The arts and architecture of the Greek island of Chios in the eastern Aegean reveal that it has long been exposed to a wide range of political and cultural influences, both through invasion and occupation and through commerce and intellectual contacts. First settled during the Neolithic period and reputed to be the birthplace of Homer, ethnically Greek Chios has been ruled by a succession of peoples, most recently the Byzantines, the Genoese, the Ottomans and, since 1912, the Greeks.

**CHIOS, GREECE**

In 1261 the Genoese settled on the island with the permission of the Byzantine emperor. By the mid-14th century they had taken control of the island and became semi-independent of Byzantium. During their time on the island, which lasted until the mid-16th century, the Genoese built numerous stone towers and several walled villages [1].

A few of the Genoese medieval walled villages in the southern part of the island remain relatively unaltered. These villages were usually built around a central defensive tower and were enclosed by a continuous wall, with only one or two gates providing access to the interior of the village. Situated far from the sea, so as to be inaccessible to marauding pirates, these fortress-villages were designed by the Genoese to protect the local Greek population, who were valued as producers of mastic. A resin laboriously obtained from the lentiscus shrub, mastic is quite expensive and cultivated commercially only in the southern part of Chios. Mastic was, and is, prized for flavoring food and drink and making varnish.
Great prosperity came to the island with Ottoman rule, which began in 1566. Chios was highly valued by the Turks, and the island had special protection as a production center for citrus and mastic, which was chewed by the women of the harem. Indeed, Chios is so abundant with sweet-smelling mastic and citrus that it is known in Greece as the “wonderful-smelling island.” The island was also a center for the transshipment of goods in the eastern Aegean.

Prosperity ended with the Massacre of Chios in 1822. In Ottoman retribution for the perceived treachery of the island’s support for the Greek Independence Movement, at least 25,000 Chians were killed, 5,000 were enslaved and most of the remainder of the original population of 120,000 were scattered as refugees. The island was liberated from the Ottoman Empire and joined the modern Greek state in 1912.

The many architectural influences on Chios range from one of the finest examples of Byzantine monastic architecture, at Nea Moni, to the Genoese fortified villages, to examples of Turkish architecture including a mosque (now an architectural museum) and Turkish baths that still stand in the eponymous capital city of Chios.

PIRGÍ, CHIOS

The site of our study is Pirgí, one of the remaining medieval villages on Chios. Adjacent to the village square stands a Genoese defensive tower, which would have been the final refuge of the inhabitants if the village had been invaded. The few streets linking the square to the gates are twisting and narrow, and small alleyways, often ending in cul-de-sacs, lead off from them. The curving of the streets and alleys, which seldom intersect at right angles, increases the labyrinthine effect of the street plan (see Article Frieze, Fig. 1). The roofs of the houses are all of the same height, vaulted and connected by bridging arches, thus allowing movement across the village at roof level for escape to the tower.

The structure of Pirgí and the other medieval villages in the southern part of Chios is fundamentally different from that of other Aegean settlements and indicates the intentional design of the entire village for defense. The plans for the village, which were sent from Genoa in the mid-14th century, are identical to plans for villages in Liguria and elsewhere in Italy. Since the Genoese plan was a standard form, the preexisting church buildings had to be accommodated during construction [2]. At the beginning of the 20th century, the village overflowed its original walls; it currently indicates the shadows of the sails by reversing the gray and white. Among the possible designs are mill sails, reeds for thread, diamonds or sawteeth, and their shadows (see Fig. 2).

Scholars have puzzled over the origin of xistá on Chios. Some, especially Koukoules [4], have suggested that the Pirgí embellishments are remnants of Byzantine art; however, Bouras [5] makes a convincing case that xistá came to Chios from Italy during the 15th century and were made more elaborate locally. The technique of xistá is remarkably similar in method to the graffiti found in Italy from the Renaissance onwards; however, Italian graffiti are scratched in a much simpler design based on the structure of the underlying stone construction. The more complex technique of xistá utilizes shapes also found in local jewelry, embroidery and weaving. Some xistá façades have tassel designs at the bottom, as would be found on the edges of woven carpets, reminiscent of the custom of hanging decorative woven dowry cloths from windows on feast days, especially the day preceding Lent [6].

The evolution from merely mimicking the form of the underlying stones to elaborative decorative art is a phenomenon that resulted from the meeting of two cultural traditions. At the time of the Greek-Turkish War in 1922, when the Asia Minor Greeks came to Chios as refugees, they brought with them their rich decorative traditions of elegant rug weaving and colorful embroidery design. This influx had a profound impact on the striking decorative style of xistá in Pirgí [7]. There had actually been a small influence from Asia Minor prior to the refugees’ arrival. At the end of the 19th
tional symmetry occurs in nature in inanimate objects such as snowflakes and in animate objects such as flowers, cross-sections of fruits, starfish, jellyfish and several other sea animals. Rotational symmetry occurs universally in decorative art, ranging from the primitive designs of prehistoric humans to the folk art of virtually all cultures to the modern designs found on wheel covers and advertising logos. In architecture, it appears in many structures, especially where stability, order and strength are represented, such as forts, government buildings and monuments. Leonardo da Vinci carried out a study of rotational symmetry in order to incorporate it in his architectural work [11].

The second category of symmetry, frieze symmetry, consists of a design that repeats itself regularly along a given direction. Friezes appear naturally in certain rock formations, in the structure and coloring of animals such as caterpillars, centipedes and snakes, and in the regular budding and branching along the stems of various plant species. Frieze symmetry enjoys popularity in the repeating designs on handrails, picket fences and the lace edging of pillowcases and occurs in many other contexts, such as the appearance of marching band members in a row, or the tracks left in dust by the tires of a car. It commonly appears in the interlacing strip designs of the ancient Celts and in the folk art of many other cultures. In architecture, frieze symmetry appears in the regular spacing of columns or windows, especially in Italian Renaissance designs. It also underlies the appeal of the regular spacing of trees and light poles along a boulevard.

The third category of symmetry, space-filling symmetry, occurs in nature wherever there is crystalline structure in a physical object, and thus it forms the basis of chemical and physical crystallography. In decorative art, this type of symmetry emerged as early as the fourth millennium BC at the temple of Erech, where enormous columns were covered with many different types of space-filling designs. It appears in textiles, quilting, wallpaper patterns, tiling and basketry. It forms the basis of Escher’s art and appears also in Islamic art and the repeated patterns of Warhol. In architecture, especially in skyscrapers, space-filling patterns form a major component of the visual impact, particularly in the exterior of the buildings.

**The Key to Understanding Symmetry: Genotype Versus Phenotype**

Mathematical group theory provides the structure for the organization of friezes into families. A useful analogy can be found in biology. The appearance of a symmetrical object corresponds to its genotype and the underlying mathematical group structure to its genotype. For example, in this paper there are several illustrations of Type 1 friezes (defined below), all differing from one another in appearance; that is, they have different phenotypes. But all are in the same family, sharing the same set of actions under which they appear invariant; that is, they have the same genotype—they share the same mathematical group structure. Comprehension of the group structure into which the individual designs fit enriches the understanding of the attractiveness of friezes.

**The Group Theory of Friezes**

The principal type of symmetry that appears in Pirgí is frieze symmetry. A frieze is a pattern that repeats regularly along a given direction, extending infinitely to the right and left, at least in the imagination. If an entire frieze were moved to the right or left one unit, it would appear exactly as before. In addition, there may be certain ways in which a frieze can be rotated or reflected so that its appearance is indistinguishable from the original. The action of moving the entire frieze to the right by the distance necessary to achieve realignment is termed a translation and is denoted by \( t \).

Each frieze is associated with a group, and the elements of the associated group are the actions that leave the frieze image invariant. The group binary operation between two actions \( a \) and \( b \) (designated \( ab \)), denotes the first action, \( a \), followed by the second action, \( b \). Since actions \( a \) and \( b \) leave the image invariant, so does action \( ab \). There are five basic actions: translation \( (t) \), vertical mirror \( (m_v) \), horizontal mirror \( (m_h) \), half turn \( (1/2) \) and glide reflection \( (g) \).

A mathematical group consists of a set of elements and an operation that com-
bines any two elements to produce a third. Four properties must be satisfied: presence of an identity element, existence of an inverse for each element, associativity and closure. The identity element for a frieze group is the (non)action of doing nothing at all, which obviously leaves the frieze unchanged. Each action has an inverse; for example, the inverse of \( m \) is itself, because \( m \) followed by \( m \) is the same as doing nothing. In addition, \( (abc) \) is identical to \( ab \), so associativity holds. Thus the only remaining requirement for a collection of elements to be a group is closure, that is, every binary operation of two elements in the collection must give a result that is also in the collection. Certain collections of the five basic actions have this property of closure, while others do not. Collections that do have this property are the ones that result in friezes, and they also form mathematical groups. As such, there exists an important connection between mathematics and friezes. Formal presentation of frieze groups can be found in many sources; particularly readable introductions include Farmer [12] and Washburn and Crowe [13].

**Symmetry Actions for Friezes**

The pattern below has a vertical mirror re- 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \]

In the next frieze type, a horizontal mirror reflection \( (m_h) \), the part above the horizontal midline mirrors the part below.

Another possible symmetry action is a half-turn \( (1/2) \). The frieze

\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \] 
\[ \ldots \]

has neither a vertical nor a horizontal mirror line; however, rotation of the entire frieze by 180° around the center of any \( S \) results in an appearance unchanged from the original.

The final symmetry action is a glide reflection \( (g) \), which consists of a horizontal reflection followed by a translation a certain distance to the right. In Fig. 4, each left footprint, when reflected across the horizontal centerline, fails to match up with a reflected right footprint directly below it; however, if the frieze is then translated to the right by an appropriate distance, the reflected left footprints will match the right footprints and vice versa.

**The Seven Standard Frieze Groups**

There are seven different frieze groups [14,15]. Translations \( (t) \) are present in all friezes—that is what makes them friezes—and therefore need not be mentioned in the list of actions (see Table 1). The presence of various other symmetry actions is what distinguishes one group type from another.

**The Friezes of Pirgi: All Seven Standard Groups Are Present**

It is interesting that all seven of the standard types of friezes appear in Pirgi (see Fig. 5). These seven frieze types occur in varying degrees of frequency. We separated the friezes into two categories: friezes on the five houses that had retained their older frieze patterns (Old Friezes), and friezes on a selection of 13 of the approximately 700 other decorated houses (New Friezes). A total of 417 friezes were analyzed; the results are summarized in Table 2, where the top number is the number of occurrences of each frieze type and beneath it is the percentage of the friezes that are of that frieze type.

**A New Approach: Color-Reversing Friezes**

The seven-group analysis presented in the preceding section provides a useful start to understanding the friezes of Pirgi; however, our analysis left us with the strong sense that the Pirgi friezes have more subtlety than can be described using only the standard seven groups. What is so unusual in Pirgi is that the outlines the artist draws to mark the design consist only of straight-line segments and arcs of circles, that is, constructs realizable using only a straight-edge and compass—a sort of homage to Euclid. We designate friezes having this property *Euclidean Friezes*.

Even more interesting is that these lines and arcs are used as perimeters of closed regions that can be filled in with alternating dark gray and white tones; if one region is filled in with gray, then all its adjacent regions are white, and vice versa. Such a restriction creates an invitation to the artist’s subconscious creativity to explore the possibilities available in color-reversal schemes of this type. This revelation provided the component that had been missing in our analysis.

A good example is the frieze of marching right triangles in Fig. 5. Since this frieze possesses no mirrors, no half-turns and no glide reflections, it is a Type 7 design, that is, it has no symmetry beyond the required translation. Still, there is some appeal that has not been taken into account in the standard seven-group analysis. This appeal is based on the fact that if a half-turn is performed at the point halfway down the slanted line, the result is the same design, but with the colors reversed.

The presence of this and other similar friezes indicated that there was more work of interest to be found. We undertook to discover the particular rules that the artists of Pirgi were intuitively obeying in this color-reversing art form. We are not the first to think that some situations call for more insight than the standard seven-group analysis can provide. Knight [16], in this journal, studied some unusual ambiguities and curiosities of frieze typology. As early as 1935, Woods analyzed color-reversing friezes in textile design [17]. Washburn and Crowe [18] distinguish between two types of color-reversing friezes: one in which the foreground figure alternates between two colors, while the background remains consistently white, and another in which there is no distinction in importance between foreground and background—both change colors equally. The artists of

---

**Table 1. The seven standard frieze types.**

<table>
<thead>
<tr>
<th>Mirrors</th>
<th>Type 1: XXXXX</th>
<th>Horizontal &amp; vertical mirrors &amp; half turns ( (m, m_h/1/2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 2:</td>
<td></td>
<td>Vertical mirrors &amp; half turns &amp; glide reflections ( (m, 1/2, g) )</td>
</tr>
<tr>
<td>Type 3:</td>
<td></td>
<td>Vertical mirrors &amp; no half turn ( (m) )</td>
</tr>
<tr>
<td>Type 4:</td>
<td></td>
<td>Horizontal mirrors &amp; no half turn ( (m_h) )</td>
</tr>
<tr>
<td>No Mirrors</td>
<td>Type 5: SSSSSS</td>
<td>Half turns ( (1/2) )</td>
</tr>
<tr>
<td>Type 6:</td>
<td></td>
<td>Glide reflections &amp; no half turns ( (g) )</td>
</tr>
<tr>
<td>Type 7:</td>
<td></td>
<td>Translation only</td>
</tr>
</tbody>
</table>

---

Fig. 4. Glide reflection symmetry. (© Loukas Kalisperis)
Pirgí utilize only the latter type of color reversal. This form is reminiscent of the art of M.C. Escher in that foreground and background play equal roles. Any frieze having this property we designate as Escher type. The friezes of Pirgí are Euclidian, Escher-type, color-reversing friezes.

FRIEZES WITH COLOR-REVERSING ACTIONS

The frieze artists of Pirgí start with a well-defined prototypical cell, which they repeat over and over to form a single long horizontal strip. The simplest type of color reversal is to insert reversed-color cells that alternate with the normal cells. This new frieze has a cell width twice that of the original. It has also a new symmetry action: translation by a half-cell-width distance to the right and color reversal. This action is denoted \( r' \), in contrast to \( r \), which is the translation by a full cell width without color reversal. Five examples of friezes with \( r' \) are illustrated in Fig. 6.

The primes indicate color reversal. For example, \( m_y' \) is a vertical mirror symmetry action where the design on one side of the mirror is reflected and color reversed when compared with the other side.

In addition to the five friezes with the \( r' \) action (translation with color reversal) illustrated in Fig. 6, there exist other color-reversing friezes not possessing \( r' \). These are illustrated in Fig. 7. For example, in Type 5 there are no places with vertical mirrors \( (m_y) \), but there are places where if a vertical reflection and color reversal \( (m_y') \) are performed, the frieze image appears identical to the original. There are also places where if a half-turn and color reversal \( (1/2') \) are performed, the appearance of the frieze is again invariant. So the Type 5 frieze has symmetry actions \( m_y, m_y', \) and \( 1/2' \) in addition to the ever-present \( r \). Note that the photographs in Fig. 7 are of entire walls, and therefore some friezes of the normal type are mixed in with the color-reversing type.

THREE CATEGORIES OF FRIEZES

There are 24 distinguishable frieze groups when color-reversing actions are permitted, as proved by Belov [19] and Grünbaum and Shephard [20]. We can collect these 24 types into three categories. There are the seven standard non-color-reversing frieze groups, or the Magnificent Seven. In addition there are the five so-called Boring Groups, which have only one generator (along with \( r \)) and so are relatively uninteresting to the modern Greek eye. See Table 3.

The remaining category was the one of most interest to the artists of Pirgí. It consisted of the 12 groups containing both color-reversal and non-color-reversal symmetry actions. These 12 mixed groups, which we designate the Grecian Groups, form the basis for the frieze art form at Pirgí.

Table 3 is arranged in decreasing degrees of symmetry from top to bottom, and also within each category from left to right, under the convention that vertical mirrors are the strongest actions and glide reflections the weakest. In Pirgí half-turns appear to be slightly more im-

<table>
<thead>
<tr>
<th>Type</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old Friezes</td>
<td>41</td>
<td>9</td>
<td>23</td>
</tr>
<tr>
<td>New Friezes</td>
<td>129</td>
<td>24</td>
<td>57</td>
</tr>
<tr>
<td>Total</td>
<td>170</td>
<td>33</td>
<td>80</td>
</tr>
</tbody>
</table>

Leonardo_37-3_175-266 5/12/04 10:17 AM Page 239
portant than horizontal mirrors, and so we have ordered the table accordingly.

Various frieze types appear with different degrees of frequency at Pirgí, as seen in Table 4. In our earlier, non-color-reversing frequency chart, all 417 of the friezes were classified as one of Type 1 through 7. In Table 4 our more sensitive color-reversing tool has moved 243 of the friezes into the color-reversing part of the table, leaving only 174 for Types 1 through 6. It is significant that in this new classification scheme, there are no longer any Type 7 (no symmetry) friezes in Pirgí—every frieze now possesses some symmetry beyond the required translation.

Note that there is a more even distribution of types among the old friezes than among the new. To be more precise, if we label a type “rare” when its frequency of occurrence is less than 1 percent, then exclude the Boring Groups and Type 7 (no symmetry), the New Friezes include six types that are rare while the Old Friezes have only two rare types.

**SUMMARY**

With their color-reversing friezes, the artists of Pirgí have invented a novel art form and brought it to near-perfection. Nowhere else, to our knowledge, has there been an entire art form based on the nearly complete exploration of all types of color-reversing friezes (beyond the Boring Friezes). The most interesting occurrence of color-reversing friezes other than at Pirgí is the rafter art of the Maori, in which five of the 17 color-reversing frieze types have been identified in their pure form (although these friezes possess superimposed extra curlicues in a third color). In a fascinating paper, Donnay and Donnay [21] show that if one erases certain selected parts of some of the Maori friezes, then what remains gives rise to four other color-reversing friezes. They conclude that the Maori have regularly added these extra parts for the “mischievous purpose of desymmetrization” [22]. The pottery of the San Ildefonso pueblo in New Mexico has been shown by Crowe and Washburn [23] to include 14 of the 17 color-reversing frieze types—but color-reversal does not form the underlying basis for their art—only a tiny percentage of their friezes possess color-reversal, and most of those that do, possess a third-color background, unlike at Pirgí. Two color-reversing frieze types repeatedly occur among the baskets of the Yurok, Karok and Hupa Indians [24]. One can also find isolated instances of color-reversing Escher-type friezes in a 16th-century Persian rug border, a Kenyan pot, a Fiji tapa and a weaving from Peru [25].

The main point is that at Pirgí the artists have gone much farther than in any other culture in exploring the area of pure dichromatic color-reversing friezes, arriving at a fully developed art form in which all but one of the non-Boring friezes are represented. The format of this art can be understood only by adding the color-reversing analysis described above to the standard seven-group analysis. Not only does the sophistication of the color-reversing analysis clarify the nature of the activities and intentions of the Pirgí artists, but also it serves as an indicator of which frieze types are most likely or unlikely to appear in this art form. Among the 17 color-reversing frieze types, the five Boring Groups are the weakest in that each contains only one symmetry action (in addition to the always-present $r$). Only one of these low-symmetry friezes appears in Pirgí. Only one other of the 17 color-reversal type friezes fails to appear, and it possesses only the three weakest symmetry actions—$m_y$, $g$ and $r'$. The closely
related group that possesses $g'$, $m_h$, and $r'$ also appears very rarely. All other stronger Grecian Groups appear, and those with vertical mirrors appear most. Simply put: For the artists of Pirgí, symmetry appeals, and more symmetry appeals more.

REMARKS

A few friezes possess the well-known optical illusionary property of reversible stability—stared at long enough, a black shape on a white background suddenly shifts, and all one can see is its complementary white shape on a black background. Two examples of this phenomenon can be seen in the frieze in Fig. 5 and the top frieze in Fig. 7.

The rules of the decorative art form invented by the artists of Pirgí are well defined and fairly rigid. It is fascinating in such situations to observe whether some artists are trying to break the rules. The presence of the undulating floral pattern as the topmost frieze in many houses is certainly an expression of freedom from the polygons and circles displayed on the rest of the friezes on the house; curiously, however, such capping friezes in Pirgí have yet to display any color-reversing aspects. On one modern house we found a loud shout for freedom from the straight-line-and-circle Euclidean dictum, in the form of waves that incorporate a color-reversal aspect. There are also recent instances of vertical friezes and of the incorporation of space-filling patterns, indicating that the art of Pirgí is not static, but a living, developing art form. A few of the modern developments are illustrated in Fig. 8. We leave it to the reader to spot the three places where the old rules are being relaxed. And finally, as we left town, we discovered a delightful and unique statue decorated in the style of Pirgí.

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References


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