

Simulation Study for a Finite Helical Axis Analysis of Tooth Movement

Kazuo Hayashi^a; Meiri Hamaya^a; Itaru Mizoguchi^b

Abstract: The description of motion using a finite helical axis (FHA) is independent of the chosen coordinate system because the values of the helical axis parameters do not change with coordinate transformations. However, an intuitive understanding of tooth movement expressed by the FHA can be slightly difficult for orthodontists who have never used it. The purpose of this study was to clarify the basic behavior of the FHA to increase our understanding of the FHA during tooth movement. Parameters of the FHA were calculated in two different simulations of canine retraction. In simulation 1, as the tipping angle of the canine was increased from 5° to 30°, the direction vector of the FHA v approached the x-axis (tipping axis), thus increasing the rotation angle about the FHA θ . These results demonstrated that the direction vector of the FHA can indicate the axis of rotation that is most affected in three-dimensional transformation. In simulation 2, bodily tooth movement of the canine was increased from one to five mm, and the shortest distance from the origin to the FHA d increased in a linear manner. In this case, the percent increase in bodily tooth movement could be determined by the d value. This study showed that the use of FHA makes it possible to determine the torques applied to the tooth and the true bodily tooth movement during orthodontic treatment, and this may lead to a better understanding of how to move teeth. (*Angle Orthod* 2005;75:350–355.)

Key Words: Simulation study; Finite helical axis; Tooth movement

INTRODUCTION

The movement of a tooth can be described through the use of a center of rotation within a plane. The ratio between the net moment and net force on a tooth (M/F ratio) with reference to the center of resistance determines the center of rotation.¹ This center of rotation can also be constructed graphically, where the intersection of the midperpendiculars on two distinct landmark displacement vectors is assessed. Although orthodontic tooth movement is generally three-dimensional (3-D), a consideration of planar movement using the finite center of rotation is still an effective approach for clinical use.

However, more detailed information on tooth movement through space is needed for a biomechanical analysis of orthodontic tooth movement. Most studies of 3-D orthodontic tooth movement have been carried out based on a

rectangular coordinate system (XYZ system) with six degrees of freedom.^{1–5} However, this representation varies depending on the position of the coordinate origin, the rotation sequence of the axis, and the timing of translation.⁶ In particular, the reliability of computing rotation based on an XYZ system is poor. In one reported example, the average rotation ranged from -2.48° to 1.31° .⁷ The distal driving behavior of the canine has also been evaluated using 3-D information that could be expressed by modified Euler angles, but the results again varied with the position of the coordinate origin and the rotation sequence of the axis.^{8,9}

A finite helical axis (FHA) can be used to solve the problem of superscription and has been applied to the analysis of joint function.^{10–14} The description of motion using a finite center of rotation for planar motion or the FHA for spatial motion is independent of the chosen coordinate system because the value of the rotation center or the helical axis and the value of the translation along the axis do not change with coordinate transformations. Thus, the FHA parameters are the same for all points on a moving body at any particular instant. This invariance makes it possible to compare tooth movement among different individuals. We previously reported a new method for calculating the FHA and applied this method to the 3-D analysis of orthodontic tooth movement.¹⁵ Although the FHA has been previously applied in an analysis of tooth movement, no previous report has provided detailed information on its basic behavior.

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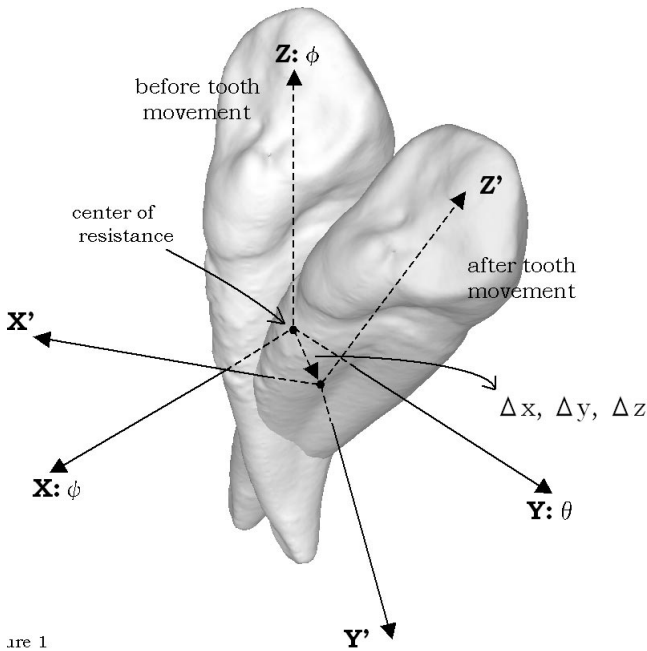


Figure 1

FIGURE 1. Schematic drawing of the coordinate system for calculating the FHA from a canine drive simulation. Translations are denoted by Δx , Δy , and Δz . Rotations are denoted by ψ , θ , and ϕ , which represent the tipping angle, flaring angle, and rotation angle, respectively.

The purpose of this study was to clarify the basic behavior of the FHA to increase our understanding of the FHA during tooth movement. Thus, we must translate simple tooth movement expressed by an XYZ system into tooth movement represented by the FHA. Parameters of the FHA were calculated from two different simulations of canine retraction.

MATERIALS AND METHODS

Simulation model

A simulation model was constructed based on the assumption of distal driving of the maxillary left canine. The origin of a rectangular coordinate system was defined at the center of resistance of the canine. In a healthy single-rooted tooth, the center of resistance is located at 0.24 times the root length measured apical to the level of the alveolar crest, according to a 3-D finite element analysis.¹⁶ To represent changes in the position of the canine in 3-D space, we must introduce six parameters, which can be defined by the translations Δx , Δy , and Δz and the rotations ϕ , ψ , and θ (Figure 1), which denote the rotation angle, tipping angle, and flaring angle, respectively.

Construction of the FHA

We previously reported a new method for calculating the FHA,¹⁵ which can be summarized as follows. Each point $P'(x', y', z')$ on the 3-D shape after movement was expressed

in the XYZ system as a function of the six parameters that describe translation and rotation. Thus, the 3-D displacement of $P(x, y, z)$ on the 3-D shape before movement was determined from the formula

$$a = Ab + c, \quad (1)$$

where A is a 3×3 rotation matrix, a is the position vector of arbitrary points after tooth movement, b is the position vector of arbitrary points before tooth movement, and c is the translation vector. Tooth movement in 3-D space was then determined as a displacement vector r with components $(x' - x)$, $(y' - y)$, and $(z' - z)$. This movement involves both translation and rotation. The displacement vector r was expressed as

$$r = (A - E)b + c, \quad (2)$$

where E is a unit matrix, which is a function of the coordinates before movement. The 3×3 matrix A is the rotation about the axis that passes through the origin and is parallel to the FHA. Only for points on the FHA, the displacement vector r is parallel to this axis and does not change by rotation, so the axis position follows from

$$r = A^T r. \quad (3)$$

Therefore, by multiplying equation 2 by A^T and applying the relation in equation 3,

$$(A - E)b + c = A^T(A - E)b + A^T c, \quad (4)$$

and equation 4 can be written as

$$(A + A^T - 2E)b + (E - A^T)c = 0. \quad (5)$$

Equation 5 also follows from

$$d/db \{((A - E)b + c)^T ((A - E)b + c)\} = 0, \quad (6)$$

which yields the point b that has minimal displacement and therefore lies on the FHA. The relation in equation 5 is valid at all points on the FHA. Although the solution is not unique, it can be obtained by substituting a value for the Z component.

When b is chosen on the FHA, then a , from equation 1, is on the same axis. The distance between a and b is a translation t along the FHA. Next, an arbitrary point d_0 outside the FHA was chosen, and its position after tooth movement d_1 was calculated again using equation 1

$$d_1 = Ad_0 + c. \quad (7)$$

Next, the orthogonal projection of d_0 and d_2 ($d_2 = d_1 - t$) on the FHA was calculated:

$$q' = q + \frac{(d - q) \cdot l}{l \cdot l} l \quad (8)$$

where q' is the position vector of a point on the FHA, q is the position vector of an arbitrary point on the FHA, d is d_0 or d_2 , and l is the direction of the FHA. Thus, the rotation

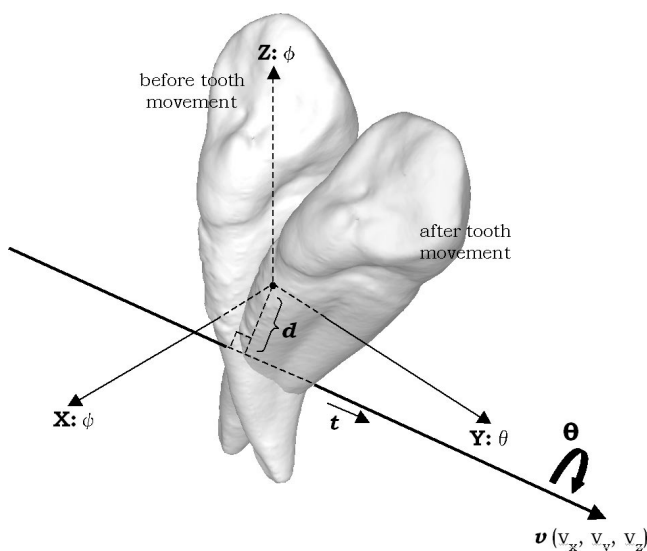


FIGURE 2. Schematic drawing of calculated FHA parameters: direction vector of the FHA $v (v_x, v_y, v_z)$, rotation angle about the FHA θ , translation along the FHA t , and shortest distance from the origin to the FHA d .

angle θ about an FHA in space is identical to the angle between d_0 , q' , and d_2 .

FHA parameters calculated two different simulations

FHA yields a comprehensive 3-D description of rigid body motion. However, investigators and clinicians who have never used it may find it difficult to interpret the FHA because it contains all the information on tooth rotation and translation that is provided by the XYZ system. The FHA can be understood in detail by conducting simulations under various conditions. Therefore, two different tooth movement simulations (rotation and translation) were carried out, which should produce different FHA behaviors.

First, FHA was defined for tipping angles (ψ) that were changed in steps of 5° from 5° to 30° (simulation 1). The other five parameters remained fixed (flaring angle $[\theta] = 0^\circ$, rotation angle $[\phi] = 5^\circ$, $\Delta x = 0.0$ mm, $\Delta y = 1.0$ mm, $\Delta z = 0.0$ mm). Next, the FHA was defined for translation (Δy) as a distal movement that was changed in steps of one mm from one to five mm (simulation 2). The other five parameters remained fixed (flaring angle $[\theta] = 0^\circ$, rotation angle $[\phi] = 5^\circ$, tipping angle $[\psi] = 5^\circ$, $\Delta x = 0.0$ mm, $\Delta z = 0.0$ mm). In each simulation, the sequence of rotation in the XYZ system was in the order X, Y, and Z, and the following FHA parameters were calculated: the direction vector of the FHA $v (v_x, v_y, v_z)$, the rotation angle about the FHA θ , the translation along the FHA t , and the shortest distance from the origin to the FHA d (Figure 2). Generally, a combination of more than two rotations in the XYZ system is equivalent to a rotation about the FHA. Thus, it was necessary to set not only a tipping angle but also a rotation

angle to calculate the FHA in this study. If there was no translation in the XYZ system, the FHA passed through the center of resistance (coordinate origin). In simulation 1, it was necessary to set distal movement (Δy) to evaluate the shortest distance from the origin to the FHA.

RESULTS

Figure 3a shows the direction vector v of the FHA in simulation 1. When the tipping angle of the canine was 5° , v_x , v_y , and v_z were 0.707, 0.031, and 0.707, respectively. As the tipping angle of the canine was increased from 5° to 30° , v_x increased from 0.707 to 0.986, v_y slightly increased from 0.031 to 0.043, and v_z decreased from 0.707 to 0.161. Components v_x , v_y , and v_z changed in a nonlinear manner. An absolute component value of one means that the FHA is parallel to the corresponding coordinate axis. A value of zero means that the FHA is perpendicular to the coordinate axis, and a value of 0.707 means that the FHA is at 45° to the coordinate axis. In this study, absolute rather than signed component values of the direction vector were used to evaluate FHA behavior. Figure 3b shows the rotation angle about the FHA θ in simulation 1. As the tipping angle of the canine increased from 5° to 30° , θ increased from 7.07° to 30.40° . This angle θ also changed in a nonlinear manner. Figure 3c shows the translation along the FHA t in simulation 1. As the tipping angle of the canine was increased from 5° to 30° , t slightly increased from 0.71 to 0.98 mm. Figure 3d shows the shortest distance from the origin to the FHA d in simulation 1. As the tipping angle of the canine was increased from 5° to 30° , d decreased from 5.74 to 0.32 mm. Both values (t and d) changed in a nonlinear manner.

Figure 4a shows the direction vector v of the FHA in simulation 2. Although the translation (Δy) was changed in steps of one mm from one to five mm, the direction vectors were kept in the same direction. The values of v_x , v_y , and v_z were 0.707, 0.031, and 0.707, respectively. Figure 4b shows the rotation angle about the FHA θ in simulation 2. The rotation angle about the FHA also had a constant value ($\theta = 7.07^\circ$) during the simulation. Figure 4c shows the translation along the FHA t in simulation 2. As the translation of the canine was increased from one to five mm, t increased from 0.71 to 3.53 mm in a linear manner. Figure 4d shows the shortest distance from the origin to the FHA d in simulation 2. As the translation of the canine was increased from one to five mm, d increased from 5.74 to 34.42 mm in a linear manner.

DISCUSSION

The disadvantage of the FHA method is that the axis is not defined for pure translations and the direction and position of the FHA are highly sensitive to landmark measurement errors, especially in the case of small rotations.^{11,12} Generally, if the rotation angle about the FHA calculated

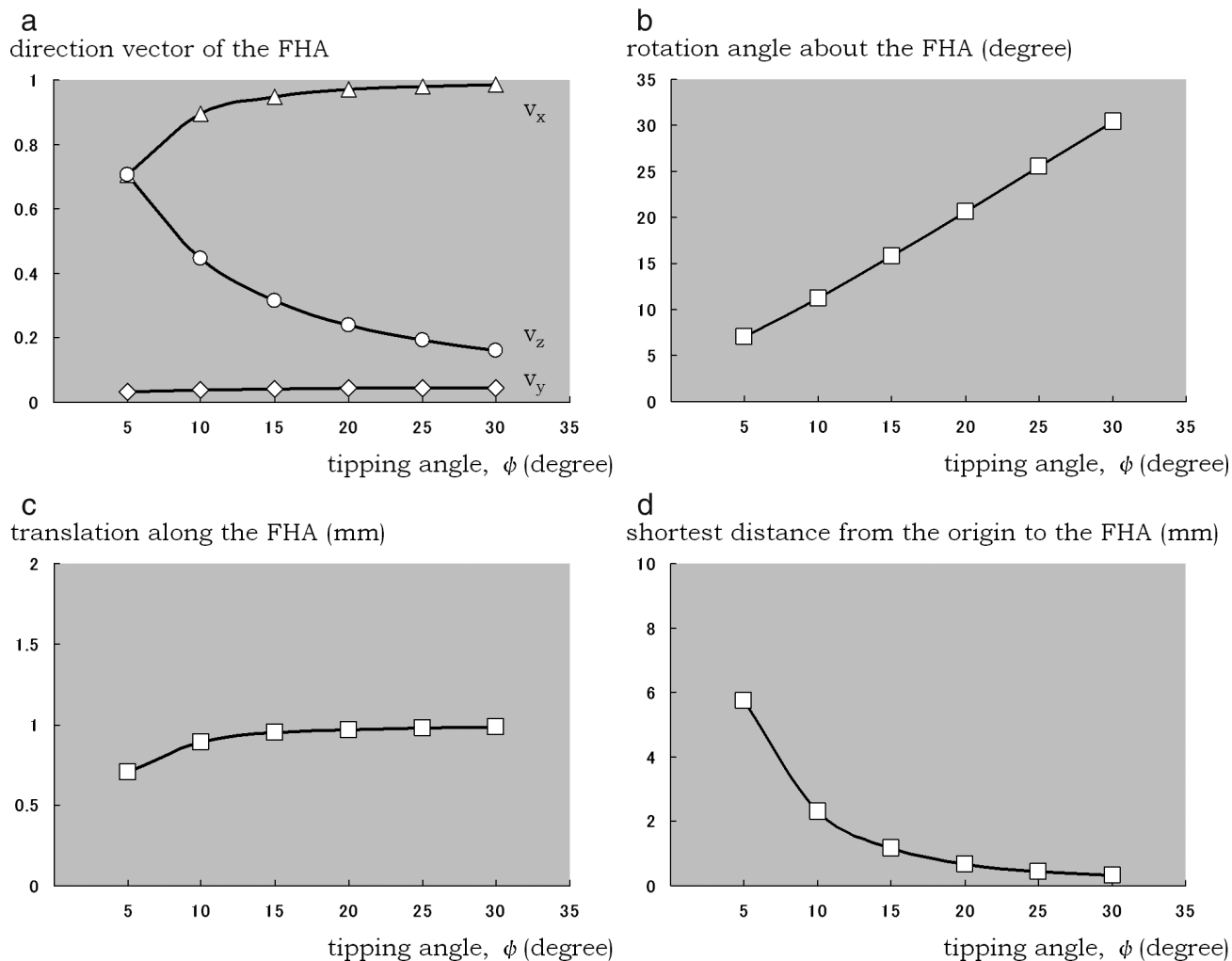


FIGURE 3. FHA parameters in simulation 1. (a) Direction vector v (v_x , v_y , v_z) of the FHA. (b) Rotation angle about the FHA θ . (c) Translation along the FHA t . (d) Shortest distance from the origin to the FHA d .

for motion analysis is small, the FHA must be normalized to apply $|\text{FHA } v| = 1$. In addition, when the rotation angle about the FHA is between 150° and 180° , the FHA is extremely prone to error.⁶ To investigate the effect of this error, FHA parameters and occlusal contacts from a simulation of jaw motion were calculated, and the results supported a 0.7° step limitation of jaw rotation.¹⁴ In this study, the minimum rotation angle in the canine drive simulation was set at 5° , and the tipping angle (ψ) was changed in steps of 5° from 5° to 30° . Therefore, the error in the calculation of FHA in this study can be ignored. However, it is necessary to consider that the accuracy of the helical axis parameters depends on the accuracy of the spatial reconstruction of the markers, the number of the markers, and the relative positions of the markers on the tooth.

In simulation 1, the FHA parameters calculated from the simple tipping movement showed nonlinear behavior. Although it is difficult to accurately systematize nonlinear behavior, it can be proven mathematically. The details will be

published in the near future. In this study, we obtained detailed information on tooth movement by considering the correspondence of the rotation angle about the FHA θ with the direction vector of the FHA v (v_x , v_y , v_z). As the tipping angle of the canine was increased from 5° to 30° , the direction vector of the FHA v (v_x , v_y , v_z) approached the x-axis, thus increasing the rotation angle about the FHA (Figure 3a,b). These results demonstrated that the direction vector of the FHA could indicate the axis of rotation that was most affected in the 3-D transformation.

When the value of the v_x component is equal to one, the FHA is parallel to the x-axis. This means that only tipping movement occurred. As the tipping angle of the canine was increased from 5° to 30° , translation along the FHA t slightly increased (Figure 3c). As the tipping angle of the canine was increased, the shortest distance from the origin to the FHA d clearly decreased (Figure 3d). These results demonstrated that the increase in the extent of rotational movement more strongly affects d than t . On the other hand, the

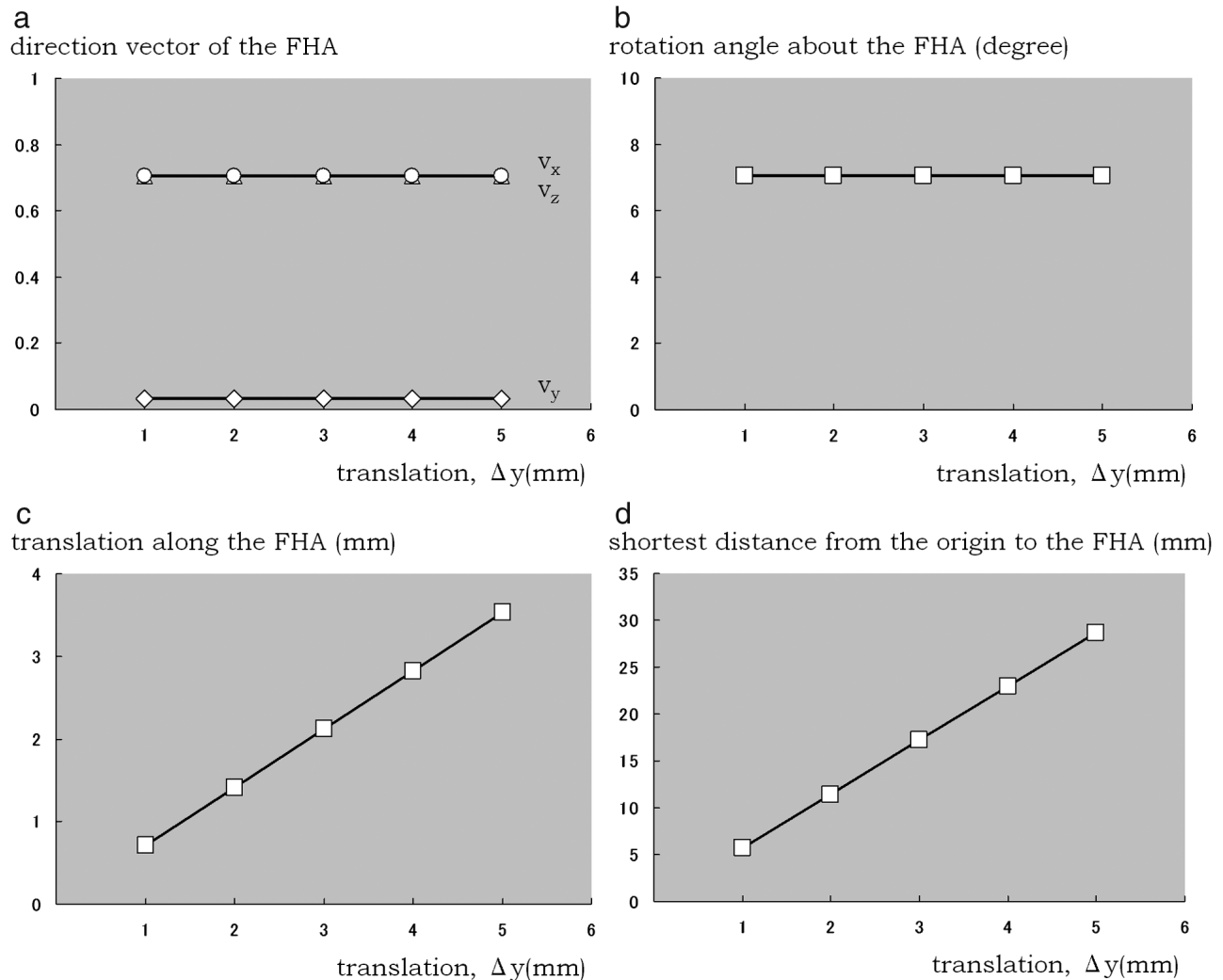


FIGURE 4. FHA parameters in simulation 2. (a) Direction vector v (v_x , v_y , v_z) of the FHA. (b) Rotation angle about the FHA θ . (c) Translation along the FHA t . (d) Shortest distance from the origin to the FHA d .

extent of bodily tooth movement is reflected more in the d value than in the t value. Although this simulation was carried out using biaxial rotation, similar behavior should be seen with triaxial rotation.

In simulation 2, the FHA parameters calculated from simple bodily movement showed linear behavior. Although translation (Δy) was changed in steps of one mm from one to five mm, the direction vectors v and the rotation angle about the FHA θ were maintained in the same direction and at the same value (Figure 4a,b). These results demonstrated that bodily tooth movement has no influence on the above two parameters. In contrast, as bodily movement of the canine was increased from one to five mm, the t and d values increased in a linear manner. These results demonstrated that an increase in the extent of bodily tooth movement strongly influences both d and t . Furthermore, the extent of bodily tooth movement is reflected more in the d value than in the t value, as seen in simulation 1.

In dentistry, numerical computation and computer simulation play essential roles.^{17,18} With respect to tooth movement, several morphometrical and theoretical studies have provided detailed information on tooth movement, although these studies were carried out based on an XYZ system.^{1-5,7} In the planar case, tooth movement has been described in terms of the center of rotation and resistance.^{1,3} Application of this procedure to the spatial case requires construction of the FHA. The FHA and XYZ methods of viewing tooth movement are complementary, not competitive. Both methods yield comprehensive 3-D descriptions of rigid body motion. In fact, the XYZ system can be converted mathematically to the FHA system. Although the XYZ system is more intuitive to the clinician, the FHA system is biomechanically correct. Furthermore, use of the FHA makes it possible to determine the torques applied to the tooth and the true bodily tooth movement during orthodontic treatment by analyzing the shortest distance from a tooth

to the FHA, and this may lead to a better understanding of how to move teeth.

REFERENCES

- Smith RJ, Burstone CJ. Mechanics of tooth movement. *Am J Orthod*. 1984;85:294–307.
- Rubin C, Krishnamurthy N, Capilouto E, Yi H. Stress analysis of the human tooth using a three-dimensional finite element model. *J Dent Res*. 1983;62:82–86.
- Pedersen E, Andersen K, Gjessing PE. Electronic determination of rotation produced by orthodontic force systems. *Eur J Orthod*. 1990;12:272–280.
- Middleton J, Jones ML, Wilson AN. The role of the periodontal ligament in bone modeling: the initial development of a time-dependent finite element model. *Am J Orthod Dentofacial Orthop*. 1996;109:155–162.
- Iwasaki LR, Haack JE, Nickel JC, Morton J. Human tooth movement in response to continuous stress of low magnitude. *Am J Orthod Dentofacial Orthop*. 2000;117:175–183.
- Richard PP. *Robot Manipulators*. Tokyo, Japan: Corona Publishing; 1984.
- Ashmore JL, Kurland BF, King GJ, Wheeler TT, Ghafari J, Ramsay DS. A 3-dimensional analysis of molar movement during headgear treatment. *Am J Orthod Dentofacial Orthop*. 2002;121:18–29.
- Yamamoto K, Toshimitsu A, Mikami T, Hayashi S, Harada R, Nakamura S. Optical measurement of dental cast profile and application to analysis of three-dimensional tooth movement in orthodontics. *Front Med Biol Eng*. 1989;1:119–130.
- Yamamoto K, Hayashi S, Nishikawa H, Nakamura S, Mikami T. Measurement of dental cast profile and three-dimensional tooth movement during orthodontic treatment. *IEEE Trans Biomed Eng*. 1991;38:360–365.
- Spoor CW, Veldpaus FE. Rigid body motion calculated from spatial co-ordinates of markers. *J Biomech*. 1980;13:391–393.
- Woltring HJ, Huiskes R, de Lange A, Veldpaus FE. Finite centroid and helical axis estimation from noisy landmark measurements in the study of human joint kinematics. *J Biomech*. 1985;18:379–389.
- Gallo LM, Airoidi GB, Airoidi RL, Palla S. Description of mandibular finite helical axis pathways in asymptomatic subjects. *J Dent Res*. 1997;76:704–713.
- Gallo LM, Fushima K, Palla S. Mandibular helical axis pathways during mastication. *J Dent Res*. 2000;79:1566–1572.
- DeLong R, Ko CC, Olson I, Hodger JS, Douglas WH. Helical axis errors affect computer-generated occlusal contact. *J Dent Res*. 2002;81:338–343.
- Hayashi K, Araki Y, Uechi J, Ohno H, Mizoguchi I. A novel method for the three-dimensional (3-D) analysis of orthodontic tooth movement—calculation of rotation about and translation along the finite helical axis. *J Biomech*. 2002;35:45–51.
- Tanne K, Koenig HA, Burstone CJ. Moment to force ratios and center of rotation. *Am J Orthod Dentofacial Orthop*. 1988;94:426–431.
- DeLong R, Ko CC, Anderson GC, Hodges JS, Douglas WH. Comparing maximum intercuspal contacts of virtual dental patients and mounted dental casts. *J Prosthet Dent*. 2002;88:622–630.
- DeLong R, Heinzen M, Hodges JS, Ko CC, Douglas WH. Accuracy of a system for creating 3D computer models of dental arches. *J Dent Res*. 2003;82:438–442.