NOTES

ASYMMETRIES IN THE CONDITIONAL MEAN DYNAMICS OF REAL GNP: ROBUST EVIDENCE

Prasad V. Bidarkota*

Abstract—We investigate asymmetries in the conditional mean dynamics of U.S. GNP. Because the statistical evidence on nonlinearities in the conditional mean could be influenced by the presence of outliers or by a failure to model conditional heteroskedasticity, we explicitly account for outliers by assuming that the innovations are drawn from the stable family, and model time-varying volatility by a GARCH(1,1) process. We also allow for the possibility of long memory in the series with fractional differencing. Our results indicate statistically significant nonlinearities in the conditional mean that persist even after accounting for these features in the data.

I. Introduction

The possible existence of asymmetries in economic fluctuations is being tested extensively using aggregate macroeconomic data. While studies such as Neflic (1984), Brunner (1992, 1997), Beaudry and Koop (1993), Potter (1995), and Ramsey and Rothman (1996) conclude that there are significant asymmetries, others (Falk (1986), Sichel (1989), DeLong and Summers (1986), and Diebold and Rudebusch (1990)) have either failed to confirm these findings or have found only weak evidence supporting them. Moreover, while Tsay (1988) demonstrates that linearity could be rejected by the presence of outliers, Balke and Fomby (1994) and Scheinkman and LeBaron (1989) actually report weakened evidence against linearity in U.S. real GNP data once outliers are taken into account. The latter study also shows a weakening of the evidence against linearity after accounting for conditional heteroskedasticity in this series.

The existence of outliers in real GNP is also demonstrated in Blanchard and Watson (1986), who conclude that fluctuations in economic activity are characterized by a mixture of large and small shocks. That homoskedastic models may not accurately portray this time series is also evidenced in French and Sichel (1993) and Brunner (1992, 1997). Of related interest is the possible characterization of this series as a fractionally integrated process displaying long memory (Sowell, 1992a).

The purpose of this paper is to investigate whether or not asymmetries exist in the conditional mean dynamics of U.S. real GNP, taking into account the possibility of the aforementioned other features in the data. Specifically, we investigate the robustness of the findings of Beaudry and Koop (1993) to the possible presence of outliers, conditional heteroskedasticity, and long memory, and corroborate our findings using other models as well.

Conditional heteroskedasticity can be accounted for with the now well-known GARCH or related class of models and long memory with the fractionally integrated extensions of standard ARIMA (ARFIMA) models. Accounting for outliers is most easily accomplished using leptokuritic distributions.1 While several candidate distributions exist, such as the Student-$t$, we use the stable distributions in this study as these are natural generalizations of Gaussian distributions. Real GNP, being an aggregate of the economywide output, can innocuously be viewed as evolving as an outcome of several individually unimportant sectoral shocks, and the Generalized Central Limit Theorem (Zolotarev (1986, ch. 1)) dictates that the limiting distribution of such a process, if it exists, must belong to the stable class.

Section II presents the general model we use, and section III presents estimation results. Section IV provides additional evidence using alternate models, threshold autoregressions, and the last section presents brief conclusions.

II. Nonlinear ARFIMA-GARCH Model with Stable Errors

The most general model we consider can be represented as

$\Phi(L)(1 - L)^d(Y_t - \mu) = \Omega(L) - 1|CDR_t + \epsilon_t,$

$(1a)$

where

$c_t^n = b_1 c_{t-1}^n + b_2|\epsilon_{t-1}|^n.$

$(1b)$

$\Delta Y_t = 100\Delta \ln \text{GDP}$ is the growth rate of real GNP, $\Omega(L)$ is the information set containing the history of the series up to time $t$, $\mu$ is the mean of the process, $d$ is the differencing parameter that takes real values, $\Phi(L)$ and $\Omega(L)$ are polynomials of orders $p$ and $r$, respectively, in the lag operator $L$ with $\Phi(0) = \Omega(0) = 1$, and $CDR_t$ is the current depth of recession, defined in Beaudry and Koop (1993) as the gap between the current level of output and the economy’s historical maximum level, that is, $CDR_t = \max \{y_t - y_{t-n} - y_t, 0\}.$

A random variable $X$ is said to have a symmetric stable distribution $S_\Delta(\delta,c)$ if its log-characteristic function can be expressed as $\ln E \exp (i \delta X) = i \delta \Delta - |\delta|^c.$

The parameters $c > 0$ and $\delta \in (-\infty, \infty)$ are measures of scale and location, respectively, and $\Delta \in (0, 2]$ is the characteristic exponent governing the tail behavior with a smaller value of $\Delta$ indicating thicker tails. The normal distribution belongs to the symmetric stable family with $\Delta = 2$ and is the only member with finite variance, equal to $2c^2$.

Equation (1b) describes the evolution of the scale of the conditional distribution. It reduces to the familiar GARCH(1,1) process for the conditional variance when $\Delta = 2$ (that is, when shocks are normal).

Consider the case when $\Omega(L) = 1$. Then, equation (1a) reduces to the standard ARIMA model with integer differencing when $d = 0$ (the

1 We do not consider any moving average (MA) terms in the specification of the model. Maximum-likelihoood estimation of mixed ARMA models with stable errors poses a challenge, although the Whittle estimator (Mikosch et al. (1995)) and minimum dispersion estimators (Brockwell and Davis (1991)) have been used in this context.

2 We do not consider any moving average (MA) terms in the specification of the model. Maximum-likelihoood estimation of mixed ARMA models with stable errors poses a challenge, although the Whittle estimator (Mikosch et al. (1995)) and minimum dispersion estimators (Brockwell and Davis (1991)) have been used in this context.
Moreover, it permits recessions to be less or more persistent than those of linear models, and invertible solution to an ARFIMA model requires that 
\[ d \] is order zero and \[ \rho > 1 \] for the theory of fractionally differenced ARMA time series with infinite-variance stable shocks, Kokoszka and Taqqu (1995) show that a unique causal MA(\( \infty \)) representation to an ARFIMA model exists if \( \alpha d + 1 < 1 \). This implies that \( d \) can be positive only when \( \alpha > 1 \). Further, for such a model to be a solution to an AR(\( \infty \)) process requires that \( \alpha > 1 \) and \( \rho < (1 - \alpha)/\alpha \). Consequently, we restrict \( \alpha \) and \( d \) in equation (1) to satisfy these constraints in order to force our estimated models to possess causal and invertible representations.

Abstracting from fractional differencing for a moment (although addition of the nonlinear CDR term into a standard ARMA framework is ad hoc), this model has the virtues of simplicity and parsimony. It nests ARMA models and does not give rise to any nuisance parameters when testing for the significance of the nonlinear terms governing the conditional mean dynamics using the likelihood-ratio (\( LR \)) test. Moreover, it permits recessions to be less or more persistent than those of linear models, and invertible solution to an ARFIMA model requires that \( d \) is order zero and \( \rho > 1 \) for the theory of fractionally differenced ARMA time series with infinite-variance stable shocks, Kokoszka and Taqqu (1995) show that a unique causal MA(\( \infty \)) representation to an ARFIMA model exists if \( \alpha d + 1 < 1 \). This implies that \( d \) can be positive only when \( \alpha > 1 \). Further, for such a model to be a solution to an AR(\( \infty \)) process requires that \( \alpha > 1 \) and \( \rho < (1 - \alpha)/\alpha \). Consequently, we restrict \( \alpha \) and \( d \) in equation (1) to satisfy these constraints in order to force our estimated models to possess causal and invertible representations.

The presence of asymmetries essentially implies that either the innovations are asymmetric or the transmission mechanism is nonlinear, or that innovations are asymmetric and the transmission mechanism is also nonlinear. It would be hard to disentangle the nonlinear effects, if any, of the innovations themselves from the propagation mechanism. Although asymmetric \( \alpha \)-stable distributions exist and are well defined, it is beyond the scope of this paper to determine whether the asymmetries in mean real GNP—if they exist at all—are caused by asymmetric impulses being propagated linearly, or by symmetric impulses being propagated nonlinearly, or by a combination of the two. Our objective here is merely to investigate whether asymmetries, regardless of how they can best be characterized, exist at all in the conditional mean dynamics of real GNP.

There is also a technical reason for restricting ourselves to symmetric stable distributions. Although the stable distribution and density may be evaluated by using Zolotarev’s (1986, p. 74, 78) proper integral representations or by taking the inverse Fourier transform of the characteristic function, McCulloch (1996) has developed a fast numerical approximation to these that applies only in the symmetric case. This approximation has an expected relative density precision of \( 10^{-6} \) for \( \alpha \in [0.84, 2] \). We therefore restrict \( \alpha \) in this range to allow us to use this approximation.

The exact full-information maximum-likelihood (ML) method for estimating ARFIMA models due to Sowell (1992b) is applicable only when the errors are i.i.d. normal. However, Baillie et al. (1996) note that implementing Sowell’s ML procedure for more-complicated models (such as nonnormal or conditionally heteroskedastic models or both as is the case here) is likely to be either computationally extremely demanding or completely intractable. Instead, they use the conditional sum of squares (CSS) estimator, originally proposed in the context of ARFIMA processes by Hosking (1984), to estimate their ARFIMA-GARCH models, with normal or Student-\( t \) errors. In our empirical work that follows in section III and IV, we too use the CSS estimator to estimate the fractionally integrated models. Restricted versions of the models that do not involve fractional differencing are estimated by conditional maximum likelihood.

The CSS estimation of ARFIMA models consists of fitting an ARMA model to the series, \( (1 - L)^p \Delta y_t - \mu \), obtained by expanding the differencing operator, \( (1 - L)^p \), with the binomial expansion and truncating the infinite series at the first available observation. The CSS estimator is discussed in the context of ARMA models by Box and

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**Table 1.** — CDR-AUGMENTED ARFIMA-GARCH MODEL ESTIMATES

<table>
<thead>
<tr>
<th>Model (iv)</th>
<th>Model (iii)</th>
<th>Model (ii)</th>
<th>Model (i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>2 (restricted)</td>
<td>1.69 (0.12)</td>
<td>1.79 (0.09)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.53 (0.19)</td>
<td>0.55 (0.16)</td>
<td>0.56 (0.14)</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>0.30 (0.08)</td>
<td>0.32 (0.06)</td>
<td>0.32 (0.07)</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>0.22 (0.09)</td>
<td>0.19 (0.08)</td>
<td>0.22 (0.08)</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>0.69 (0.05)</td>
<td>0.57 (0.05)</td>
<td>0.95 (0.39)</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>3.8e-3 (8.4e-3)</td>
<td>1.2e-3 (6.7e-3)</td>
<td>3.9e-3 (6.7e-3)</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.07 (0.07)</td>
<td>0.91 (0.06)</td>
<td>0.04 (0.03)</td>
</tr>
<tr>
<td>( \omega_1 )</td>
<td>0.32 (0.09)</td>
<td>0.34 (0.08)</td>
<td>0.26 (0.08)</td>
</tr>
<tr>
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</tr>
</tbody>
</table>

1. \( LR (\alpha = 0) \) is the log-likelihood ratio statistic; standard errors are in parentheses for the parameter estimates; for model (iv), the standard errors in parentheses are robust to normality, obtained using the covariance matrix \( T^{-1} \) when testing for the significance of the nonlinear terms governing the conditional mean dynamics using the likelihood-ratio (\( LR \)) test. 4
2. \( LR (d = 0) \) gives the \( x^2 \) test for normality. The distribution of the test statistic is nonstandard because the null hypothesis lies on the boundary of admissible values for \( \beta \).
3. \( LR (\alpha = 2) \) is a test for linearity. The distribution of the test statistic is nonstandard (see footnote 4 in the main text).
4. \( LR (\alpha = 2) \) is a test for normality. The distribution of the test statistic is nonstandard (see footnote 4 in the main text).
5. \( LR (\alpha = 2) \) is a test for normality. The distribution of the test statistic is nonstandard because the null hypothesis lies on the boundary of admissible values for \( \alpha \).
Jenkins (1976). Its asymptotically normal distribution for the ARFIMA case, when the mean is known and the errors are normal, is derived in Li and McLeod (1986). The CSS procedure is asymptotically equivalent to full MLE. Some properties of the CSS estimator in the context of ARFIMA models, particularly with respect to its bias, are discussed by Baillie et al. They conclude that the CSS estimator performs quite satisfactorily in comparison to Sowell’s (1992b) exact MLE, while being computationally feasible for more-complex models.

III. Estimation Results

We estimate model (i) in equation (1) above and three of its restricted versions using the chain-weighted, seasonally adjusted, quarterly U.S. GNP data spanning the period 1947:1 through 1996:4.\(^5\) Imposing \(d = 0\) gives model (ii). A homoskedastic version of model (ii) is model (iii). Imposing \(\alpha = 2\) on model (iii) gives the Gaussian model (iv).

Summary statistics for the GNP growth rate indicate a sample skewness of \(-0.12\) (\(p\)-value for skewness = 0.76) and a kurtosis measure of 4.04 (\(p\)-value for kurtosis = 3 < 0.01). The Jarque-Bera test easily rejects normality (\(p\)-value < 0.001).\(^6\) Sample autocorrelations at the first, second, and fifth lags appear statistically significant at the 0.10 level using the approximate 1/\(n\) sample size standard errors. A specification search for each of the models (i) through (iv) with lag orders \(p \leq 3\) and \(r \leq 3\) (for parsimony) fails to select any single model consistently using the Akaike and the Schwarz information criteria. However, the parameterization \(p = 2\) and \(r = 1\) is picked by at least one of the two criteria in all four models, and by both criteria in two of the models. The Box-Ljung test fails to indicate any residual serial correlation with this parameterization, although it should be noted that the asymptotic distribution for this test applies only when the second moments of the error distribution are assumed finite.

Estimation results for models (i) through (iv) with the above parameterization are given in table 1. Column 1 gives the estimates for the Beaudry/Koop model with the updated and revised (chain-weighted) data. Accounting for conditional heteroskedasticity increases the estimate of the stable index \(\alpha\) from 1.69 to a more nearly normal value of 1.79, indicating that some of the leptokurtosis is captured by GARCH. A test for normality based on \(\alpha = 2\) rejects easily, however, in all cases using the Monte Carlo small-sample null distribution. (See footnote 5 to the table.)

Homoskedasticity is easily rejected versus the GARCH(1,1) alternative using the test labeled LR (no GARCH).\(^7\) This contrasts with the analysis of Balke and Fomby (1994), who report weakened evidence against homoskedasticity versus GARCH alternatives, once outliers are taken into account. An LR test for ARCH(1) (not shown) is also easily rejected versus this alternative (\(p\)-value = 3.06 < 0.05).

The LR test statistic for a unit root versus the fractional alternative \(d = 0\) is a very weak 0.74. The Monte Carlo simulations in Sowell (1992a) suggest that the true \(p\)-value may in fact be higher than the asymptotic \(\chi^2\) \(p\)-value of 0.39 reported there for this test. Sowell (1992a) and Baillie (1996) point out that GNP data are not sufficiently i.i.d. and the preceding skewness and kurtosis tests are approximate because they fail to reflect the non-i.i.d. nature of the data.

5 The data is taken from table 2A of the Survey of Current Business, May 1997. I thank Pok-sang Lam for bringing my attention to this revised data set and David Brasington for his help in collecting it.

6 As an anonymous referee correctly points out, the \(p\)-values for this test and the preceding skewness and kurtosis tests are approximate because they fail to reflect the non-i.i.d. nature of the data.

7 However, both the Goldfeld-Quandt test and the Box-Ljung test on squared residuals from models (iii) and (iv) fail to reject homoskedasticity at conventional significance levels.

8 More generally, the switch between regimes in this class of models is determined by whether \(\Delta y_{t-2} > s\), and \(l\) and \(s\) are estimated along with other parameters of the model.

9 In general SETAR models (as in footnote 8) when the delay and threshold parameters are estimated, testing the null hypothesis of a single regime results in unidentified nuisance parameters, and standard asymp-
strong, with the p-value for the LR test being better than 0.06 in all cases with two exceptions: SETAR model (i) fails to reject linear mean dynamics marginally at the 0.10 level, and the SETAR model (ii) fails to reject at even the 0.20 level. A more extensive specification search with SETAR models, incorporating GARCH and fractional differencing, could reconcile this discrepancy.

V. Conclusions

Our results indicate that nonlinearities in the conditional mean of real GNP reported in earlier studies are robust to the possible presence of outliers, conditional heteroskedasticity, and long memory in the series. Nonlinearities and nonnormalities beyond what can be accounted for by conditionally heteroskedastic GARCH models with Gaussian innovations are important measures of persistence of shocks to GNP based on linear models are likely to be biased. Given the findings in this paper, real business-cycle theorists may need to do more than merely match the first two moments and the autocorrelations and cross-correlations of different macroeconomic variables in order to validate their theories.

REFERENCES


Table 2.—Switching Regression Models

<table>
<thead>
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</tr>
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<tbody>
<tr>
<td>Number of parameters</td>
<td>8</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>LogL</td>
<td>−269.10</td>
<td>−265.92</td>
<td>−256.07</td>
</tr>
<tr>
<td>LR (one regime)</td>
<td>18.08 (0.001)</td>
<td>16.85 (0.002)</td>
<td>9.64 (0.047)</td>
</tr>
<tr>
<td>LR (α = 2)</td>
<td>6.38 (&lt;0.003)</td>
<td>3.60 (&lt;0.011)</td>
<td>3.02 (&lt;0.013)</td>
</tr>
<tr>
<td>LR (no GARCH)</td>
<td>19.69 (1.2e-4)</td>
<td>0.05 (0.819)</td>
<td></td>
</tr>
<tr>
<td>LR (d = 0)</td>
<td></td>
<td></td>
<td>0.05 (0.819)</td>
</tr>
</tbody>
</table>

Panel 2 — SETAR model

| Number of parameters | 8 | 9 | 12 | 13 |
| LogL | −273.24 | −266.38 | −256.38 | −256.77 |
| LR (one regime) | 9.80 (0.044) | 15.92 (0.003) | 6.07 (0.280) | 7.65 (0.105) |
| LR (α = 2) | 13.72 (<0.001) | 4.64 (<0.003) | 7.44 (<0.002) |
| LR (no GARCH) | 16.05 (0.001) | | | |
| LR (d = 0) | | 3.18 (0.075) |

Notes:
1. As in table 1.
2. LR (one regime) is a test for linear conditional mean dynamics. The joint null hypothesis is \( \mu_1 = \mu_2, \theta_1 = \theta_2, \theta_3 = \theta_5, \) and \( \gamma = 1. \) The \( \chi^2 \) p-values are given in parentheses.


Hansen, B. E., “Inference When a Nuisance Parameter is Not Identified under the Null Hypothesis,” Econometrica 64(2) (1996), 413–430.


INFLATION AND ASYMMETRIC PRICE ADJUSTMENT

Robert A. Buckle and John A. Carlson*

Abstract—Using a unique micro data set, we find pervasive evidence of price asymmetry that is systematically related to inflation. An ordered probit model of pricing by manufacturing, building and merchandising firms shows that inflation: (i) increases the probability of a price increase in response to cost increases and (ii) decreases the probability of a price decrease in response to decreases in demand. Predicted inflation-induced asymmetries also show up for price responses to cost decreases and demand increases but not as overwhelmingly. Similar asymmetries are evident in firm’s expectations of price changes, with a slight optimistic bias relative to actual changes.

I. Introduction

The nature of nominal price rigidity has a crucial influence on the real effects of monetary policy and the characteristics of business cycles. Some economists have argued that these price rigidities are likely to be asymmetric (for example, Tobin (1972)) and some textbooks have captured this idea with a convex aggregate supply curve (for instance, Lipsey (1983, chapter 41)) while Ball and Mankiw (1994) have recently suggested a microfoundational theory for price asymmetry in which price asymmetry depends on inflation. However, to date there has been little empirical research evaluating price asymmetry, and there has been no previous evidence that price asymmetries depend on inflation. The purpose of this paper is to fill that gap.

We make use of a unique micro data set to test for price asymmetries at the firm level and to evaluate the Ball/Mankiw proposition that price asymmetry depends on general inflation. In the conclusion to their paper, Ball and Mankiw comment that “an aspect of our model that might be examined in future empirical work is the relation between price adjustment and inflation” (p. 261). This is the principal objective of this paper.

The data are obtained from a survey of New Zealand firms that provide information about changes to their selling prices, costs, and demand. Each firm’s response to each question can be identified, so we can readily identify those firms that report increases and those that report decreases in prices, costs, and demand in order to compare the relative sensitivity of prices to increases and decreases in costs and demand across different inflation environments. Furthermore, firms can be identified by industry type, and the survey provides information about expected changes to prices, cost, and demand, as well as actual changes to these variables.

The remainder of the paper is structured as follows. Section II describes in more detail the Ball/Mankiw pricing model and how the interaction of menu costs and inflation can induce an asymmetric response to shocks to desired price. Section III explains the survey data used to evaluate these ideas empirically. Section IV describes the way we have arranged the survey data, the estimation procedure, and the results. Section V concludes.

II. Explanations for Price Asymmetries

Some of the most well developed ideas providing microfoundations for nominal price rigidities have followed the “menu cost” approach, as reflected in the collection of papers in Sheshinski and Weiss (1993). A feature of this approach is that, if it is costly to change price, firms will delay changes until the private benefits outweigh the private costs. If there is general inflation, a firm’s real price will automatically fall—thereby possibly offsetting a need to lower its nominal price.

Ball and Mankiw use these ideas to argue that nominal price adjustments are asymmetric. They consider a model, based on the monopolistic competition models of Blanchard and Kiyotaki (1987) and Ball and Romer (1989), which combines elements of time-contingent pricing, in which a firm adjusts prices on a regular time schedule, and state-contingent pricing, in which a firm has the option of changing prices whenever economic circumstances warrant a change. If midway between regular price changes shocks are large enough, the firm will pay a menu cost and make an additional price change. This set-up enables Ball and Mankiw to avoid the complications created by cumulative shocks over several periods and to concentrate on whether or not a firm should change price in response to a single shock.

Formally, let θ be an exogenous shock to a firm’s desired price in the absence of any menu costs, π the general rate of inflation, and C the