      (4) Anteroposterior position of A relative to Na (Prog.): A to Na, measured parallel to palatal plane. The measure is positive if A is anterior to Na.
      (5) Vertical position of condyle relative to palatal plane (Hc): Cs to palatal plane, measured perpendicular to palatal plane.

   c. Maxillomandibular Relation (measurements are positive if mandible is anterior to maxilla)
      (1) Skeletal
         (a) Position of A point relative to B point (A/B): A to B, measured parallel to natural occlusal plane.
         (b) Position of A point relative to B point ($\underline{A}/\overline{B}$): A to B, measured parallel to palatal plane.
      (2) Dental
         (a) Position of $\underline{e}$ point relative to $\bar{e}$ point (e/e): $\underline{e}$ to $\bar{e}$, measured parallel to natural occlusal plane.
         (b) Difference between maxillary and mandibular sagittal dental arch lengths: ($\underline{e}$-A) minus ($\bar{e}$-B).

2. *Angular Measures* (see Figure 3)
   a. Angle of cranial base flexure (∠Flex.): Ba-S-Na.
   b. Gonial angle (∠Gon.): Cp-Go-Me.
   c. Palatal plane angle (∠Pal.): angulation of palatal plane relative to S-Na.
   d. Occlusal plane angle (∠Occ.): angulation of natural occlusal plane relative to palatal plane.

e. Mandibular plane angle (∠Mand.): angulation of mandibular plane relative to palatal plane.
f. Maxillary prognathism (∠X): Na-S-PNS
g. Pterygoid plane angle (∠Ptyg.): angulation of internal pterygoid process relative to S-Na.
h. Maxillomandibular relation (∠Z): Gn-S-PNS

3. *Area Measures* (see Figure 4)
   a. Septal area (Sp): area of the closed figure S-Na-A proj.-PNS-S. "A proj." is the intersection of Na-A with palatal plane.
   b. Mandibular area (Mn): the area of the mandibular outline including the dentition.
   c. Total facial area (Σf): the sum of Sp and Mn.
   d. Maxillomandibular relation (Mn/Sp): ratio of mandibular to maxillary area.

4. *Proportional Measures*
   a. Maxillary
      (1) Sp/S-Na
      (2) Sp/S-Ba
      (3) Sp/Ha
      (4) Sp/Hp
      (5) Sp/A-PNS
      (6) Sp/S-PNS
      (7) Sp/$\underline{e}$-A
   b. Mandibular
      (1) Mn/M
      (2) Mn/Hi
      (3) Mn/Hc
      (4) Mn/Cp-S
      (5) Mn/$\bar{e}$-B

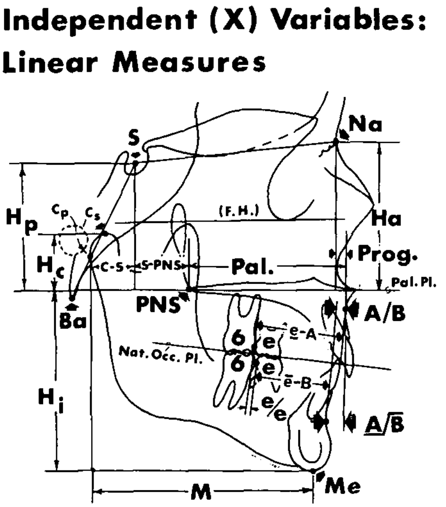
Independent (X) Variables: Linear Measures

Figure 2

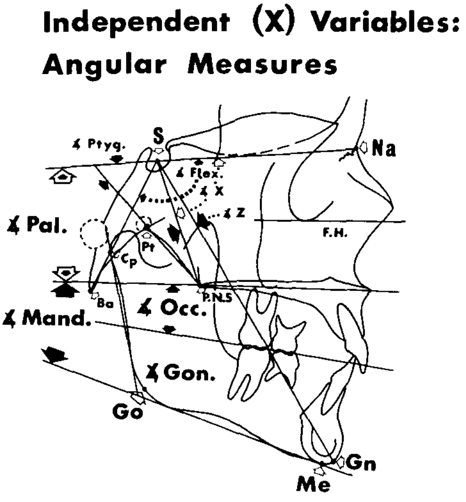
Independent (X) Variables: Angular Measures

Figure 3

TABLE 2

Dependent Variables

1. *Future Size*
   a. Maxillomandibular relation:
      (1) A/B
      (2) $\underline{A}/\overline{B}$
      (3) ∠Z
      (4) e/e
      } As defined for independent variables
      (5) A/B_1: measured parallel to a plane having the same angulation relative to S-Na as had the natural occlusal plane in the earlier cephalogram.
      (6) e/e_1: as with A/B_1.
   b. Maxillary prognathism: Prog., as defined for independent variables.
2. *Change in Size:* (original measurement) minus (subsequent measurement).

Independent (X) Variables: Area Measures

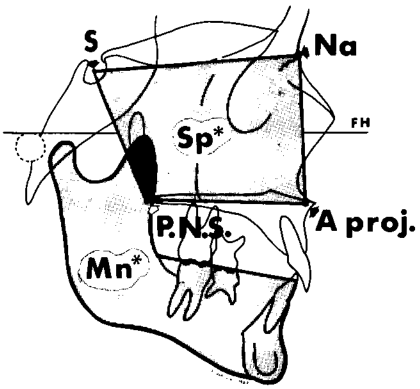

Figure 4

Dependent (Y) Variables:

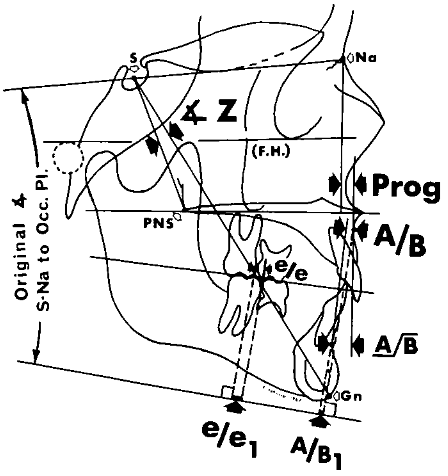

Figure 5

To obtain a clearer picture of the relative importance of the relationship, the statistic r^2, the coefficient of determination, was used:

$$r_{xy}^2 = \frac{\text{Variability of y explained by x}}{\text{Total variability of y}}$$

In order to determine whether the simultaneous consideration of *more than one* independent variable would account for a greater proportion of the variability in a given dependent variable, the technique of stepwise multiple regression was employed. Accordingly, it should be noted that for the measures employed, an assumption of normality does not seem out of order. The resulting "prediction" equations were of the form

$$\hat{y}_{ij} = a_i + b_{i1}x_{ij1} + b_{i2}x_{ij2} + \ldots + b_{ik}x_{ijk}.$$

The individual elements of these equations, for either males or females, are as follows:

1. $\hat{y}_{ij}$ is the predicted value of the ith dependent variable (i=1, 2, . . ., 14) for the jth individual. For males, j=1, 2, . . ., 50, and for females j=1, 2, . . ., 40.

2. a_i represents the numerical value of the "Y-intercept" of the multiple regression plane, and is constant for the ith equation.
3. $x_{ij1}, x_{ij2}, \ldots, x_{ijk}$ represent the values of the k independent variables which are found to contribute *significant unduplicated* information relative to the estimation of y_{ij}.
4. $b_{i1}, b_{i2}, \ldots, b_{ik}$ are regression coefficients which, when multiplied by their appropriate "x" ($b_{i1}x_{ij1}$, $b_{i2}x_{ij2}$, etc.), express the relative importance of each of the k significant independent variables.

With the aid of the IBM 7090 computer, a total of 28 such equations, 14 each for males and females, were calculated.

To test the practical value of each prediction equation the following factors were examined:[18,31,61]

1. The coefficient of multiple correlation, $R_{y\hat{y}}$, to determine the degree of linear relationship between the actual values (y_{ij}) and the predicted values ($\hat{y}_{ij}$) of a given future measure.
2. The coefficient of multiple determination, $R^2_{y\hat{y}}$, to show the proportion of the total variability of y which is explained by the regression equation, where

$$R^2_{y\hat{y}} = \frac{\text{Explained variability of y}}{\text{Total variability of y}}$$

3. The standard error of estimate (S.E.Y), to determine the absolute size (in mm or degrees) of the standard deviation of the differences between the observed and predicted values of y.
4. A coefficient of prediction efficiency (E), to estimate, for this sample, the per cent reduction in standard errors of estimate (improvement of accuracy) when one method of prediction is used as opposed to another:

$$E = \frac{(S.E.Y_A) - (S.E.Y_R)}{(S.E.Y_A)} \cdot 100, \text{ where}$$

   $S.E.Y_A$ is the standard error of estimate, for a given dependent variable, obtained from an alternate method of prediction, and $S.E.Y_R$ is the standard error of estimate, for the same dependent variable, obtained from the appropriate multiple regression equation.[19,39]
5. The coefficient of reliability r_{xx}, measuring the extent of linear correlation between two separate sets of measurements (double-determinations) of A/B and e/e obtained from each of a separate series of 90 cephalograms, traced by a number of workers. This statistic is an estimate of the maximum valid linear correlation possible for the prediction of these two dependent variables.
6. The standard deviation of the differences between the 90 double-determinations (S.D.D.), an estimate of the minimum valid standard error possible for A/B and e/e.

FINDINGS

Preliminary Observations

Means and standard deviations for all measures, as well as range of change for dependent variables, are presented in Tables 3-5. Because this investigation was predicated upon the assumption that individual changes may occur in the pattern of facial growth, the data in Table 5 are of particular interest. It may be noted that, while the mean five-year changes were relatively slight, the sample estimates for range and standard deviation were of sufficient magnitude to indicate that the extent of *individual* change, in at least some subjects, is of considerable interest.

Simple Correlation

In Tables 6 and 7 are presented the coefficients of linear correlation between

TABLE 3

Independent Variables

Means and Standard Deviations for 50 Males

MEASURE	MEAN	S.D.	MEASURE	MEAN	S.D.
Cp-S	14.59	2.83	∠X	65.56	4.02
S-PNS	21.86	3.11	∠Z	–1.69	2.92
Cp-PNS	36.43	2.78	∠Flex.	127.62	4.51
S-Na	71.39	2.98	∠Ptyg.	45.40	4.85
S-Ba	42.67	2.62	∠Gon.	133.50	4.03
M	64.37	5.74	Sp	385.00	37.58
Pal.	45.33	2.48	Mn	413.10	42.27
Hi	58.46	3.54	Σf	798.10	77.17
Hc	20.32	3.57	Mn/Sp	1.08	0.076
Hp	38.04	2.80	Mn/Cp-S	29.48	7.46
Ha	44.98	3.80	Mn/Hi	7.07	0.52
$\underline{e}$-A	27.70	1.64	Mn/Hc	20.86	3.71
$\bar{e}$-B	27.10	1.72	Mn/M	6.43	0.56
Prog.	–3.67	2.87	Mn/$\bar{e}$-B	15.29	1.99
$\underline{A}/\overline{B}$	-10.06	2.97	Sp/S-PNS	17.93	2.81
A/B	–0.09	2.32	Sp/Pal.	8.48	0.54
e/e	0.43	1.70	Sp/Hp	10.13	0.78
($\underline{e}$-A) - ($\bar{e}$-B)	0.57	1.63	Sp/Ha	8.57	0.36
∠Pal.	5.52	2.76	Sp/Na-S	5.39	0.41
∠Occ.	16.29	3.27	Sp/S-Ba	9.03	0.61
∠Mand.	30.55	4.21	Sp/$\underline{e}$-A	13.97	1.78

TABLE 4

Independent Variables

Means and Standard Deviations for 40 Females

MEASURE	MEAN	S.D.	MEASURE	MEAN	S.D.
Cp-S	13.92	2.92	∠X	66.11	4.11
S-PNS	21.45	2.90	∠Z	–0.57	3.20
Cp-PNS	35.37	2.68	∠Flex.	127.18	4.71
S-Na	69.46	3.17	∠Ptyg.	45.97	5.56
S-Ba	41.89	2.81	∠Gon.	132.18	5.48
M	64.99	6.38	Sp	370.15	40.99
Pal.	44.99	2.71	Mn	406.45	46.28
Hi	57.30	2.87	Σf	776.60	82.70
Hc	20.05	3.26	Mn/Sp	1.10	0.08
Hp	36.97	2.81	Mn/Cp-S	30.37	6.69
Ha	44.56	4.08	Mn/Hi	7.08	0.69
$\underline{e}$-A	27.66	1.97	Mn/Hc	20.73	3.69
$\bar{e}$-B	26.67	1.79	Mn/M	6.27	0.60
Prog.	–2.47	2.71	Mn/$\bar{e}$-B	15.28	2.06
$\underline{A}/\overline{B}$	–8.31	3.17	Sp/S-PNS	17.49	2.68
A/B	0.02	2.62	Sp/Pal.	8.21	0.61
e/e	0.92	1.52	Sp/Hp	10.00	0.68
($\underline{e}$-A) - ($\bar{e}$-B)	0.99	1.96	Sp/Ha	8.30	0.41
∠Pal.	6.21	2.67	Sp/Na-S	5.32	0.43
∠Occ.	14.17	3.45	Sp/S-Ba	8.83	0.82
∠Mand.	29.36	4.49	Sp/$\underline{e}$-A	13.44	1.73

TABLE 5

Dependent Variables

MALES

Future Size			*Five-year Change*		
MEASURE	MEAN	S.D.	MEASURE	MEAN	S.D.
1. $\underline{A}/\overline{B}$	–8.61	3.31	8. $\underline{A}/\overline{B}$	1.45	2.25
2. A/B	–0.20	2.44	9. A/B	–0.04	1.75
3. A/B_1	0.99	2.69	10. A/B_1	1.08	1.86
4. e/e	0.39	1.73	11. e/e	–0.05	1.24
5. e/e_1	0.64	1.79	12. e/e_1	0.22	1.30
6. $\angle Z$	1.75	2.99	13. $\angle Z$	3.45	1.90
7. Prog.	–2.95	3.39	14. Prog.	0.75	1.53

FEMALES

Future Size			*Five-year Change*		
MEASURE	MEAN	S.D.	MEASURE	MEAN	S.D.
1. $\underline{A}/\overline{B}$	–7.56	4.25	8. $\underline{A}/\overline{B}$	0.77	2.39
2. A/B	0.20	3.16	9. A/B	0.15	1.91
3. A/B_1	0.97	3.93	10. A/B_1	0.95	2.43
4. e/e	1.05	1.37	11. e/e	0.03	1.23
5. e/e_1	1.14	1.44	12. e/e_1	0.22	1.27
6. $\angle Z$	2.24	3.37	13. $\angle Z$	2.81	1.71
7. Prog.	–1.40	3.69	14. Prog.	1.11	2.36

the 42 independent variables and the future size or five-year change of the seven measures of maxillomandibular relation and maxillary prognathism. Due to the large number of coefficients calculated (1176), only those which were significant at the 5% level of confidence are presented here. It may be seen that a large number of independent variables were significantly related to future size, the highest correlation in each instance being reserved for the original size of the same measure. Relatively few, however, bore a significant relation to the five-year change. Thus, when characteristics of the craniofacial skeleton were considered separately, much more information was available concerning the extent of stability than the degree of change.

Multiple Regression and Correlation

The prediction equations and standard errors of estimate obtained from a simultaneous consideration of all 42 independent variables along with the coefficients of multiple correlation and determination (R and R^2) may be seen in Tables 8 and 9. All values of R were significant at the 5% level. Obviously, when only one significant independent variable could be employed to generate a given equation ($k = 1$), the values of $R_{y\hat{y}}$ were equal to r_{xy}. Thus, an improvement in accuracy obtained only when more than one significant variable was available.

The values of R^2 ranged from .029 to .812, and were approximately .5 to .8 for the prediction of future size and from .1 to .4 for the prediction of change. Unfortunately, these results easily can be misinterpreted. It should, for that reason, be noted that in terms of standard errors it makes no essential difference which quantity—future size or five-year change—is chosen for estimation: given the original value of the variable in question, the final result is the same. This assertion may be verified by comparing the magnitude of S.E.Y for "size" with the corresponding value for "change" (Tables 8 and 9).

TABLE 6

Significant Coefficients of Linear Correlation Between Separate Independent and Dependent Variables

MALES

Dependent Variables: Future Size

Independent Variables	$\underline{A}/\sqrt{B}$	r	A/B	r	A/B_1	r	e/e	r	e/e_1	r	∠Z	r	Prog.	r
Linear	$\underline{A}/\sqrt{B}$	.75	$\underline{A}/\sqrt{B}$	.73	A/B	.73	e/e	.74	e/e	.73	S-PNS	–.54	Prog.	.89
	A/B	.55	$\underline{A}/\sqrt{B}$	.55	e/e	.54	A/B	.58	A/B	.56	Cp-S	.38	S-PNS	.50
	e/e	.51	e/e	.51	$\underline{A}/\sqrt{B}$	.49	$\underline{A}/\sqrt{B}$	.56	$\underline{A}/\sqrt{B}$	.53	$\underline{A}/\sqrt{B}$	.34	Pal.	.39
	Hp	–.36	$\underline{e}$-A	–.31	($\underline{e}$-A) - ($\bar{e}$-B)	–.46			Cp-PNS	–.28	Pal.	.33	M	.37
	M	.33	S-Na	–.30	Ha	–.38					Hc	.31	Ha	.35
	Hc	–.30			Cp-PNS	–.36							Cp-S	–.28
	($\underline{e}$-A) - ($\bar{e}$-B)	–.30			Pal.	–.31								
					$\bar{e}$-B	.28								
					S-Na	–.28								
Angular	∠Occ.	–.40	($\underline{e}$-A) - ($\bar{e}$-B)	–.52			∠Flex.	–.36	∠Flex.	–.35	∠Z	.79	∠Pal.	.36
											∠X	.57	∠Mand.	–.35
	∠Mand.	–.33	∠Z	.28							∠Ptyg.	.40	∠Occ.	–.33
											∠Gon.	–.29		
Area & Proportion	Mn/M	–.53	Mn/Sp	.39	Sp/S-Ba	–.43					Sp/S-PNS	.63	Sp/Hp	.35
	Sp/Ha	–.33	Sp/S-Ba	–.35	Sp	–.40					Mn/Hi	.31	Mn/Hi	.34
					Mn/Hi	–.39					Sp/S-Na	.31	Sp/S-Na	.34
					Sp/S-Na	–.36					Sp/S-Ba	.29	Mn/Hc	.31
					Sp/Pal.	–.35							Sp/S-PNS	–.30
					Σf	–.31							Sp	.29
					Mn/(e-B)	–.30								

TABLE 6 continued

Dependent Variables: Five-year Change

Independent Variables	A/$\overline{B}$	r	A/B	r	A/B_1	r	e/e	r	e/e_1	r	∠Z	r	Prog.	r
All Types	Sp/Pal.	–.36	Cp-S	.47	Ha	–.38	e/e	–.34	e/e	–.32	∠Z	–.28		
			A/B	–.31	Sp/Pal.	–.34	Sp	.28						
			e/e	–.29	∠Ptyg.	–.32	Σf	.28						
			Cp-PNS	.28	Sp/S-Na	–.31	Sp/S-Na	.28						
							∠Ptyg.	.28						

In Table 10 are presented coefficients of prediction efficiency (E) for multiple regression equations as compared with alternative methods or assumptions which could be used by the clinical orthodontist. These alternatives were:

1. The assumption that the future size of a given measure (for any individual in the 4-8 years age-group) may be estimated by the mean size of that same measure in the 9-13 year age-group. Under this assumption S.E.Y_A equals the standard deviation of the given measure in the 9-13 year age-group.
2. The assumption that the future size of a given measure (for any individual in the 4-8 year age-group) is forecast by the original size of that measure plus the mean five-year change. Under this assumption S.E.Y_A equals the standard deviation of the change in y over the five-year interval between cephalograms.

It may be seen that the use of specific prediction equations provided a marked reduction in standard errors of estimate when compared with the results of the assumption that the future mean would be achieved by the individual. However, when compared with the results of the assumption that just a relatively slight mean change would occur, the multiple regression equations generally produced only a modest reduction in the standard errors of estimate. The only marked exception to this generalization was the prediction of e/e relation for females. In this instance the multiple regression equation provided confidence intervals which were about 30% smaller than those obtained from the next most accurate method of prediction.

The measures of technical accuracy, r_{xx} and S.D.D., were obtained mostly for purposes of discussion and will be reported in that context.

TABLE 7

Significant Coefficients of Linear Correlation Between Separate Independent and Dependent Variable

FEMALES

Dependent Variables: Future Size

Independent Variables	$\underline{A}/\bar{B}$	r	A/B	r	A/B_1	r	e/e	r	e/e_1	r	∠Z	r	Prog.	r
Linear	$\underline{A}/\bar{B}$	.84	A/B	.80	A/B	.80	e/e	.66	e/e	.64	Hp	.47	Prog.	.76
	A/B	.70	($\underline{e}$-A) - ($\bar{e}$-B)	–.78	($\underline{e}$-A) - ($\bar{e}$-B)	–.78	A/B	.52	A/B	.54	$\bar{e}$-B	.45	S-PNS	.37
	($\underline{e}$-A) - ($\bar{e}$-B)	–.59	$\underline{A}/\bar{B}$	.59	$\underline{A}/\bar{B}$	.57	$\underline{A}/\bar{B}$	.47	$\underline{A}/\bar{B}$	.48	Pal.	.45	Cp-S	–.35
	M	.54	$\underline{e}$-A	–.45	$\bar{e}$-B	.44	Ha	–.42	Ha	–.35	$\underline{A}/\bar{B}$	.43	Pal.	.31
	e/e	.46	e/e	.41	$\underline{e}$-A	–.39	Hp	–.36	Hp	–.32	M	.41		
	Hc	–.45	$\bar{e}$-B	.33	e/e	.36	S-Na	–.31	Hc	–.32	S-PNS	–.38		
	$\bar{e}$-B	.31	Cp-PNS	–.31	Cp-PNS	–.35					($\underline{e}$-A) - ($\bar{e}$-B)	–.35		
											A/B	.33		
Angular	∠Mand.	–.61	∠Z	.31	∠Z	.41	∠X	–.36			∠Z	.87	∠Mand.	–.30
	∠Z	.49					∠Flex.	–.31			∠Gon.	–.41		
	∠Gon.	–.33									∠X	.38		
	∠Occ.	–.31									∠Mand.	–.33		
Area & Proportion	Mn/Hc	.57	Mn/Hc	.34	Mn/Hc	.33	Sp/Pal.	–.46	Sp/Pal.	–.43	Sp/S-PNS	.51	Mn/Cp-S	.36
	Mn/M	–.44					Sp/S-Na	–.37	Mn/M	–.37	Mn/Hi	.45		
							Mn/M	–.35			Mn	.38		
											Sp/Ha	.36		
											Σf	.35		
											Sp/S-Na	.35		

TABLE 7 continued

Dependent Variables: Five-year Change

Independent Variables

All Types

A̲/B̄	r	A/B	r	A/B$_1$	r	e/e	r	e/e$_1$	r	∠Z	r	Prog.	r
(e̱-A) - (ē-B)	–.35	∠Gon.	–.34	S-PNS	–.41	e/e	–.48	e/e	–.48	∠X	–.40	(e̱-A) - (ē-B)	–.43
S-PNS	–.31	Ha	–.31	(e̱-A) - (ē-B)	–.39					∠Pal.	–.38	A/B	.37
∠Gon.	–.31			Cp-PNS	–.35					S-Ba	–.38		
				Sp/Hp	–.31					Sp/Hp	–.37		
				∠Z	.31					Ha	–.37		
										∠Ptyg.	–.32		
										Σf	–.32		
										Mn/Hi	–.31		
										Sp/S-Na	–.31		
										Mn	–.31		

DISCUSSION

For the orthodontist, prediction involves not one, but a sequence of procedures and/or assumptions, each of which may contribute a measure of accuracy or inaccuracy to the final estimation. It will be the purpose of this discussion to analyze these various components in light of the present findings.

In Figure 6 is shown a plot of "variability accounted for" (r^2 or R^2 times 100) as a function of the various steps employed, or conditions encountered. This graph provides an overview of the various aspects of the problem of clinical prediction and, as such, will be referred to frequently. At this juncture a note of caution is in order: the *relative* position of the elements in Figure 6 is mathematically defensible; the *absolute* magnitude of any estimated values, of course, is not. However, it is hoped that all estimates are as conservative as the current literature and the present findings will allow.

As a first approximation, the orthodontist could presume that the future size of A/B is forecast best by an estimate of the appropriate population mean. As far as the individual is concerned, however, this is equivalent to no prediction at all, inasmuch as none of the variability of A/B has been accounted for (Figure 6, step A).

Fortunately, the original size of A/B is significantly related to its future size ($r = .80$). Thus, $(.80)^2 \cdot 100$, or 64% of the variability in future A/B may be eliminated merely by knowing the magnitude of the original dimension (Figure 6, B). It would seem, then, that the individual pattern may be expected to demonstrate not only a clinically interesting amount of change (Table 5), but also a useful amount of stability. As stated by Horowitz and Hixon:[24]

TABLE 8

Multiple Regression Equations, R, R^2, and S.E.Y

MALES

	$\hat{y}$ =	a +	(b_1 •	x_1) +	(b_2 •	x_2) +	(b_3 •	x_3) +	(b_4 •	x_4)	R	R^2	S.E.Y
Future Size	$\underline{A}/\overline{B}$	12.48	0.84	$\underline{A}/\overline{B}$	–1.49	Sp/Pal.					.785	.616	2.11
	A/B	–0.07	0.88	A/B	0.32	Cp-S	–0.11	∠Ptyg.			.811	.657	1.49
	A/B_1	10.20	0.81	A/B	–0.20	Ha					.788	.621	1.71
	e/e	0.06	0.76	e/e							.742	.550	1.19
	e/e_1	0.31	0.76	e/e							.725	.526	1.26
	∠Z	3.12	0.81	∠Z							.792	.628	1.87
	Prog.	0.89	1.05	Prog.							.887	.787	1.60
Five-year Change	$\underline{A}/\overline{B}$	28.21	–1.87	Sp/Pal.	–0.40	$\bar{e}$-B					.469	.220	2.06
	A/B	–0.49	0.35	Cp-S	–0.10	∠Ptyg.					.543	.295	1.52
	A/B_1	9.53	–0.19	Ha							.384	.148	1.75
	e/e	0.06	–0.25	e/e							.339	.115	1.19
	e/e_1	0.33	–0.25	e/e							.322	.104	1.25
	∠Z	3.14	–0.19	∠Z							.285	.081	1.86
	Prog.												

TABLE 9

Multiple Regression Equations, R, R^2, and S.E.Y

FEMALES

	$\hat{y}$ =	a +	(b_1 •	x_1) +	(b_2 •	x_2) +	(b_3 •	x_3) +	(b_4 •	x_4)	R	R^2	S.E.Y
Future Size	$\underline{A}/\overline{B}$	9.78	1.22	$\underline{A}/\overline{B}$	–0.34	S-PNS					.864	.746	2.23
	A/B	14.49	1.18	A/B	–0.22	Ha	–0.63	e/e	–0.13	∠Gon.	.883	.780	1.58
	A/B_1	19.83	1.58	A/B	–0.90	e/e	–0.26	S-Na			.865	.748	2.08
	e/e	0.25	0.46	e/e	–1.42	Sp/Pal.	0.17	S-Na			.800	.641	0.86
	e/e_1	11.66	0.51	e/e	–1.47	Sp/Pal.	0.31	Sp/$\underline{e}$-A			.755	.570	0.99
	∠Z	10.19	0.92	∠Z	–0.30	∠Pal.	–0.88	Mn/M			.901	.812	1.54
	Prog.	1.77	1.08	Prog.	–0.51	($\underline{e}$-A) - ($\bar{e}$-B)					.809	.654	2.26
Five-year Change	$\underline{A}/\overline{B}$	1.19	–0.43	($\underline{e}$-A) - ($\bar{e}$-B)							.353	.125	2.30
	A/B	13.93	–0.14	∠Gon.	–0.20	Ha	–0.42	e/e			.596	.355	1.62
	A/B_1	7.98	–0.31	S-PNS	–0.43	($\underline{e}$-A) - ($\bar{e}$-B)					.532	.283	2.14
	e/e	0.14	–0.51	e/e	–1.33	Sp/Pal.	0.16	S-Na			.675	.456	0.96
	e/e_1	11.66	–0.49	e/e	–1.47	Sp/Pal.	0.31	Sp/$\underline{e}$-A			.669	.447	0.99
	∠Z	51.75	–0.73	∠Z	0.97	Sp/		Sp/Ha			.606	.367	1.44
	Prog.	1.62	–0.51	($\underline{e}$-A) - ($\bar{e}$-B)		S-PNS	–2.10				.428	.183	2.19

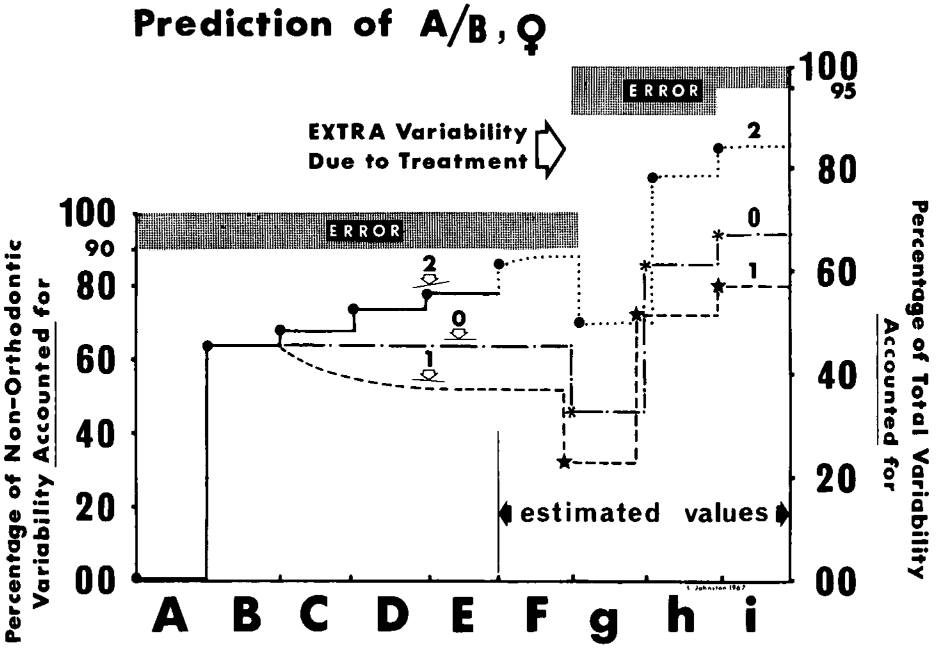

Figure 6 Cumulative plot of variability accounted for as a function of the prediction steps employed, or conditions encountered. Line "0", no attempt to predict individual five-year growth changes. Line "1", attempt based on spurious assumptions. Line "2", attempt based on the appropriate "prediction equation." In the absence of validation on other samples this must be viewed as a maximal estimate. Steps A-F refer to the prediction of growth in the absence of orthodontic intervention; steps g-i refer to prediction when growth and treatment changes are confounded. The additional treatment variability is represented by an increase in the height of the right ordinate. The stippled area, labeled "error," represents an estimate (r_{xx}^2 times 100) of that random variability inherent in the cephalometric method.

TABLE 10

Prediction Efficiency

Assumption: the individual will attain the mean for ages 9-13.			Assumption: the individual will undergo the mean five-year change.		
Dependent Variable	Males	Females	Dependent Variable	Males	Females
$\underline{A}/\overline{B}$	36%	48%	$\underline{A}/\overline{B}$	7%	7%
A/B	39%	50%	A/B	15%	17%
A/B_1	36%	47%	A/B_1	8%	14%
e/e	32%	37%	e/e	4%	30%
e/e_1	30%	31%	e/e_1	3%	22%
∠Z	38%	54%	∠Z	2%	10%
Prog.	53%	39%	Prog.	-4%	4%

> ". . . The best estimate that can be made for the facial pattern of any given individual is to begin with the pattern presented by the patient and add on the average growth change."

Because the standard deviation of a measure at time T_2 is usually greater than that of the change during the interval T_1 to T_2, this method results in a prediction efficiency of $[(S.D._{future} - S.D._{change})/S.D._{future}]$ times 100. For A/B, this amounts to an efficiency of 40%.

Although a 40% reduction in confidence intervals is useful, its theoretical basis is relatively trivial; it still is not, in any strict sense, "growth prediction," inasmuch as the only unknown quantity, the five-year change, is still estimated by a mean. Ideally, one would prefer an individual prediction for change as well. It is at this stage that the various prediction schemes begin to go their separate ways (Figure 6, C).

Patently, the orthodontist may or may not choose to attempt the prediction of change. Contrary to much that has been written, failure to attempt this step does not necessarily constitute "neglect of the patient." In the absence of proof to the contrary, omission might actually represent the most prudent course of action: should a prediction be attempted on the basis of nonsignificant variables, the result is usually less accurate than that obtained from the use of the mean change alone.[22] Specifically, if r_{xy} does not differ significantly from zero, the use of x to achieve a predicted value of y which differs from the mean will, in most cases, result in a larger sum of squares.[37] The result of an indeterminate number of such pseudo-predictions is depicted in Figure 6 by line "1", steps C-F.

Even if statistically significant predictors are available, one must have reasonably well-defined rules, such as are provided by regression equations, in order to use them properly. In dealing with numbers, hunches or clinical experience alone can waste much valuable information. Thus, the problem of prediction reduces to two basic questions:

1. Are the variables employed significant?
2. If significant, exactly *how* should each be used?

With reference to the common practice of using a single favorite cephalometric measure (e.g., gonial or mandibular plane angle) no evidence could be found to indicate that any of the 42 measures investigated could provide, by itself, sufficient information upon which to base a highly useful prediction of A/B change or, for that matter, any of the other dependent variables (Figure 6, C-2). Those measures which were at least statistically significant were clinically useless, their coefficients of determination (r^2) being generally less than .10 (Tables 6 and 7). This result is in basic agreement with a number of other size-gain correlation studies cited earlier. There are, of course, an infinity of measures which could yet be investigated. Unfortunately, the large number, varied nature (linear, angular, area, and proportional), and slight significance of those which have already been examined, both here and elsewhere, would seem not to foreshadow an imminent breakthrough. Parenthetically, one wonders how long, in the absence of reinforcement by positive results, this line of research should be continued.

If a single variable is not enough, why not several? In Tables 6 and 7 it may be seen that a rather large number of measures were found to bear a slight, but statistically significant, linear relation to the five-year change in a given dependent variable. Thus, it might seem reasonable to assume that the use of a sufficient number of these "10% predictors" might yield a rather accurate estimation of future growth. For the present sample, however, the general efficacy of such a procedure could not be substantiated. It was found that many of the statistically significant variables were merely abstractions of the same craniofacial characteristic (e.g., maxillomandibular relation) and, as a result, provided redundant information. Because the stepwise multiple regression program used in this study rejects such repetitious information, in only four equations (for "change") were as many as three variables combined to form the predictive system at the 5% level of confidence (Tables 8 and 9). Thus, for the prediction of a given five-year gain, the pertinent information contained in all 42 independent variables could actually be obtained from a linear combination of not more than three. It should be noted that the specific group of k variables employed in each equation is probably not unique for this sample, inasmuch as other groups having equal (but not greater) accuracy could, in theory, be obtained. For A/B change (Table 9) three variables were sufficient: gonial angle (∠Gon.), upper anterior face height (Ha), and molar relation (e/e). The progressive improvement in accuracy attendant on their use is shown in Figure 6: C-2, D-2, E-2, respectively.

Recently, DeVries[15] has applied similar statistics to a sample of 30 Class II, Division 1 females, measured at ages 8 and 12. Although for A/B his results tend to corroborate the present study (values of R for future size and four-year change were .91 and .48, respectively), they must be evaluated with caution: sample selection was, in part, based on the magnitude of one of the variables chosen for prediction, A/B at age 12. Specifically, choice was exercised to ensure that " 'A' plane was in a prognathic relationship to B point as measured on the Cephalometric Template *at age 12* [italics mine]." The following statement by Snedecor[58] is pertinent: "Though X may be be chosen at will, there must be no selection of Y; it must be taken at random from all the Y's corresponding to a given X."

As may be seen in Table 10, an individual prediction based on one of the calculated equations is likely to be, at best, only 30% more efficient than one obtained from the sum of the original measurement and an estimate of the mean change for the population. Unfortunately, in the most interesting cases—those in which the actual change proves to be relatively great—the mean is, by definition, always inaccurate. In contrast, an appropriate multiple regression equation could be used in the prediction of change as well. It is suggested that equations of the efficiency presented here, if validated, could be employed "qualitatively" in a certain proportion of clinical cases. Using A/B change for females, this approach may be illustrated.

For A/B, the mean five-year change was +0.2 mm and the standard error of the regression was ±1.6 mm. If, for a given individual, a change of more than [(+0.2) + (+1.6)] mm were forecast, one would have a fair chance of being correct in the assumption that a positive change of at least 0.2 mm might occur. Conversely, if a negative change of [(+0.2 + (—1.6)] mm or more were predicted, one would assume that a positive change as great as +0.2 might *not* occur. It is obvious that such "qualitative" predictions would be possible in only about one third of the females to which it might be applied, and perhaps not at all for males (Table 10). Whether this limited knowledge would be worth the effort is a moot point.

Several generalizations concerning the predictive significance of the various types of measures employed in this study are perhaps in order (Tables

6-9). Venerable "predictors" such as the mandibular, palatal, and occlusal plane angles, as well as the linear size of discrete anatomical structures, were seldom found to be of significance. Instead, maxillary and mandibular position, maxillomandibular relation, area, and proportional measures involving area could most frequently be used in generating the various equations. Obviously, the specific types of measures which prove useful are a function of the dependent variables chosen for prediction, in this case maxillomandibular relation and maxillary position. It is not, however, immediately apparent just what else one would like to predict. For example, Maj *et al.*[35,36] have recently presented equations for the prediction of mandibular growth increments. Although these equations are as accurate as those presented here, one is still left with the problem of estimating: 1) direction of estimated mandibular growth, 2) future position of the mandible as a whole, 3) future maxillary growth, and 4) future maxillary position. Additional error is the inevitable consequence. Thus, the prediction of dimensions which are not useful, in and of themselves, would seem merely to beg the question.

Perhaps the most concerted attempt to apply methods of growth prediction to clinical orthodontics has been that of Ricketts.[50,51,53,54] In his method most aspects of clinical estimation are considered in quite some detail. It is, however, unfortunate that his procedures are generally thought to constitute a system of "growth prediction"—unfortunate in that relative to the prediction of individual, as opposed to mean growth changes several objections may be raised:

1. There is little evidence as to the exact significance of each of the many variables employed.
2. There is little reason to believe that these numerous variables are not, to a greater or lesser extent, redundant (ten are obtained from the mandible alone).
3. There are no objective rules available for the use of these variables.
4. Estimates of confidence intervals or of prediction efficiency for a consecutive, nonorthodontic series are not available.

Although the results of the present study indicate that these objections are potentially very significant, a more direct verification is, of course, desirable, inasmuch as *any* method, complex or simple, which has a dubious theoretical basis and which is unsupported by acceptable proof could easily be less accurate than no prediction at all (Figure 6, compare lines 0 and 1). As Galton has said, "General impressions are never to be trusted."

Obviously, if the prediction of change is to be improved dramatically (Figure 6, F-2), new sources of information may have to be found. Cephalometric data from the P.A. film,[55] as well as from siblings,[25,26,27,57] might prove useful depending on the characteristic to be estimated.

There is, however, one important aspect to prediction for which the computer can supply no formulae: the estimation of orthodontically-induced changes in the craniofacial complex.[11,23,49,52,62] This is uniquely the province of the individual orthodontist. For the anteroposterior position of A point, treatment changes of up to 8.0 mm reported by Ricketts[53] far exceed any growth changes reported here (4.8 mm —see Prog., Table 5). Failure to account for such additional* variability would, of course, result in a generalized

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* For reasons of simplicity an additive model for growth and treatment effects is assumed here. The actual nature of any interaction is, however, unknown.

decrement in accuracy regardless of method (Figure 6, g). In contrast, the ability to estimate in advance the effects of treatment may well result in a significant improvement in accuracy (Figure 6, h).

There is, unfortunately, a limit to the degree of accuracy possible. This ceiling is imposed by the errors inherent in the cephalometric technique. The values of r_{xx} and S.D.D. calculated for A/B and e/e indicate that coefficients of determination (r^2, R^2) higher than .897 and .943 and standard errors smaller than 1.11 and 0.42 mm, respectively, are highly unlikely, regardless of the predictive scheme employed. If values such as these are not clinically useful, either error must be reduced (Figure 6, i) or the concept of cephalometric prediction abandoned. Whether or not the steps necessary to reduce error[21,56] would exceed the point of diminishing returns is entirely another matter.

In view of the findings of this investigation it would appear that there may be at least three components to be dealt with in the prediction of many craniofacial measures:

1. The extent of stability
2. The extent of change due to growth
3. The extent of change due to treatment

Because of the relative magnitude of each of these components, it is probable that a "clinically useful" forecast of the posttreatment face could obtain in the complete absence of any valid prediction of individual growth changes (Figure 6, h-0 and h-1). Whether or not cephalometric prediction is capable of a more sophisticated role in clinical orthodontics is, perhaps appropriately, a question for the future. In the meantime, as first stated by Cicero, "I shall always consider the best guesser the best prophet."

CONCLUSIONS

For the present sample the following conclusions may be drawn:

1. Certain craniofacial characteristics, measured at one point in time (4, 5, 6, 7, or 8 years of age), appear to be related to changes which may occur in maxillomandibular relation and maxillary prognathism during the succeeding five years.
   a. However, no single independent variable (of the 42 which were investigated) would seem to provide sufficient information upon which to base a prediction of individual change.
   b. Of these variables, measures of craniofacial relationships (position, proportion, and certain angles) are of greater predictive significance than the linear size of discrete anatomical structures.
   c. Popular measures such as mandibular plane angle, occlusal plane angle, and the angle of cranial base flexure would seem to be of little predictive significance.
2. By employing multiple regression and correlation a limited number of independent variables may sometimes be combined to yield an improved prediction of maxillomandibular relation and maxillary prognathism.
   a. In terms of millimeters or degrees these predictions are only somewhat (10-30%) more accurate, for a given individual, than the assumption that the mean change will obtain.
   b. If properly validated, equations of this efficiency could be employed clinically in a qualitative rather than strictly quantitative sense.
3. The ultimate accuracy of cephalometric prediction may be limited, not so much by the availability of

significant information, as by the error intrinsic to the method itself.

4. There appear to be two sources of considerable accuracy available to any contemporary method of growth prediction: the extent to which the individual pattern remains stable and the extent to which the individual orthodontist is capable of predetermining the effects of his own treatment procedures.
5. Unfortunately, it cannot be substantiated that contemporary methods are generally capable of providing an efficient estimate of individual changes attributable only to growth.

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BIBLIOGRAPHY

1. Arkin, Herbert and Colton, R.R.: Tables for statisticians. New York, Barnes and Noble, 1961.
2. Balbach, D. R.: Cephalometric study of the relationship between the morphology of the mandible and its future occlusal position. *Univ. of Michigan School of Dentistry,* 1965.
3. Björk, Arne: *The face in profile.* Lund, Berlingska Boktryckeriet, 1947.
4. ————: The significance of growth changes in facial pattern and their relationship to changes in occlusion. *D. Rec.,* 71: 197-208, 1951.
5. ————: Estimating of the age changes in overjet and sagittal jaw relation. *Europ. Orthodont. Soc. Trans.,* 29: 240-256, 1953.
6. Björk, Arne and Palling, Mogens: Adolescent age changes in sagittal jaw relation, alveolar prognathy and incisal inclinations. *Acta Odont. Scand.,* 12: 201-232, 1955.
7. Björk, Arne: Cranial base development. *Am. J. Orthodont.,* 41: 198-225, 1955.
8. ————: Facial growth in man studied with the aid of metallic implants. *Acta Odont. Scand.,* 3: 9-34, June 1955.
9. ————: Variations in the growth pattern of the human mandible: Longitudinal radiographic study by the implant method. *J. Dent. R.,* 42: 400-411 (suppl.), 1963.
10. Brodie, A. G.: On the growth pattern of the human head from the third month to the eighth year of life. *Am. J. Anat.,* 68: 209-262, 1941.
11. Buchin, I. D.: An appraisal of the effect of the edgewise arch appliance in modifying the dentofacial profile. *Am. J. Orthodont.,* 43: 801-818, 1957.
12. Burstone, C. J.: Process of maturation and growth prediction. *Am. J. Orthodont.,* 49: 907-919, 1963.
13. Carroll, J. L., Knott, V. B., and Meredith, H. V.: Changes in several calvariofacial distances and angles during the decade of childhood following age 5 years. *Growth,* 30: 47-78, 1966.
14. Coben, S. E.: The integration of facial skeletal variants. *Am. J. Orthodont.,* 41: 407-434, 1955.
15. DeVries, S. O.: A cephalometric study of craniofacial growth prediction of Class II, Division 1 females. *Univ. of Michigan School of Dentistry,* 1966.
16. Donovan, R. W.: Recent research for diagnosis. *Am. J. Orthodont.,* 40: 591-609, 1954.
17. Downs, W. B.: Variations in facial relationships: Their significance in treatment and prognosis. *Am. J. Orthodont.,* 34: 812-840, 1948.
18. Ezekiel, Mordecai and Fox, K. A.: *Methods of correlation and regression analysis.* New York, John Wiley, 1959.
19. Guilford, J. P.: *Fundamental statistics in psychology and education.* New York, McGraw-Hill, 1950.
20. Harvold, Egil: Some biologic aspects of orthodontic treatment in the transitional dentition. *Am. J. Orthodont.,* 49: 1-14, 1963.
21. Hatton, M. E. and Grainger, R.M.: Reliability of measurements from cephalograms at the Burlington Orthodontic Research Centre. *J. Dent. Res.,* 37: 853-859, 1958.
22. Hill, A. B.: *Principles of medical statistics,* 6th ed., New York, Oxford Univ. Press, 1955.
23. Holdaway, R. A.: Changes in relationship of points A and B during orthodontic treatment. *Am. J. Orthodont.,* 42: 176-193, 1956.
24. Horowitz, S. L. and Hixon, E. H.: *The nature of orthodontic diagnosis.* St. Louis, Mosby, 1966.

25. Howells, W. W.: Correlations of brothers in factor scores. *Am. J. Phys. Anthropol.*, 11: 121-140, 1953.
26. ————: Variability in family lines vs. population variability. *N.Y. Acad. Sci. Ann.*, 134(2): 624-631, 1966.
27. Hunter, W. S.: A study of the inheritance of craniofacial characteristics as seen in lateral cephalograms of 72 like-sexed twins. *Europ. Orthodont. Soc. Trans.*, 41: 59-70, 1965.
28. Jenkins, D. H.: A study of the dentofacial anatomy in normal and abnormal individuals employing lateral cephalometric radiography. abstr. *Am. J. Orthodont.*, 41:149-151, 1955.
29. ————: Analysis of orthodontic deformity employing lateral cephalostatic radiography. *Am. J. Orthodont.*, 41: 442-452, 1955.
30. Johnston, L. E.: A statistical evaluation of cephalometric prediction. *Univ. of Michigan School of Dentistry*, 1964.
31. Kelly, Truman: *Fundamentals of statistics*. Cambridge, Mass., Harvard Univ. Press, 1947.
32. Kendall, M. G. and Buckland, W. R.: *Dictionary of statistical terms*. Edinburgh, Oliver and Boyd, 1957.
33. Krogman, W. M. and Sassouni, Viken: *Syllabus in roentgenographic cephalometry*. Philadelphia, Philadelphia Center for Research in Child Growth, 1957.
34. Lande, M. J.: Growth behavior of the human bony facial profile as revealed by serial cephalometric roentgenology. *Angle Orthodont.*, 22: 78-90, 1952.
35. Maj, G., Luzi, C., and Lucchese, P.: Relations between mandibular growth and mandibular features. *Europ. Orthodont. Soc. Trans.*, 39: 303-308, 1963.
36. Maj, G. and Luzi, C.: Longitudinal study of mandibular growth between nine and thirteen years as a basis for an attempt of its prediction. *Angle Orthodont.*, 34: 220-230, 1964.
37. Mather, Kenneth: *Statistical analysis in biology*. London, Methuen, 1965.
38. Meredith, H. V.: Growth in bizygomatic face breadth during childhood. *Growth*, 18: 111-134, 1954.
39. ————: A longitudinal study of change in size and form of the lower limbs on North American white schoolboys. *Growth*, 19: 89-106, 1955.
40. ————: A time series analysis of growth of the nose height during childhood. *Child Develop.*, 29: 19-34, Mar. 1958.
41. Meredith, H. V. and Meredith, E. M.: Concomitant variation of measures of body size on boys 5 years of age and measures of growth rate for the ensuing sexennium. *Growth*, 22: 1-8, 1958.
42. Meredith, H. V., Knott, Virginia B., and Hixon, E. H.: Relation of the nasal and subnasal components of facial height in childhood. *Am. J. Orthodont.*, 44: 285-294, 1958.
43. Meredith, H. V.: Recent studies on growth of the body and face. *Am. J. Orthodont.*, 45: 110-124, 1959.
44. ————: A longitudinal study of growth in face depth during childhood. *Am. J. Phys. Anthropol.*, 17: 125-135, 1959.
45. Merow, W. W.: A cephalometric statistical appraisal of dentofacial growth. *Angle Orthodont.*, 32: 205-243, 1962.
46. Nanda, R.S.: The rates of growth of several facial components measured from serial cephalometric roentgenograms. *Am. J. Orthodont.*, 41: 658-673, 1955.
47. Newman, Katherine and Meredith, H. V.: Individual growth in skeletal bigonial diameter during the childhood period from 5 to 11 years of age. *Am. J. Anat.*, 99: 157-187, 1956.
48. Pelton, W. J. and Elsasser, W. A.: Studies of dentofacial morphology. IV. Profile changes among 6,829 white individuals according to age and sex. *Angle Orthodont.*, 25: 199-207, 1955.
49. Ricketts, R. M.: Facial and denture changes during orthodontic treatment as analyzed from the temporomandibular joint. *Am. J. Orthodont.*, 41: 163-179, 1955.
50. ————: Planning treatment on the basis of the facial pattern and an estimate of its growth. *Angle Orthodont.*, 27: 14-37, 1957.
51. ————: Foundations for cephalometric communication. *Am. J. Orthodont.*, 46: 330-357, 1960.
52. ————: The influence of orthodontic treatment on facial growth and development. *Angle Orthodont.*, 30: 103-131, Disc. 131-133, 1960.
53. ————: Cephalometric synthesis. An exercise in stating objectives and planning treatment with tracings of the head roentgenogram. *Am. J. Orthodont.*, 46: 647-673, 1960.
54. ————: Cephalometric analysis and synthesis. *Angle Orthodont.*, 31: 141-156, 1961.
55. Savara, B. S.: A method for measuring facial bone growth in three dimensions. *Human Biol.*. 37: 245-255, 1965.

56. Savara, B. S., Tracey, W. E., and Miller, P. A.: Analysis of errors in cephalometric measurements of three-dimensional distances on the human mandible. *Arch. Oral Biol.*, 11: 209-217, 1966.

57. Schreider, E.: Anthropometric correlations between adult brothers. *Nature*, 192: 1311, 1961.

58. Snedecor, G. W.: *Statistical methods applied to experiments in agriculture and biology.* 4th ed., Ames, Iowa, Iowa State Univ. Press, 1946.

59. Tweed, C. H.: The Frankfort-mandibular plane angle in orthodontic diagnosis, classification, treatment planning, and prognosis. *Am. J. Orthodont. and Oral Surg.*, 32: 175-230, 1946.

60. ————: Treatment planning and therapy in the mixed dentition. *Am. J. Orthodont.*, 49: 881-906, 1963.

61. Walker, Helen and Lev, Joseph; *Statistical inference.* New York, Henry Holt, 1953.

62. Wieslander, L.: The effect of orthodontic treatment on the concurrent development of the craniofacial complex. *Am. J. Orthodont.*, 49: 15-27, 1963.