Holistic Polyhedrons: A New Concept of Making Mobile Members

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M obiles constitute a special form of kinetic art. Although mobiles have commanded a large viewing public, their design and construction have attracted hardly any scientific interest. In an introductory paper on geometric mobiles [1], I described a set of rules that I have adopted for the conceptualization, design and construction of my mobiles; in particular I presented my use of mathematical modeling to develop quantitative design data so as to avoid trial-and-error construction and to add an intellectual dimension to mobile-making.

This note reports on three-dimensional mobile members derived from delineation of polyhedrons. A polyhedron consists of ridges (lines), intersections of the ridges (points) and surfaces enclosed by the ridges. For a given polyhedron, these ridges, points and surfaces are all geometrically defined in space. Our first visual impression of a polyhedron is from the outside, and few would consider how a polyhedron would look if one were to scientifically integrate its internal as well as external attributes (ridges, points and surfaces), or even omit certain such attributes. These ridges, points and surfaces in space, both outside and inside a polyhedron, may therefore be considered available parts for a mobile designer to use or omit in creating configurations of interest. I shall show two examples, the tetrahedron and the octahedron, from which such holistic effects can be derived.

The balancing and hanging of mobile members and the design of high-sensitivity beams are described in my previous paper [2] and more fully in a monograph [3] and will not be entered into in the present paper.

TETRAHEDRON

Let us start with a modification of the simplest polyhedron, the tetrahedron, consisting of four equilateral-triangular surfaces with four exterior points and six ridges. Inside the tetrahedron, radiating from its center of gravity, there are four axes joined to the four points and inclined to each other at an angle calculated to be about 109.5°. Any two of the four axes together with an opposite ridge form a triangular inner surface. There are six such inner surfaces, inclined to each other at an angle of 120°.

Now let us cut these six inner surfaces from two aluminum disks, as shown in the upper diagram of Fig. 1, three surfaces to a disk, each with a vertex angle (measured at the center of the disk) of 109.5°. Because we are using a disk as the parent material, we are converting the straight-line ridge of a tetrahedron into an arc of the disk, and the triangular surface of a tetrahedron into a sector. The three vertex angles add up to 328.5°, leaving a remnant sector of 31.5°, which will be used to counter-balance our mobile member, as shown in the bottom diagram of Fig. 1. Note that we do not cut apart the three surfaces from the disks, but, instead, we fold the three surfaces, mountain-and-valley style, into a reverse-obverse trihedral of 120°. From the second disk, we cut a similar trihedral, but bend this second disk into the mirror image of the first.

Let us drill a hole at each corner of the six sectors in order to sew together three sectors as a group at each of the four points of the tetrahedron. When the two trihedrals are thus sewn together, the vertices of the six sectors will meet at a common point, the center of gravity of the tetrahedron, from which this mobile member is suspended.

When properly constructed, this mobile member will respond to air currents with four types of motion: it will not only rotate, but also roll and pitch, as well as revolve against the counterbalancing 31.5°, two-sector drive. While rotating on its own vertical axis, this mobile member will display configurations showing either three or four of its sectors (hence its name, 3-4 Transform [Fig. 2]), with the sectors oriented either concavely or convexly. These configurations repeat themselves as the mobile rotates every 360° in the following order:

3-lobe concave → 4-lobe concave → 3-lobe concave
→ 3-lobe convex → 4-lobe convex

OCTAHEDRON

Now we shall see how a variety of holistic configurations can be derived from the ridges, points and surfaces of an octahedron.

Diagram A of Fig. 3 shows an octahedron formed from eight triangular surfaces with six points and 12 ridges. We select six of the 12 ridges to form a contiguous six-sided frame, UABTCDU. We can easily make this contiguous frame out of an aluminum welding rod. Diagram B shows the six-sided frame transposed by deleting the lines not to be used in the mobile construction. Now, let us lace together the corresponding sides with strings, that is, side BT against AU and side TC against UD, by joining corresponding points, 1 with 1',

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Kwauk, Holistic Polyhedrons

Fig. 1. Following the orientation of the six internal surfaces of a tetrahedron, this 3D mobile member is made from six sectors cut from two parent disks.

Fig. 2. 3-4 Transform. This mobile member, made from two identical disks, describes the six internal surfaces of the simplest polyhedron, the tetrahedron.

2 with 2', etc. through 7 with 7'. The lacing forms a carpet extending from side AB to side CD. Closer examination will show that the upper side of the carpet next to side AB turns counter-clockwise as we weave the carpet from left to right; when the carpet ends at side CD, it has turned 180°, that is, upside down. The six-sided frame actually twists the progressive positioning of the individual threads of the lace; thus I call this mobile member Carpet Twister (Fig. 4).

In designing the Carpet Twister, I consider it desirable to maintain the same width of the carpet at the starting side AB, the terminal side CD and the midpoint TU. This makes the four top and four bottom ridges all $\sqrt{3}/2$ times the length of the horizontal ridges, which is taken to be unity. Accordingly, both the top angle BTC and bottom angle AUD each become 109.472°, and the side angles, $ABT$, $TCD$, $CDU$ and $UAB$, are each 54.736°.

If the string used in lacing the corresponding sides $BT$ and $TC$ against $AU$ and $UD$ is replaced by a series of stiff rod connectors (shown in Diagram C) spaced equidistantly along the sides, then we need only four sides (i.e. ridges) (instead of the six needed for the Carpet Twister), which are supported in space by the stiff rod connectors. I call this configuration Rectilinear Rod Twister (Fig. 5). Two of these four sides, $TC$ and $UD$, are shown to lie in the $y = 0$ and $x = 0$ planes, respectively. The equations of the upper and lower sides are: for $TC$, $z = z_0(1 - x)$, and for $UD$, $z = z_0(-1 + y)$, where $z_0$ is the intercept on the $z$-axis: $z_0 = 1$ for an octahedron made of eight equilateral triangles, and $z_0 = \sqrt{2}/2$ for the case in which the central connector has the same length as the two terminal connectors. Then the lengths of the connectors can readily be written from the two-point-in-space formula:

$$d = \sqrt{(x - 0)^2 + (0 - y)^2 + z_0[(1 - x) - (-1 + y)]^2}$$

Since both ends of each connector are always joined to the two opposite sides, equidistant from the ends of the sides ($x = y$), the above equation simplifies to the following, for the respective cases of $z_0 = 1$ and $z_0 = \sqrt{2}/2$:

$$d = \sqrt{6x^2 - 8x + 4} \quad d = \sqrt{4x^2 - 4x + 4}$$

What the above equations say is that for either case the connectors are not of equal lengths.

The unequal lengths of the rod connectors between the top and bottom
A. Extracting a 6-sided frame from an octahedron, $UABTCDU$.

Fig. 3 (A–E). Holistic configurations derived from an octahedron.

B. Lacing corresponding points to form a carpet by joining 1 to 1’, 2 to 2’ through 7 to 7’. While the carpet extends from side $AB$ to side $CD$, it is twisted upside down. Therefore it is called Carpet Twister.

C. Rectilinear Rod Twister, for which the connecting strings (in B) are replaced by stiff rods. This reduces the six sides (ridges) seen in Carpet Twister to four. The connecting rod lengths are unequal.

D. Elliptical Rod Twister. When the upper and lower members $BTC$ and $AUD$ are replaced by half ellipses with major:minor axis ratios of $\sqrt{2}:1$, then all rod connections are equal in length.

E. Skeletal Octahedron. The six-sided construction seen in Diagram B can also be rendered as a sheet-metal frame using two interlocking C-shaped members cut from a parent square. The remaining two square corners are used as counter-balancing drives.

Fig. 4. Carpet Twister. This mobile is made from six of the 12 ridges of an octahedron formed from eight triangular surfaces. The six ridges are bent from an aluminum welding rod.

Fig. 5. Rectilinear Rod Twister. The strings lacing the top and bottom sides, as seen in Carpet Twister, have been replaced with aluminum rods, so that the number of ridges is further reduced to four.
The six-sided wire frame for Carpet Twister can be converted into a corresponding sheet-metal frame as follows. Cut two C-shaped components of unit shank width, as shown in Diagram E, from a parent square measuring $4 \times 4$ times the shank width. Fold each of the two components at the right-angled corners, valley-and-mountain style, that is, $90^\circ$ down and up. The bent C-shaped components are then lap-joined to each other at their corresponding terminals, $A1$ to $B1$ and $A2$ to $B2$, at $90^\circ$ shift in position, and then stitched together through their respective four holes. The two $1 \times 1$ corners cut from the top right and bottom left of the parent square are used as counter-balancing drives. Due to the ample surfaces of the sheet metal, oriented at right angles to one another, this mobile member is multi-dimensionally responsive to air currents. I call this fourth configuration Skeletal Octahedron (Fig. 7).

References

2. See Kwauk [1].

Glossary

gromatic mobiles—mobiles made of geometrically definable members: rods, both straight and curved; plates shaped as triangles, squares, rectangles, circles or parts cut from a circle, e.g. rings, sectors, segments, etc. Geometrically definable parts facilitate their modeling to yield mathematical equations for their balancing as well as their orientation in space.

holistic—as used in the title of this paper, this word denotes all the attributes of polyhedrons: their ridges (lines); the intersections of the ridges (points) and surfaces enclosed by the ridges, both outside and inside; the lengths of the ridges; and the angles between ridges and between surfaces. These attributes, all geometrically defined in space, constitute the available parts for a mobile designer to pick, to use or omit in creating configurations of interest.

Thus I have created a third configuration, the Elliptical Rod Twister (Fig. 6). When this mobile member rotates, it displays the following four major configurations: