CONTROL OF MILK FILLING OPERATIONS[1]

R. Burt Maxcy

Department of Dairy Husbandry,
University of Nebraska, Lincoln

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Work was undertaken to determine the applicability of statistical techniques for the evaluation and control of milk filling operations. It was shown that the components of the variation followed a so-called normal distribution. Thus, well developed statistical techniques can be applied. The use of gross weight appeared to be a logical control measure. The extent of deviation was found to be a characteristic of the individual filling operation. The machine filling larger containers showed a greater variation but not in proportion to the quantity filled. These statistical techniques were found applicable for the evaluation, adjustment, and routine control of filling operations for milk. Furthermore, these findings can be used as a helpful guide in establishing tolerances to be used for regulatory purposes.

Modern filling operations are designed for high speed production. At the present filling rate a considerable quantity of milk is involved, and with each new model of machine there is generally an increase in the filling rate. Adequate control measures for filling accuracy are therefore needed. The public should be protected against shortages in the individual cartons. Yet, the control system should not seriously penalize the producing plant.

There is a lack of well defined, adequate standards for regulating filling operations. Legal standards are in terms of volume while routine control is in terms of weight. Often the control standards, including tolerances, etc., are different for various regulatory groups because of a lack of understanding of the variation that is normal and to a certain degree inevitable.

The necessity for a plant to overfill is taken for granted. The amount of overfill is in a great degree determined by the extent of variation. A plant may experience considerable loss in complying with the variety of standards. For example, one of our most efficient plants of a national organization estimates its daily loss from the above lack of well defined uniform standards in excess of $100 per day.

Only a limited amount of work has been published pertaining to the evaluation of filling operations. Farmer *(3)* reviewed the commercial application of some statistical quality control measures as adapted to the receiving of materials and supplies and also to the packaging process in an ice cream operation. Some further work pertaining to the application of statistical quality control techniques was carried out by Smallwood and Roberts *(5)*. Tarver and Schenck *(6)* explored the use of extreme value control charts in the operation of canning machinery. Jensen *(4)* of the Bureau of Standards published a Handbook on the checking of prepackaged commodities. It dealt primarily with the personal behaviorial aspects and management of the official checking program. The paucity of published work especially on milk filling operations plus the value of product involved points to the need for additional work along these lines.

METHODS

In determining the quantity of milk in a package the weight was used. Approximately 200 containers were taken for each sample from a continuous uninterrupted operation. For the early evaluations the weight was determined to the nearest 0.1 g. After the initial pattern of distribution was found the weights were recorded to the nearest gram. In those trials where the net weight was to be determined the milk was emptied and the residue was rinsed from the package by using distilled water. The cartons were air dried and weighed individually to allow the calculation of the net weight of milk by the difference. Weight was chosen as the unit of measure for the contents since weighing constituted the only practical system of non-destructive checking for routine control purposes.

EXPERIMENTAL

The filling operations evaluated consisted of the University dairy plant and five rather large commercial plants. The commercial plants were chosen to represent the three large manufacturers of common commercial filler machines. Half-pint, quart, and half-gallon operations were evaluated.

To determine the nature and extent of variation that could be expected with a common packaging machine for paper bottles, a group of half-pint units was obtained from the University dairy plant. The gross weight of each package, and the weight of the rinsed and dried carton was determined. The net weight of milk was determined by the difference. The nature of the distribution of these weights can be seen in the accompanying Figures. The mean

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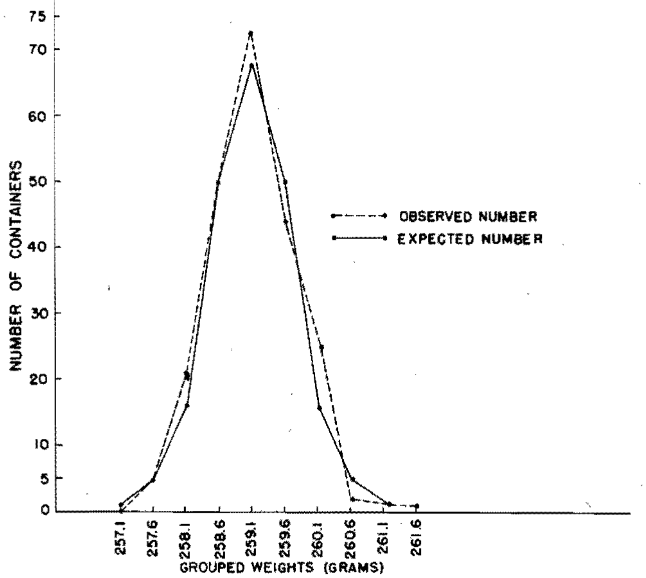

FIGURE I. THE OBSERVED AND THEORETICAL DISTRIBUTION OF WEIGHTS OF FILLED HALF-PINT CONTAINERS OF MILK

was taken as the midpoint, and the interval of grouping was chosen for convenience of plotting. Figure 1 shows the grouped distribution of gross weights. The variance of these weights was determined by the following formula:

$$S^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N - 1}$$

From the mean and variance a normal theoretical curve was established. This curve is shown as the expected distribution in Figure 1. The similarity of plot of the observed distribution and the calculated curve showed the filling operation had a variation that exemplified a normal distribution. A good fit was indicated by the Chi-square test.

A similar approach was taken for the two major components of the gross weight, e.g., the container, which included fiberboard, glue, wax, and wire clip, and the net weight of milk. The distribution of the weights of the containers is shown in Figure 2. The expected curve based on the calculated variance for the containers is also shown. Here too the appearance and the Chi-square test indicated a normal distribution. The distribution of the net weights of milk is shown in Figure 3 along with the theoretical curve calculated from the variance. As would be expected, the appearance and the Chi-square test indicated a normal curve.

It is apparent from the presentation of the above work, which was corroborated by further experiments, that the deviation of the gross weights and components are normal. Thus, one is permitted the use of the well developed mathematical operations of a normal distribution. For example, Anderson and Bancroft *(1)* pointed out that the component deviations of an overall system can be expressed as follows:

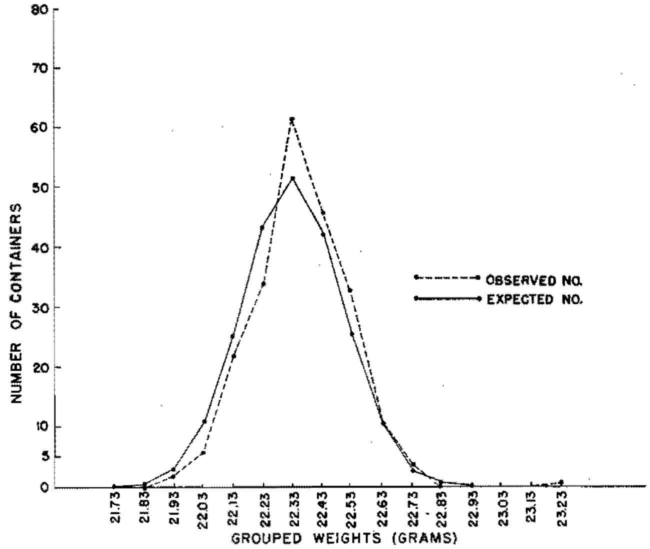

FIGURE 2. THE OBSERVED AND THEORETICAL DISTRIBUTION OF WEIGHTS OF EMPTY HALF-PINT CONTAINERS FROM WHICH MILK WAS REMOVED

$$S_x^2 = S_1^2 + S_2^2 + S_3^2 + \ldots + S_i^2$$

The total variance is equal to the sum of the variance of the mutually independent components. Since the variance of the net weight is large as compared to the variance of the carton (in this instance 0.4314 compared to 0.0277), the gross weight appears to be a logical means of routine control.

A careful look at Figure 3 gives an example of the applicability of this technique to the evaluation of an operation. The mean net weight of milk was 236.8 g and none of the packages contained the ex-

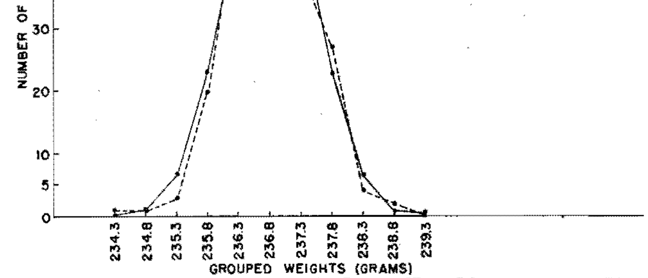

FIGURE 3. THE OBSERVED AND THEORETICAL DISTRIBUTION OF NET WEIGHTS OF HALF-PINT CONTAINERS OF MILK

TABLE 1. CHARACTERISTICS OF OPERATIONS FILLING HALF-PINT PACKAGES

Operation	Mean gross weight (grams)	Standard deviation
1	259	0.6
1a	278	0.9
2	261	2.1

pected 244.15 g (based on 946.33 cc per quart and a density of 1.032 g per cc). It was obvious that there was a continuous shortage and action should be taken to obtain adjustment of the filling operation.

The above operation was evaluated approximately a year later when the operating personnel were different. No advance warning was given. In addition, an outside commercial operation was evaluated. The mean gross weights of two evaluations of the University operation filling half-pints and the similar commercial operation are given in Table 1 as operation 1, 1a, and 2, respectively. There was a mean difference of approximately 18.4 g of milk in each half-pint the consumer received from operations 1 and 1a. This quantity is of obvious importance and indicates a lack of adequate control.

The one quart filling operation of the University was evaluated, but with the use of a different approach to establish the net weight of milk. A series of 202 packages was completed except that no milk was put into them. The mean weight of the packages was 46.5 g and the standard deviation was 0.5. A series of 220 filled packages was taken to determine the gross weight. The mean gross weight was found to be 1,028 g and the standard deviation was 3.4 g. The mean net weight was calculated to be 981 g. Similar results were obtained with other packaging operations. The results indicated this to be a satisfactory method of evaluating an operation. It was also apparent that the gross weight was an adequate criterion for the expression of the characteristics of the filling operation, since the distribution was normal and the standard deviation of the gross weight was seven times that of the package.

TABLE 2. CHARACTERISTICS OF OPERATIONS FILLING QUART PACKAGES

Operation	Mean gross weight (grams)	Standard deviation
1	1,019	7.7
2	1,019	6.0
3[a]	1,019	5.8
4	1,021	5.0
5	1,028	3.4
6[b]	1,018	3.6

[a]Over a one year period and 314 packages.

[b]The components of the two individual filling mechanisms showed a standard deviation of 0.8 and 0.9.

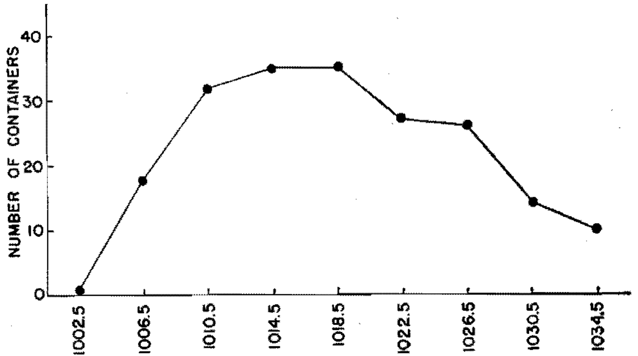

FIGURE 4. THE DISTRIBUTION OF GROSS WEIGHTS OF QUART CONTAINERS OF MILK FROM A COMMON FILLING OPERATION.

A number of other quart filling operations were evaluated. An example of the distribution of an operation with a standard deviation of 7.7 g - an extremely wide variation - is given in Figure 4. Some other operations showed considerably less variation. A comparison of six operations is given in Table 2. Operation number 1 showed more than twice the variation of operation number 5, and 1.5 times the variation of number 4. Thus, it follows that considerably more milk would have to be included in the average package of operation number 1 in order to meet a prescribed standard. It is also evident that each operation should have its own control standards.

Operation number 6 had an unusual characteristic that was not apparent from the compilation of data, but became apparent on plotting the data. This is shown in Figure 5. The two humps show that there are independent variables within this operation. This might be expected since it is a dual operation. It is also worthy of note that the standard deviation as represented by the two individual humps was 0.8 and 0.9 g.

The above work was extended to the evaluation of half-gallon fillers to determine the similarity and extent of variation as compared to the quart machine. An example of the distribution is given in Figure 6, which is based on weights in ounces and shows the unit of weight is relative and can be converted. Comparative data in grams from four operations are given in Table 3. These figures indicate the extent of variation that may be expected. The variation is in the general range of the one-quart machines. It is far from being doubled as might be expected from the quantity of milk involved. It is apparent that

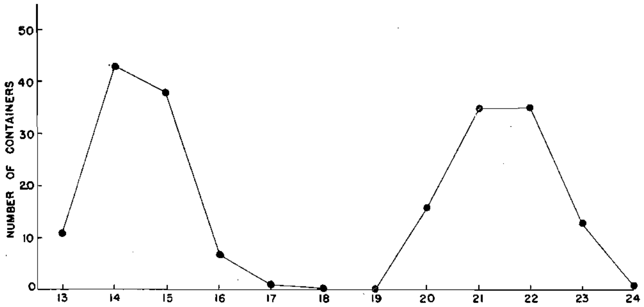

FIGURE 5. THE DISTRIBUTION OF GROSS WEIGHTS OF PACKAGES FROM A QUART MACHINE WITH TWO INDIVIDUAL FILLING MECHANISMS

TABLE 3. CHARACTERISTICS OF OPERATIONS FILLING HALF-GALLON PACKAGES

Operation	Mean gross weight (grams)	Standard deviation
1	2,027	7.6
2[a]	2,025	7.0
3	2,041	2.8
4	2,041	4.7

[a]Over a one year period and 288 packages.

each filling operation and size of container must be considered separately, because the extent of deviation is a characteristic of the individual filling operation.

Some observations were made to determine the applicability of these techniques to other operations in the dairy plant. Figure 7 serves as an example and shows results on paper cartons of cottage cheese, which is known to be an extremely difficult product to control. The data are presented as the grouped distribution around the mean gross weight of 378.85 g with a group interval of 10 g. The plot on these data shows that the technique is applicable though a close examination of the data shows that the variation is much greater than is found in milk filling operations previously given.

To compare the continuous run samples with routine random samples, two operations were evaluated by taking throughout a year the weights of periodic individual samples of approximately two per day of production. The data from the continuous runs were compared to data obtained by compiling the weights of the periodic single individual cartons for routine control purposes. The same general degree of variations was obtained.

The above results are based on the individual weights of a large number of packages. For routine control work sample size must be selected in harmony with the degree of accuracy desired. In order to exemplify the type of variation and the approach to selecting a sample size, Figure 8 was constructed using a theoretical mean of 975 g and a standard deviation of 5. This figure shows that 95.5 per cent of the individual weights would be between 965 and 985 g. If multiple units are taken the deviation becomes less. The relation is $s_{\bar{x}} = s/\sqrt{N}$, where $s_{\bar{x}}$ is the standard deviation of the mean weight of a sample of N units and s is the standard deviation of the individual units in the entire operation. It is apparent that the mean weight of a 4 unit sample would fall between 970 and 980 g with a 95.5% frequency, and a 10 unit sample would have a mean

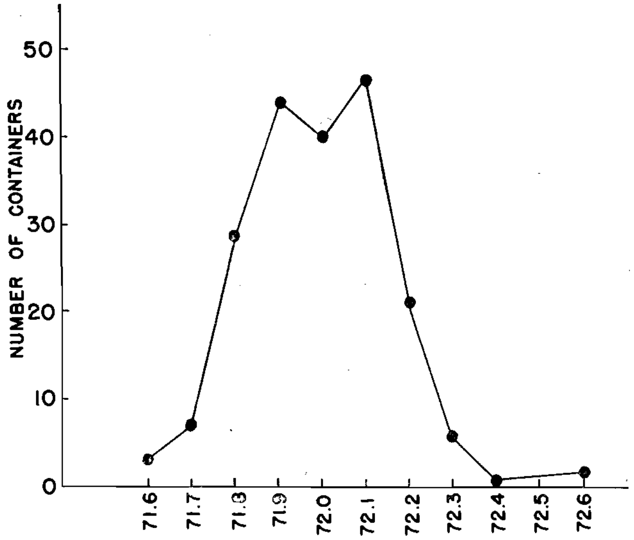

FIGURE 6. THE DISTRIBUTION OF GROSS WEIGHTS OF HALF-GALLON CONTAINERS OF MILK.

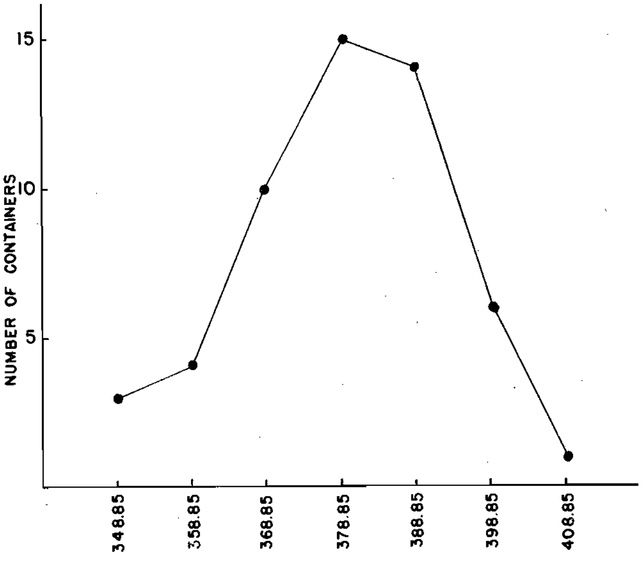

FIGURE 7. THE DISTRIBUTION OF GROSS WEIGHTS OF PACKAGES OF COTTAGE CHEESE.

weight between 971.84 and 978.16 g with the same frequency. Similar calculations show a 25 unit sample would have a mean weight between 974.0 and 976.0 g with this frequency. From these figures it is apparent that the routine control might be based on a fairly small sample depending on the accuracy demanded and the frequency of sampling, or control might be based on a number of individual periodic samples. For the basic adjustment of machines, however, a sample of 25 or more should be used.

The considerable variation involved in a milk filling operation makes it necessary to take logical precautions in controlling unfair practices against the consuming public and to prevent serious economic loss to the producing plants. It is apparent that the examination of a single package may tell if the individual consumer is receiving the quantity stipulated on the package. On the other hand, the examination of an individual package may tell little about the next package from that machine or the general characteristic of the filling operation. For example, a single package from the operation of machine 1 shown in Table 2 might vary from 1003.6 to 1034.4 and be within a perfectly normal operation. One would expect to get a package weighing between these limits 95.5% of the time, or beyond these limits 4.5% of the time.

All of the above results show the necessity of overfilling to maintain a reasonable assurance that individual packages will not be underfilled. The extent of overfill is directly related to the characteristic of the filling operation. This is expressed by the standard deviation, and a large standard deviation is depicted by a wide distribution on a graph. The wide distribution and overfill are the source of loss to the dairy plant. For example, even if the average net weight is correct, to have no more than 5% of the packages underfilled, it is necessary to put 1.6 times the standard deviation as an average overfill; and to have no more than 1% of the packages underfilled, it is necessary to put 2.3 times the standard deviation as an overfill. Control standards as well as the selection of machinery and conditions of operations should be based on this sort of finding.

While working with this type problem it is constantly apparent there is one major handicap in the control of packaging of milk products. Milk is labeled by volume. As the law now stands the official method of control is by volume. Most routine control work, both in the plant and regulatory bodies, is done by weight. For routine control purposes it is highly desirable to have a non-destructive type of control and weight is the only logical means available. More efficient plant control means more reliable regulatory control. It is therefore suggested that a new approach be taken to labeling. One logical way would be to label the containers as average net weight with the wording approximately one pint, quart, or half gallon as appropriate. The deviation allowed would be based on normal operating conditions. It would be a more logical method of assuring the consumer the quantity stipulated on the container, a more practical method for routine control, and a more satisfactory method for the producing plant.

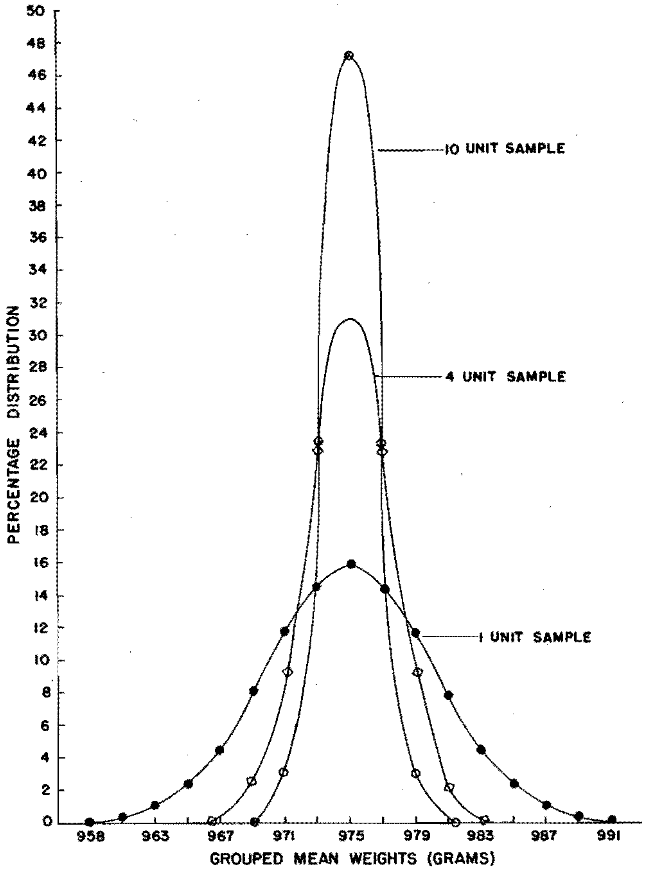

FIG. 8 THE THEORETICAL DISTRIBUTION AROUND A MEAN FOR VARIOUS SIZE SAMPLE

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