# The form of the human dental arch

**Stanley Braun, DDS, MME; William P. Hnat, PhD; Dana E. Fender, DMD; Harry L. Legan, DDS**

For some time researchers and clinicians have been attempting to describe and classify the human dental arch form. It is commonly believed that the dental arch form is initially shaped by the configuration of the supporting bone[1] and, following eruption of the teeth, by the circumoral musculature and intraoral functional forces.[2] The dental arches were once described by investigators in simple geometric terms, such as ellipse,[3-5] parabola,[6,7] segments of circles joined to straight lines,[8,9] or modified spheres.[10] It is interesting to note that Stanton,[11] an early investigator, concluded that "any plan of arch determination must be flexible enough to produce arches varying in form through an ellipse, parabola, cubic parabola, horseshoe, parallel sides, etc."

More recently, Pepe[12] and Germane et al.[13] related the dental arch form to a catenary curve. This curve describes the shape of a series of linked chains or flexible cable or wire freely suspended between two supports. As the two end supports are brought together, the curve begins to look somewhat like a dental arch. The curve shape is dependent on the weight of each chain or wire segment and the distance between the supports. It would be serendipitous if the catenary curve described the dental arch with any degree of accuracy. It is notable, however, that a recent investigator[14] found the catenary curve a better representation of the mandibular dental arch than either the ellipse or parabola, while the

**Abstract**

The human dental arch form is shown to be accurately represented mathematically by the beta function. The average correlation coefficient between measured arch-shape data and the mathematical arch shape, expressed by the beta function, is 0.98 with a standard deviation of 0.02. Forty sets of casts—15 Class I, 16 Class II, and 9 Class III—were examined. A precision machine tool device was used to record the X-, Y-, and Z-coordinates of selected dental landmarks on all casts to 0.001 mm accuracy. The coordinates were processed through a computer curve-fitting program. The Class III mandibular arches had smaller arch depth and greater arch width (beginning in the premolar area) than the Class I arches. The Class II mandibular arches exhibited generalized reduced arch width and depth compared with the Class I arches. Maxillary arch depths were similar in all three groups. However, the Class III maxillary arch widths were greater from the lateral incisor–canine area distally compared with the Class I maxillary arch, and the Class II maxillary arch form was narrower than the Class I arch form from the lateral incisor–canine area distally. The beta function more accurately described the dental arch form than representations previously reported.

**Key Words**

Arch form • Mathematical formula • Beta function

**Submitted:** March 1996 **Revised and accepted:** September 1996 Angle Orthod 1998;68(1):29-36.

Downloaded from http://meridian.allenpress.com/doi/pdf/10.1043/0003-3219(1998)068<0029:TFOTHD>2.3.CO;2 by guest on 26 June 2022

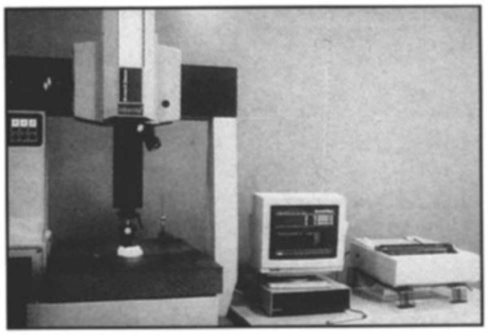

Figure 1

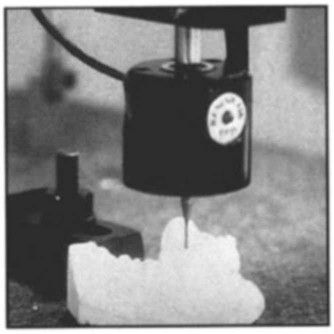

Figure 2

**Figure 1**
**MicroVal coordinate measuring machine**

**Figure 2**
**Oriented cast and touch probe recording device**

**Figure 3**
**Dental arch shape with corresponding variables**

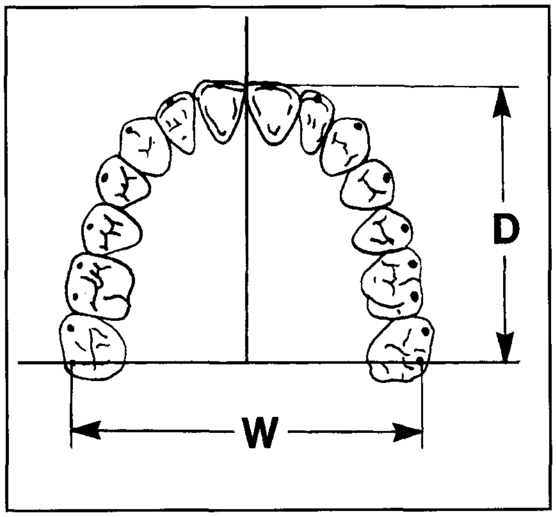


Figure 3

reverse was true for the maxillary dental arch. White[15] attempted to evaluate the accuracy of the catenary curve by subjective evaluation of the curve fit to occlusal tracings of casts. He found the catenary curve was a "good fit" in only 27% of the samples studied.

Recently, mathematical formulas such as the cubic spline[16,17] and other polynomial functions[18,19] have been adapted to describe and study the dental arch. The results have been mixed, although several have been used to produce commercial arch forms.[20-23]

This study was undertaken to determine if a highly accurate measuring device used in the machine tool industry can be applied to record specific landmarks on a sample of untreated dental casts, followed by the application of a computer curve-fitting program to develop a generalized equation describing the dental arch form, and then to evaluate whether the arch forms of Class I, Class II, and Class III occlusions are materially different from each other. If a sufficiently accurate mathematical function can be developed, useful dynamic relationships between arch depth, width, and perimeter may yield important clinical applications.

## Materials and methods

Forty sets of pretreatment orthodontic models (80 casts) of patients were selected from the clinics at the Vanderbilt University Orthodontic Center and the Orthodontic Department, University of Illinois at Chicago. Fifteen sets of casts exhibited Class I occlusion, 16 sets were Class II, and 9 sets were Class III. Subdivision occlusions were not included in the study. Casts exhibiting incisal or cuspal attrition, fracture of teeth, ectopically erupted teeth, or deciduous teeth were excluded from this study; only casts of fully developed adult dentitions (including second molars) were included.

Each cast was oriented in a Brown and Sharp Micro Val coordinate measuring machine (North Kingston, RI), seen in Figures 1 and 2. This device is used extensively in the precision machine tool industry. A frictionless air bearing probe recorded the coordinates of a point in space in each of the three orthogonal axes to 0.001 millimeters. Linear accuracy was 0.006 mm, and reproducibility 0.004 mm. The casts were each secured to a fixed plane and the touch trigger probe used to identify each measurement point, recording the corresponding X-, Y-, and Z-coordinates automatically to a computer data file.

Coordinates were recorded at the center of each incisor incisal edge, at the cusp tips of the canines and premolars, and at the mesiobuccal and distobuccal cusp tips of each molar. Eighteen points were recorded in each dental arch. The coordinate components of the dental arch form in three dimensional space for each of the 80 casts were recorded in this manner. A planar projection of each arch was subsequently obtained by reducing the Z-coordinate of each recorded landmark to zero. A reproducibility study using one cast was conducted to evaluate the variation in the measurements. It revealed the percent accuracy, based on the actual measurements in each of the three coordinate axes, to be less than 0.5%.

An analytical equation of the dental arch shape is necessary to describe the relationships between arch width, depth, and perimeter. Many mathematical functions were investigated as to fit. The beta function most closely represents the dental arch shape.[24] Two measurements, i.e., independent variables, are required to generate the dental arch shape. The independent variables molar width and arch depth are readily determined from the cast coordinate measurements. Molar width is the measured distance between the second molar distobuccal cusp tips in millimeters. Arch depth is the average perpendicular distance



Downloaded from http://meridian.allenpress.com/doi/pdf/10.1043/0003-3219(1998)068<0029:TFOTHD>2.3.CO;2 by guest on 26 June 2022

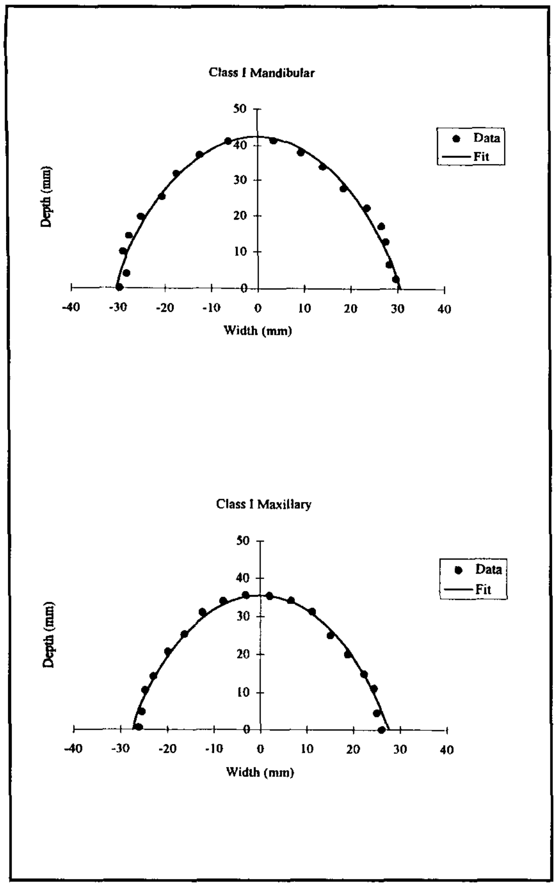


Figure 4

**Figure 4**
**Angle Class I maxillary and mandibular arch with beta function curve fit**

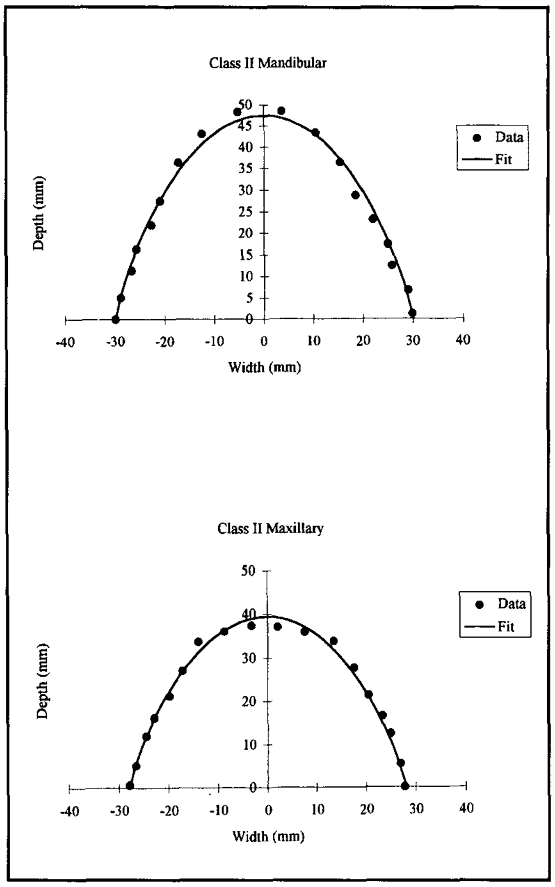


Figure 5

**Figure 5**
**Angle Class II maxillary and mandibular arch with beta function curve fit**

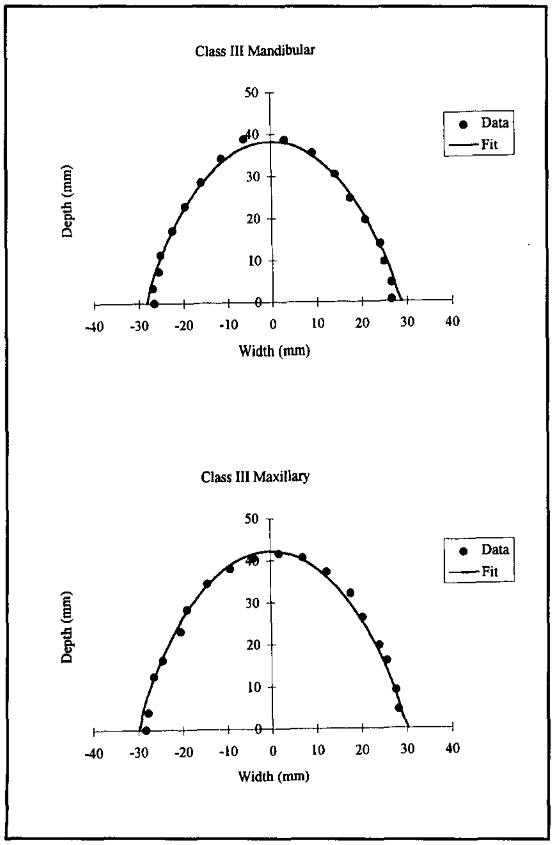


Figure 6

**Figure 6**
**Angle Class III maxillary and mandibular arch with beta function curve fit**

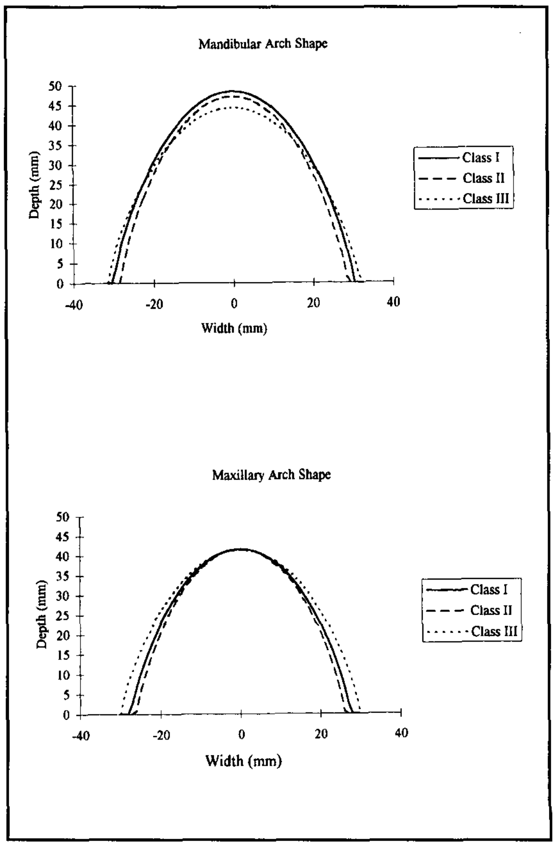


Figure 7

**Figure 7**
**Average (typical) maxillary and mandibular arch shapes for Class I, II, and III occlusions**

Downloaded from http://meridian.allenpress.com/doi/pdf/10.1043/0003-3219(1998)068<0029:TFOTHD>2.3.CO;2 by guest on 26 June 2022

**Table 1**
**Class I mandibular and maxillary measurements with curve fit data**

| Cast | Measured values (mm) Depth | Measured values (mm) Width | Curve fit values (mm) Depth | Curve fit values (mm) Width | Correlation coefficient r |
|---|---|---|---|---|---|
| Class I - mandibular arch | | | | | |
| 1a | 40.0 | 59.3 | 41.3 | 60.1 | 0.98 |
| 2a | 43.4 | 61.4 | 45.2 | 61.5 | 0.99 |
| 3a | 45.9 | 57.7 | 47.9 | 58.8 | 0.98 |
| 4a | 42.0 | 56.8 | 45.2 | 56.9 | 0.99 |
| 5a | 44.8 | 53.3 | 50.1 | 56.2 | 0.95 |
| 6a | 46.9 | 54.3 | 49.8 | 54.6 | 0.99 |
| 7a | 44.2 | 60.2 | 47.8 | 60.9 | 0.99 |
| 8a | 42.7 | 58.0 | 46.3 | 59.5 | 0.99 |
| 9a | 51.3 | 63.8 | 52.9 | 64.6 | 0.99 |
| 10a | 48.6 | 58.7 | 52.7 | 60.8 | 0.97 |
| 11a | 39.5 | 56.7 | 42.3 | 57.5 | 0.99 |
| 12a | 47.5 | 59.0 | 49.0 | 62.9 | 0.95 |
| 13a | 46.0 | 59.0 | 49.1 | 60.0 | 0.99 |
| 14a | 42.5 | 56.9 | 42.6 | 56.9 | 0.99 |
| 15a | 45.5 | 58.1 | 48.7 | 58.4 | 0.99 |
| Average | 44.7 | 58.2 | 47.4 | 59.3 | 0.98 |
| Std. Dev. | 3.2 | 2.6 | 3.5 | 2.7 | 0.01 |
| Maximum | 51.3 | 63.8 | 52.9 | 64.6 | 0.99 |
| Minimum | 39.5 | 53.3 | 42.3 | 54.6 | 0.95 |
| Class I - maxillary arch | | | | | |
| 1b | 35.2 | 52.2 | 35.3 | 53.2 | 0.99 |
| 2b | 40.3 | 57.8 | 40.1 | 57.8 | 0.99 |
| 3b | 40.7 | 50.1 | 42.7 | 50.2 | 0.99 |
| 4b | 36.8 | 49.4 | 39.1 | 49.4 | 0.98 |
| 5b | 38.8 | 52.5 | 40.0 | 53.3 | 0.96 |
| 6b | 41.2 | 49.0 | 40.9 | 50.1 | 0.97 |
| 7b | 38.8 | 55.0 | 40.6 | 55.1 | 0.99 |
| 8b | 35.4 | 51.3 | 37.4 | 51.6 | 0.99 |
| 9b | 44.5 | 58.4 | 44.8 | 58.5 | 0.98 |
| 10b | 42.9 | 46.4 | 44.5 | 50.6 | 0.87 |
| 11b | 35.1 | 51.0 | 36.1 | 51.0 | 0.98 |
| 12b | 42.3 | 57.7 | 43.5 | 59.1 | 0.99 |
| 13b | 41.1 | 51.1 | 44.2 | 51.5 | 0.98 |
| 14b | 37.3 | 50.7 | 36.7 | 50.8 | 0.98 |
| 15b | 42.4 | 51.7 | 44.0 | 52.2 | 0.98 |
| Average | 39.5 | 52.3 | 40.7 | 53.0 | 0.97 |
| Std. Dev. | 3.0 | 3.5 | 3.2 | 3.2 | 0.03 |
| Maximum | 44.5 | 58.4 | 44.8 | 59.1 | 0.99 |
| Minimum | 35.1 | 46.4 | 35.3 | 49.4 | 0.87 |

from the central incisors to the molar cross-arch dimension in millimeters. The beta function representing the dental arch shape is given by the general formula

$$Y = \frac{a\left[\frac{X - b + cm}{c}\right]^{d-1}\left[1 - \frac{X - b + cm}{c}\right]^{e-1}}{\left[m^{d-1}n^{e-1}\right]} \quad (1)$$

where Y is the arch depth at width X, and a, b, c, d, and e are parameters, and

$$m = \frac{d-1}{d+e-2}; \quad n = \frac{e-1}{d+e-2}.$$

If b=0 and d=e= 1.8, the beta function becomes symmetrical about the centerline of the teeth. The beta function then becomes

$$Y = 3.0314 * D * \left[\frac{X}{W} + \frac{1}{2}\right]^{0.8}\left[\frac{1}{2} - \frac{X}{W}\right]^{0.8} \quad (2)$$

where W represents the cross-arch distance between the second molar distobuccal cusp tips in millimeters, and D the perpendicular distance from the most anterior point between the two central incisors to the molar cross-arch dimension in millimeters, as illustrated in Figure 3. The perpendicular distance, D, is calculated by averaging the perpendicular distance from each of the two central incisors to the molar cross-arch dimension.

## Results

Using the least squares method, a beta function curve was fitted to each of the 80 casts. The Table Curve 2D curve-fitting program (AISN Software, Inc, Redmond, Wash) was used for all curve fits and calculations. The results are shown in Tables 1, 2, and 3, and reveal an average correlation coefficient of 0.98 with a standard deviation of 0.02. A sample curve fit with measured data points for each Angle classification of mandibular and maxillary arches are shown in Figures 4, 5, and 6. Measured arch width is based on the coordinate distance between the second molar distobuccal cusp tips, which as seen from the curve fit analysis, underestimates the arch width at the second molars by approximately 1 mm. The measured arch depth underestimates the true arch depth by approximately 1.5 mm. Consequently, when using the mathematical relationship (equation 2) based on these measurements, the measured value for W should be increased 1 mm and the measured value of D increased 1.5 mm (from 80 casts, reference Tables 1, 2, and 3). The resulting equation is an excellent representation of the dental arch shape, including the second molars, and is an excellent

Downloaded from http://meridian.allenpress.com/doi/pdf/10.1043/0003-3219(1998)068<0029:TFOTHD>2.3.CO;2 by guest on 26 June 2022

generalized equation of the maxillary and mandibular arch shapes for each of the Angle classification occlusions.

The average (typical) maxillary and mandibular arch shapes for each Angle classification are shown in Figure 7. The curves were generated by substituting the mean depths and widths into equation 2. The mean maxillary and mandibular dental arch-shape equations for each Angle classification are given by the formulas:

$$Y=146.721\left[\frac{X}{60.427}+\frac{1}{2}\right]^{0.8}\left[\frac{1}{2}-\frac{X}{60.427}\right]^{0.8}, \text{Class I Mandibular Arch}$$

$$Y=125.905\left[\frac{X}{54.933}+\frac{1}{2}\right]^{0.8}\left[\frac{1}{2}-\frac{X}{54.933}\right]^{0.8}, \text{Class I Maxillary Arch}$$

$$Y=142.817\left[\frac{X}{57.034}+\frac{1}{2}\right]^{0.8}\left[\frac{1}{2}-\frac{X}{57.034}\right]^{0.8}, \text{Class II Mandibular Arch}$$

$$Y=125.774\left[\frac{X}{52.225}+\frac{1}{2}\right]^{0.8}\left[\frac{1}{2}-\frac{X}{52.225}\right]^{0.8}, \text{Class II Maxillary Arch}$$

$$Y=134.235\left[\frac{X}{63.268}+\frac{1}{2}\right]^{0.8}\left[\frac{1}{2}-\frac{X}{63.268}\right]^{0.8}, \text{Class III Mandibular Arch}$$

$$Y=125.168\left[\frac{X}{59.980}+\frac{1}{2}\right]^{0.8}\left[\frac{1}{2}-\frac{X}{59.980}\right]^{0.8}, \text{Class III Maxillary Arch}$$

## Discussion and conclusions

The beta function has been shown to be an accurate representation of the human dental arches. The mean correlation coefficient of curve fit was found to be 0.98, with a standard deviation of 0.02, for the complete sample of 80 casts.

When comparing mandibular arch shapes (Figure 7), it is evident that the dental arches associated with Class III occlusions exhibit a smaller arch depth (D) than the Class I occlusions by an average of 3.3 mm. Additionally, the mandibular dental arches associated with Class III occlusions are, on average, 2.1 mm wider, beginning in the premolar area, than the Class I mandibular arches. The diminished arch depth of the Class III dental arches may be related to possible increased lip pressure in the incisor area. It is not uncommon to find retroclined incisors in patients displaying Class III occlusion. A possible explanation for the increased arch width (W) of the casts associated with Class III dental arches is that the sum of all the mesiodistal widths of the dental units around an arch represent a specific dimension. When the arch depth (D) is decreased, this peripheral dimensional excess must be resolved in one or more ways: by an increased

**Table 2**
**Class II mandibular and maxillary measurements with curve fit data**

| Cast | Measured values (mm) Depth | Measured values (mm) Width | Curve fit values (mm) Depth | Curve fit values (mm) Width | Correlation coefficient r |
|---|---|---|---|---|---|
| Class II - mandibular arch | | | | | |
| 16a | 44.4 | 59.1 | 46.0 | 59.1 | 0.98 |
| 17a | 47.9 | 59.9 | 44.8 | 62.3 | 0.97 |
| 18a | 39.8 | 55.4 | 38.6 | 59.3 | 0.95 |
| 19a | 47.7 | 57.5 | 48.0 | 57.9 | 0.99 |
| 20a | 44.4 | 57.0 | 43.7 | 57.1 | 0.99 |
| 21a | 44.8 | 56.2 | 44.1 | 56.2 | 0.99 |
| 22a | 46.4 | 56.0 | 47.3 | 56.1 | 0.99 |
| 23a | 38.4 | 56.1 | 41.3 | 56.3 | 0.99 |
| 24a | 42.7 | 58.2 | 41.3 | 58.3 | 0.98 |
| 25a | 46.9 | 52.1 | 45.3 | 53.1 | 0.99 |
| 26a | 39.5 | 51.8 | 40.2 | 52.3 | 0.99 |
| 27a | 45.7 | 57.5 | 48.8 | 58.8 | 0.99 |
| 28a | 41.7 | 55.0 | 43.1 | 55.1 | 0.97 |
| 29a | 38.4 | 58.0 | 42.7 | 59.2 | 0.98 |
| 30a | 48.6 | 52.8 | 50.1 | 53.9 | 0.98 |
| 31a | 45.9 | 57.2 | 47.6 | 57.3 | 0.99 |
| Average | 43.9 | 56.2 | 44.4 | 57.0 | 0.98 |
| Std. Dev. | 3.5 | 2.4 | 3.3 | 2.7 | 0.01 |
| Maximum | 48.6 | 59.9 | 50.1 | 62.3 | 0.99 |
| Minimum | 38.4 | 51.8 | 38.6 | 52.3 | 0.95 |
| Class II - maxillary arch | | | | | |
| 16b | 41.6 | 54.2 | 41.9 | 54.2 | 0.99 |
| 17b | 36.9 | 55.7 | 38.9 | 55.8 | 0.99 |
| 18b | 39.4 | 48.9 | 40.9 | 49.3 | 0.99 |
| 19b | 39.2 | 51.6 | 42.1 | 52.5 | 0.99 |
| 20b | 36.8 | 51.7 | 39.6 | 52.1 | 0.99 |
| 21b | 39.9 | 50.6 | 39.9 | 50.7 | 0.99 |
| 22b | 38.9 | 50.2 | 40.6 | 50.3 | 0.99 |
| 23b | 38.1 | 49.1 | 41.0 | 49.3 | 0.99 |
| 24b | 34.4 | 52.7 | 34.1 | 52.8 | 0.99 |
| 25b | 39.8 | 46.6 | 37.2 | 55.3 | 0.86 |
| 26b | 34.6 | 47.5 | 33.5 | 47.6 | 0.99 |
| 27b | 39.6 | 50.6 | 41.7 | 50.6 | 0.99 |
| 28b | 37.6 | 47.6 | 40.6 | 48.5 | 0.98 |
| 29b | 37.5 | 51.3 | 39.6 | 54.1 | 0.97 |
| 30b | 40.2 | 49.0 | 40.7 | 49.1 | 0.97 |
| 31b | 39.1 | 51.2 | 41.1 | 51.2 | 0.99 |
| Average | 38.4 | 50.5 | 39.5 | 51.5 | 0.98 |
| Std. Dev. | 2.0 | 2.5 | 2.6 | 2.6 | 0.03 |
| Maximum | 41.6 | 55.7 | 42.1 | 55.8 | 0.99 |
| Minimum | 34.4 | 46.6 | 33.5 | 47.6 | 0.86 |

Downloaded from http://meridian.allenpress.com/doi/pdf/10.1043/0003-3219(1998)068<0029:TFOTHD>2.3.CO;2 by guest on 26 June 2022

**Table 3**
**Class III mandibular and maxillary measurements with curve fit data**

| Cast | Measured values (mm) Depth | Measured values (mm) Width | Curve fit values (mm) Depth | Curve fit values (mm) Width | Correlation coefficient r |
|---|---|---|---|---|---|
| Class III - mandibular arch | | | | | |
| 32a | 38.5 | 53.1 | 39.6 | 53.9 | 0.99 |
| 33a | 43.0 | 52.6 | 44.4 | 55.5 | 0.95 |
| 34a | 43.4 | 60.1 | 44.4 | 61.9 | 0.99 |
| 35a | 47.1 | 65.5 | 49.3 | 67.0 | 0.99 |
| 36a | 37.6 | 59.8 | 40.1 | 59.9 | 0.97 |
| 37a | 40.8 | 62.0 | 43.7 | 63.9 | 0.98 |
| 38a | 43.1 | 65.8 | 43.0 | 65.9 | 0.98 |
| 39a | 41.5 | 60.6 | 43.7 | 61.3 | 0.99 |
| 40a | 45.9 | 63.6 | 48.8 | 63.6 | 0.98 |
| Average | 42.3 | 60.3 | 44.1 | 61.4 | 0.98 |
| Std. Dev. | 3.1 | 4.8 | 3.3 | 4.4 | 0.01 |
| Maximum | 47.1 | 65.8 | 49.3 | 67.0 | 0.99 |
| Minimum | 37.6 | 52.6 | 39.6 | 53.9 | 0.95 |
| Class III - maxillary arch | | | | | |
| 32b | 36.2 | 51.6 | 38.2 | 52.0 | 0.99 |
| 33b | 35.9 | 54.5 | 36.9 | 55.9 | 0.98 |
| 34b | 38.9 | 56.9 | 40.3 | 57.7 | 0.98 |
| 35b | 41.4 | 58.0 | 44.2 | 58.9 | 0.97 |
| 36b | 35.5 | 58.1 | 37.3 | 59.9 | 0.97 |
| 37b | 37.5 | 60.7 | 37.1 | 60.7 | 0.98 |
| 38b | 41.7 | 61.7 | 39.9 | 61.8 | 0.99 |
| 39b | 38.4 | 55.4 | 40.7 | 57.1 | 0.97 |
| 40b | 42.5 | 58.5 | 44.4 | 58.9 | 0.99 |
| Average | 38.7 | 57.3 | 39.9 | 58.1 | 0.98 |
| Std. Dev. | 2.7 | 3.1 | 2.9 | 2.9 | 0.01 |
| Maximum | 42.5 | 61.7 | 44.4 | 61.8 | 0.99 |
| Minimum | 35.5 | 51.6 | 36.9 | 52.0 | 0.97 |

curve of Spee (not seen in Class III malocclusions), by dental units blocked out of the general dental arch form, or by increased arch width. The data indicate that arch width increase is the option exhibited. However, it should be noted that this may be a biased outcome because casts exhibiting ectopically positioned dental units were excluded from the sample. Another possible explanation for the increase in arch width (W) seen in the Class III dental arches may be the adaptability of the tongue to the decrease in available arch depth (D) reflected in an increased lateral tongue dimension.

When Class II mandibular arches are compared with Class I arches, an average generalized reduced arch width (W) of 2.3 mm is evident, and an average reduced arch depth (D) of 3.0 mm is observed. Perhaps this can be explained by the fact that some Class II relationships result from a small mandibular body. It may be that the dental arch form is a reflection of this underlying bony morphology. This suggests further investigation.

When comparing maxillary arch shapes (Figure 7), it is apparent that arch depths for all Angle classifications are essentially the same. However, Class III dental arch widths are greater by an average 5.1 mm than Class I widths. This begins in the lateral incisor–canine area and proceeds distally. When Class II maxillary arch widths (W) are compared with Class I widths, they are found to be an average 1.5 mm narrower, beginning in the lateral incisor–canine area.

The finding of Class III maxillary arch widths (W) being generally wider than Class I maxillary arch widths may be surprising because lingually positioned maxillary posterior crossbites are of-

Downloaded from http://meridian.allenpress.com/doi/pdf/10.1043/0003-3219(1998)068<0029:TFOTHD>2.3.CO;2 by guest on 26 June 2022

ten seen in the Class III malocclusion. This is frequently related to an anteroposterior skeletal discrepancy, with the mandibular arch advanced relative to the maxillary arch. Consequently, the corresponding interarch widths are not correctly matched. However, this study has shown that greater widths exist in the maxillary posterior region (compared with Class I related dentitions) beginning in the lateral incisor–canine areas and continuing posteriorly. One may speculate that some compensating mechanism related to the anteroposterior Class III discrepancy results in the observed increase in the arch width (W). Alterations in the curve of Wilson may be a reflection of this third-order dental compensation. (Clinicians sometimes say that maxillary teeth or mandibular teeth seem to be "reaching toward each other.")

Class II maxillary dental arches, W= 38.4 mm, are narrower than Class I dental arches, W= 39.5 mm. This may be related to a similar compensating mechanism seen in Class III dental arches, except in the reverse direction, as a narrower portion of the mandibular dental arch articulates with a wider portion of the maxillary dental arch in Class II occlusions. It would be of interest to study a larger sample in this regard.

A mathematical function has been found to accurately represent the human dental arch form, and additional studies to evaluate the influence of growth and maturation on the dental arch form would be desirable. Further studies could address the question of gender influences on the above findings. Earlier studies have focused primarily on linear dimensional changes related to this issue.[25-28] Recently, Ferrario et al.[29] used a fourth-order polynomial function, based on a combination of the parabolic and elliptical shapes, to study gender differences in dental arch shapes. It would be well to repeat this study now that a highly accurate representative function has been developed.

Additionally, there are a number of orthodontic treatment philosophies that promote the concept of dental arch expansion to resolve arch length deficiencies.[30-33] Some studies have been done relating peripheral arch dimensional changes to expansion.[34-37] These have been based on relatively inaccurate approximations of the dental arch. The beta function has been shown to be an accurate representation of the dental arch. Therefore, additional studies need to be done to determine the peripheral dental arch alterations as related to arch width changes. The beta function has opened the door to studying the above-cited relationships with greater accuracy and, therefore, to achieving better understanding of the human dental arch form.

**Author Address**

Dr. Stanley Braun
7940 Dean Road
Indianapolis, IN 46240
Phone: (317) 845-8780
Fax: (317) 845-9009

*Stanley Braun, clinical professor of orthodontics, Vanderbilt University Medical Center and University of Illinois.*

*William P. Hnat, associate professor of mechanical engineering, Speed Scientific School, University of Louisville.*

*Dana E. Fender, former resident in orthodontics, Vanderbilt University Medical Center.*

*Harry L. Legan, professor and chairman of orthodontics, Vanderbilt University Medical Center.*

Downloaded from http://meridian.allenpress.com/doi/pdf/10.1043/0003-3219(1998)068<0029:TFOTHD>2.3.CO;2 by guest on 26 June 2022

## References


1. Brash JC. The aetiology of irregularity and malocclusion of the teeth, 2d ed. London: Dental Board of the United Kingdom, 1956.
2. Weinstein S, Haack DC, Morris LY, Snyder BB, Attaway HE. On an equilibrium theory of tooth position. Angle Orthod 1963; 33:1-25.
3. Wheeler RC. A textbook of dental anatomy and physiology, 3d ed. Philadelphia: Saunders, 1958.
4. Currier JH. A computerized geometric analysis of human dental arch form. Philadelphia: Temple Univ. 1967. Thesis.
5. Bonwill WGA. Geometrical and mechanical laws of articulation. Odontol Soc Penn Trans 1984-85; 119-30.
6. Sicher H. Oral anatomy, 3rd ed. St. Louis: Mosby, 1960; 269-70.
7. Broomell IN. Anatomy and histology of the mouth and teeth, 2nd ed. Philadelphia: Blakiston, 1902: 99.
8. Hawley CA. Determination of the normal arch and its application to orthodontia. Dent Cosmos 1905; 47: 541-52.
9. Williams PN. Determining the shape of the normal arch. Dent Cosmos 1917; 695-708.
10. Sved A. The application of engineering methods to orthodontics. Am J Orthod 1952; 38: 399-421
11. Stanton FL. Arch predetermination and a method of relating the predetermined arch to the malocclusion to show the minimum tooth movement. Int J Orthod 1922; 8: 757-78.
12. Pepe SH. Polynomial and catenary curve fits to human dental arches. J Dent Res 1975; 54:1124-32
13. Germane N, Lindauer SJ, Rubenstein LK, Revere JH, Isaacson RJ. Increase in arch perimeter due to orthodontic expansion. Am J Orthod Dentofac Orthop 1991; 100: 421-27.
14. Neilans LC. A computerized analysis of human dental arch form as compared to the catenary curve. Philadelphia: Temple Univ, 1968. Thesis
15. White LW. Accurate arch discrepancy measurements. Am J Orthod 1977; 72:303-8.
16. BeGole EA. Application of the cubic spline function in the descriptiion of dental arch form. J Dent Res 1980; 59: 1542-56.
17. Diggs DB. The quantification of arch form. Seattle: University of Washington, 1962. Thesis.
18. Lu KH. Analysis of dental arch symmetry (abstract). J Dent Res 1964; 43: 780.
19. Sanin C, Savara BS, Thomas DR, Clarkson OD. Arch length of the dental arch estimated by multiple regression. J Dent Res 1970; 49: 885.
20. Engel GA. Preformed arch wires: reliability of fit. Am J Orthod 1979; 76: 497-504.
21. Ricketts RM. Research in factors of appliance design and arch form. Pacific Palisades, Calif: Foundation for Orthodontic Research, 1979.
22. Roth RH. Straight wire mechanics syllabus. Burlingame, Calif: Foundation for Advanced Continuing Education, 1978.
23. McKelvain GD. An arch form designed for use with a specific straight wire orthodontic appliance. Baylor College of Dentistry, 1982. Thesis.
24. Rektorys K. Survey of applicable mathematics. Cambridge, Mass: Massachusetts Institute of Technology Press, 1969.
25. Siatkowski RE. Arch dimensional changes with age. Presented at the Edward H. Angle Society of Orthodontists, North Atlantic Component meeting, Nashville, Tenn, Apr. 1995
26. Moorres CF. The dentition of the growing child: a longitudinal study of dental development ages 3-18. Cambridge, Mass: Harvard Univ Press, 1959.
27. Moyers RE, van der Linden FPGM, Riolo ML, McNamara JA. Standards of human occlusal development. Monograph #5, Craniofacial Growth Series. Ann Arbor: Univ of Michigan, 1976
28. Sillerman JH. Dimensional changes of the dental arch: longitudinal study from birth to 25 years. Am J Orthod 1964; 50: 824-42
29. Ferrario VF, Sforza C, Miani A Jr.,Tartaglia G. Mathematical definition of the shape of dental arches in human permanent healthy dentition. Eur J Orthod 1994; 16: 287- 94.
30. O'Connor BMP. Contemporary trends in orthodontic practice: a national survey. Am J Orthod Dentofac Orthop 1993; 103: 163-70.
31. Gottlieb EL, Nelson AG, Vogels DS III. Study of orthodontic diagnosis and treatment procedures: Part I - results and trends. J Clin Orthod 1991; 25: 145-56.
32. Haas AJ. Palatal expansion: just the beginning of dentofacial orthopedics. Am J Orthod 1970; 57: 219-55.
33. Harvold EP. The activator in interceptive orthodontics. St. Louis: Mosby, 1974.
34. Walter DC. Changes in the form and dimensions of dental arches resulting from orthodontic treatment. Angle Orthod 1953; 23: 3-18.
35. Berlocher WC, Mueller BH, Timanoff N. The effect of maxillary palatal expansion on the primary dental arch circumference. J Pediatric Dent 1980; 2: 27-30.
36. Adkins MD, Nanda RS, Currier GF. Arch perimeter changes on rapid palatal expansion. Am J Orthod Dentofac Orthop 1990; 97: 194-9.
37. Ricketts RM, Roth RH, Chaconnas SJ, Schulhof RJ, Engel GA. Orthodontic diagnosis and planning. Denver, Colo: Rocky Mt Data Systems, 1982: 194-200.


Downloaded from http://meridian.allenpress.com/doi/pdf/10.1043/0003-3219(1998)068<0029:TFOTHD>2.3.CO;2 by guest on 26 June 2022