NOTES

ASYMMETRIES IN THE CONDITIONAL MEAN DYNAMICS OF REAL GNP: ROBUST EVIDENCE

Prasad V. Bidarkota*

Abstract—We investigate asymmetries in the conditional mean dynamics of U.S. GNP. Because the statistical evidence on nonlinearities in the conditional mean could be influenced by the presence of outliers or by a failure to model conditional heteroskedasticity, we explicitly account for outliers by assuming that the innovations are drawn from the stable family, and model time-varying volatility by a GARCH(1, 1) process. We also allow for the possibility of long memory in the series with fractional differencing. Our results indicate statistically significant nonlinearities in the conditional mean that persist even after accounting for these features in the data.

I. Introduction

The possible existence of asymmetries in economic fluctuations is being tested extensively using aggregate macroeconomic data. While studies such as Neftci (1984), Brunner (1992, 1997), Beaudry and Koop (1993), Potter (1995), and Ramsey and Rothman (1996) conclude that there are significant asymmetries, others (Falk (1986), Sichel (1989), DeLong and Summers (1986), and Diebold and Rudebusch (1990)) have either failed to confirm these findings or have found only weak evidence supporting them. Moreover, while Tsay (1988) demonstrates that linearity could be rejected by the presence of outliers, Balke and Fomby (1994) and Scheinkman and LeBaron (1989) actually report weakened evidence against linearity in U.S. real GNP data once outliers are taken into account. The latter study also shows a weakening of the evidence against linearity after accounting for conditional heteroskedasticity in this series.

The existence of outliers in real GNP is also demonstrated in Blanchard and Watson (1986), who conclude that fluctuations in economic activity are characterized by a mixture of large and small shocks. That homoskedastic models may not accurately portray this time series is also evidenced in French and Sichel (1993) and Brunner (1992, 1997). Of related interest is the possible characterization of this series as a fractionally integrated process displaying long memory (Sowell, 1992a).

The purpose of this paper is to investigate whether or not asymmetries exist in the conditional mean dynamics of U.S. real GNP, taking into account the possibility of the aforementioned other features in the data. Specifically, we investigate the robustness of the findings of Beaudry and Koop (1993) to the possible presence of outliers, conditional heteroskedasticity, and long memory, and corroborate our findings using other models as well.

Conditional heteroskedasticity can be accounted for with the now well-known GARCH or related class of models and long memory with the fractionally integrated extensions of standard ARIMA (ARFIMA) models. Accounting for outliers is most easily accomplished using leptokurtic distributions.[1] While several candidate distributions exist, such as the Student-*t*, we use the stable distributions in this study as these are natural generalizations of Gaussian distributions. Real GNP, being an aggregate of the economywide output, can innocuously be viewed as evolving as an outcome of several individually unimportant sectoral shocks, and the Generalized Central Limit Theorem (Zolotarev (1986, ch. 1)) dictates that the limiting distribution of such a process, if it exists, must belong to the stable class.

Section II presents the general model we use, and section III presents estimation results. Section IV provides additional evidence using alternate models, threshold autoregressions, and the last section provides brief conclusions.

II. Nonlinear ARFIMA-GARCH Model with Stable Errors

The most general model we consider can be represented as

$$\Phi(L)(1-L)^d(\Delta y_t - \mu) = [\Omega(L) - 1]\mathrm{CDR}_t + \epsilon_t, \qquad \epsilon_t | I_{t-1} \sim z_t c_t, \quad z_t \sim \text{i.i.d. } S_\alpha(0, 1) \tag{1a}$$

where

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 |\epsilon_{t-1}|^\alpha. \tag{1b}$$

$\Delta y_t \equiv 100\Delta \ \ln \mathrm{GNP}_t$ is the growth rate of real GNP, I_t is the information set containing the history of the series up to time t, μ is the mean of the process, d is the differencing parameter that takes real values, $\Phi(.)$ and $\Omega(.)$ are polynomials of orders p and r, respectively, in the lag operator L with $\Phi(0) = \Omega(0) = 1$, and *CDR* is the current depth of recession, defined in Beaudry and Koop (1993) as the gap between the current level of output and the economy's historical maximum level, that is, $CDR_t = \max\{y_{t-j}\}_{j\geq 0} - y_t$.[2]

A random variable X is said to have a symmetric stable distribution $S_\alpha(\delta, c)$ if its log-characteristic function can be expressed as $\ln E \exp(iXt) = i\delta t - |ct|^\alpha$.

The parameters $c > 0$ and $\delta \in (-\infty, \infty)$ are measures of scale and location, respectively, and $\alpha \in (0, 2]$ is the characteristic exponent governing the tail behavior with a smaller value of α indicating thicker tails. The normal distribution belongs to the symmetric stable family with $\alpha = 2$ and is the only member with finite variance, equal to $2c^2$.

Equation (1b) describes the evolution of the scale of the conditional distribution. It reduces to the familiar GARCH(1, 1) process for the conditional variance when $\alpha = 2$ (that is, when shocks are normal).

Consider the case when $\Omega(L) = 1$. Then, equation (1a) reduces to the standard ARIMA model with integer differencing when $d = 0$ (the

Received for publication May 19, 1997. Revision accepted for publication February 22, 1999.

* Kansas State University

This paper is based on the fourth chapter of my Ph.D. dissertation. I would like to thank J. H. McCulloch (without whose ideas this project would not have been initiated), G. S. Maddala, M. J. Silvapulle, D. Prentice, the seminar participants at Ohio State, Kansas State, University of Colorado—Denver, La Trobe Universities, and the SNDE symposium participants at NYU for helpful suggestions. Comments from J. Stock (editor) and three anonymous referees led to substantial improvements in the paper. Financial support from Ohio State and La Trobe Universities is gratefully acknowledged. All remaining errors and omissions are my sole responsibility.

[1] While GARCH models may account for some of the outliers, it remains to be investigated whether GARCH-normal models can account for all the observed leptokurtosis. It is conceivable that real GNP is better described by a conditionally heteroskedastic model driven by leptokurtic shocks.

[2] We do not consider any moving average (*MA*) terms in the specification of the model. Maximum-likelihood estimation of mixed ARMA models with stable errors poses a challenge, although the Whittle estimator (Mikosch et al. (1995)) and minimum dispersion estimators (Brockwell and Davis (1991)) have been used in this context.

The Review of Economics and Statistics, February 2000, 82(1): 153–157

TABLE 1.—CDR-AUGMENTED ARFIMA-GARCH MODEL ESTIMATES

$(1 - \phi_1 L - \phi_2 L^2)(1 - L)^d(\Delta y_t - \mu) = \omega_1.\text{CDR}_{t-1} + \epsilon_t, \quad \epsilon_t | I_{t-1} \sim z_t c_t, \quad z_t \sim \text{i.i.d. } S_\alpha(0, 1) \quad c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 |\epsilon_{t-1}|^\alpha$

	Model (iv)	Model (iii)	Model (ii)	Model (i)
α	2 (restricted)	1.69 (0.12)	1.79 (0.09)	1.81 (0.12)
μ	0.53 (0.19)	0.55 (0.16)	0.56 (0.14)	0.65 (0.10)
ϕ_1	0.30 (0.08)	0.32 (0.06)	0.32 (0.07)	0.56 (0.22)
ϕ_2	0.22 (0.09)	0.19 (0.08)	0.22 (0.08)	0.22 (0.09)
c_0	0.69 (0.05)	0.57 (0.05)	0.95 (0.39)	1.02 (0.37)
b_1			3.8e-3 (8.4e-3)	1.2e-3 (6.7e-3)
b_2			0.89 (0.07)	0.91 (0.06)
b_3			0.05 (0.03)	0.04 (0.03)
ω_1	0.32 (0.09)	0.34 (0.08)	0.26 (0.08)	0.27 (0.09)
d				−0.25 (0.22)
log$L$	−272.97	−267.35	−256.54	−256.17
LR ($\omega_1 = 0$)	10.35 (<0.001)	13.98 (<0.001)	8.70 (<0.001)	8.84 (<0.001)
LR ($\alpha = 2$)		11.23 (<0.003)	6.78 (<0.003)	6.50 (<0.003)
LR (no GARCH)			21.62 (7.8e-5)	
LR ($d = 0$)				0.74 (0.39)

1. log L is the log-likelihood; LR is the likelihood-ratio statistic; standard errors are in parentheses for the parameter estimates; for model (iv), the standard errors in parentheses are robust to normality, obtained using the covariance matrix $T^{-1}(I_2 I_{op}^{-1} I_2)^{-1}$, where I_2 is the Hessian, I_{op} is the outer product of the gradient, and T the sample size.

2. LR ($d = 0$) gives the χ_1^2 p-values in parentheses.

3. LR ($\omega_1 = 0$) is a test for linear conditional mean. The distribution of the test statistic is nonstandard (see footnote 4 in the main text). p-values obtained using 1,000 Monte Carlo replications with an estimated homoskedastic linear Gaussian AR(2) model are in parentheses.

4. LR (no GARCH) tests for homoskedasticity using a test of $b_2 = b_3 = 0$. Under this null, b_1 and c_0 are trivial transforms of one another. Therefore, it is not clear whether the LR statistic has an asymptotic χ^2 distribution with two or three degrees of freedom. Conservative χ_3^2 p-values are in parentheses.

5. LR ($\alpha = 2$) is a test for normality. The distribution of the test statistic is nonstandard because the null hypothesis lies on the boundary of admissible values for α. p-values obtained using 1,000 Monte Carlo replications with an estimated homoskedastic linear AR(2) model are in parentheses.

unit root case), whereas $d = -1$ indicates overdifferencing or trend stationarity.

In the case of Gaussian errors, the existence of a stationary causal and invertible solution to an ARFIMA model requires $|d| < 0.5$. (See Brockwell and Davis (1991).) With α-stable shocks, Kokoszka and Taqqu (1995) show that a unique causal $MA(\infty)$ representation to an ARFIMA model exists if $\alpha(d - 1) < -1$.[3] This implies that d can be positive only when $\alpha > 1$. Further, for such a model to be a solution to an AR(∞) process requires that $\alpha > 1$ and $|d| < (1 - 1/\alpha)$. Consequently, we restrict α and d in equation (1) to satisfy these constraints in order to force our estimated models to possess causal and invertible representations.

Abstracting from fractional differencing for a moment (although addition of the nonlinear *CDR* term into a standard ARMA framework is ad hoc), this model has the virtues of simplicity and parsimony. It nests ARMA models and does not give rise to any nuisance parameters when testing for the significance of the nonlinear terms governing the conditional mean dynamics using the likelihood-ratio (LR) test.[4] Moreover, it permits recessions to be less or more persistent than expansions, depending on the parameter estimates. For instance, when $\Phi(.)$ is of order zero and $\Omega(.)$ is order one, a positive ω_1 (refer to table 1 for the definition of ω_1) implies that negative shocks are less persistent, whereas a negative ω_1 implies the opposite. $\omega_1 = 0$ reduces to a random walk model with drift.

The presence of asymmetries essentially implies that either the innovations are asymmetric but the impulse transmission mechanism linear, or that innovations are symmetric but the transmission mechanism is nonlinear, or that innovations are asymmetric and the transmission mechanism is also nonlinear. It would be hard to disentangle the nonlinear effects, if any, of the innovations themselves from the propagation mechanism. Although asymmetric α-stable distributions exist and are well defined, it is beyond the scope of this work to determine whether the asymmetries in mean real GNP—if they exist at all—are caused by asymmetric impulses being propagated linearly, or by symmetric impulses being propagated nonlinearly, or by a combination of the two. Our objective here is merely to investigate whether asymmetries, regardless of how they can best be characterized, exist at all in the conditional mean dynamics of real GNP.

There is also a technical reason for restricting ourselves to symmetric stable distributions. Although the stable distribution and density may be evaluated by using Zolotarev's (1986, p. 74, 78) proper integral representations or by taking the inverse Fourier transform of the characteristic function, McCulloch (1996) has developed a fast numerical approximation to these that applies only in the symmetric case. This approximation has an expected relative density precision of 10^{-6} for $\alpha \in [0.84, 2]$. We therefore restrict α in this range to allow us to use this approximation.

The exact full-information maximum-likelihood (ML) method for estimating ARFIMA models due to Sowell (1992b) is applicable only when the errors are i.i.d. normal. However, Baillie et al. (1996) note that implementing Sowell's ML procedure for more-complicated models (such as nonnormal or conditionally heteroskedastic models or both as is the case here) is likely to be either computationally extremely demanding or completely intractable. Instead, they use the conditional sum of squares (CSS) estimator, originally proposed in the context of ARFIMA processes by Hosking (1984), to estimate their ARFIMA-GARCH models, with normal or Student-*t* errors. In our empirical work that follows in section III and IV, we too use the CSS estimator to estimate the fractionally integrated models. Restricted versions of the models that do not involve fractional differencing are estimated by conditional maximum likelihood.

The CSS estimation of ARFIMA models consists of fitting an ARMA model to the series, $(1 - L)^{\text{d}}(\Delta y_t - \mu)$, obtained by expanding the differencing operator, $(1 - L)^d$, with the binomial expansion and truncating the infinite series at the first available observation. The CSS estimator is discussed in the context of ARMA models by Box and

[3] Since long-memory ARFIMA models are frequently defined in terms of the rate of decay of their autocovariances, their extension to infinite-variance stable shocks is not immediate. See Kokoszka and Taqqu (1995) for the theory of fractionally differenced ARMA time series with infinite-variance stable innovations.

[4] However, Hess and Iwata (1997) show that the asymptotic distribution of the *t*-test for the significance of the nonlinear *CDR* term in model (1a) is nonstandard, both in the case when the dependent variable is nonstationary (that is, integrated of order one $I(1)$) and when it is stationary ($I(0)$).

Jenkins (1976). Its asymptotically normal distribution for the ARFIMA case, when the mean is known and the errors are normal, is derived in Li and McLeod (1986). The CSS procedure is asymptotically equivalent to full MLE. Some properties of the CSS estimator in the context of ARFIMA models, particularly with respect to its bias, are discussed by Baillie et al. They conclude that the CSS estimator performs quite satisfactorily in comparison to Sowell's (1992b) exact MLE, while being computationally feasible for more-complex models.

III. Estimation Results

We estimate model (i) in equation (1) above and three of its restricted versions using the chain-weighted, seasonally adjusted, quarterly U.S. GNP data spanning the period 1947:1 through 1996:4.[5] Imposing $d = 0$ gives model (ii). A homoskedastic version of model (ii) is model (iii). Imposing $\alpha = 2$ on model (iii) gives the Gaussian model (iv).

Summary statistics for the GNP growth rate indicate a sample skewness of -0.12 (p-value for skewness = 0 is 0.76) and a kurtosis measure of 4.04 (p-value for kurtosis $= 3 < 0.01$). The Jarque-Bera test easily rejects normality (p-value < 0.001).[6] Sample autocorrelations at the first, second, and fifth lags appear statistically significant at the 0.10 level using the approximate $1/\sqrt{\text{sample size}}$ standard errors. A specification search for each of the models (i) through (iv) with lag orders $p \leq 3$ and $r \leq 3$ (for parsimony) fails to select any single model consistently using the Akaike and the Schwarz information criteria. However, the parameterization $p = 2$ and $r = 1$ is picked by at least one of the two criteria in all four models, and by both criteria in two of the models. The Box-Ljung test fails to indicate any residual serial correlation with this parameterization, although it should be noted that the asymptotic distribution for this test applies only when the second moments of the error distribution are assumed finite.

Estimation results for models (i) through (iv) with the above parameterization are given in table 1. Column 1 gives the estimates for the Beaudry/Koop model with the updated and revised (chain-weighted) data. Accounting for conditional heteroskedasticity increases the estimate of the stable index α from 1.69 to a more nearly normal value of 1.79, indicating that some of the leptokurtosis is captured by GARCH. A test for normality based on $\alpha = 2$ rejects easily, however, in all cases using the Monte Carlo small-sample null distribution. (See footnote 5 to the table.)

Homoskedasticity is easily rejected versus the GARCH(1,1) alternative using the test labeled LR (no GARCH).[7] This contrasts with the analysis of Balke and Fomby (1994), who report weakened evidence against homoskedasticity versus GARCH alternatives, once outliers are taken into account. An LR test for ARCH(1) (not shown) is also easily rejected versus this alternative (p-value $= 3.0e - 5$).

The LR test statistic for a unit root versus the fractional alternative LR ($d = 0$) is a very weak 0.74. The Monte Carlo simulations in Sowell (1992a) suggest that the true p-value may in fact be higher than the asymptotic χ^2_1 p-value of 0.39 reported there for this test. Sowell (1992a) and Baillie (1996) point out that GNP data are not sufficiently informative about the nature of its long-run properties. The likelihood surface for d is relatively flat and typically includes both $d = 0$ and $d = -1$, and so it is hard to say whether the GNP series is best described as a trend stationary, unit root, or a fractionally differenced process.

The LR test for linear conditional mean dynamics LR ($\omega_1 = 0$) easily rejects using all four models. As the editor points out, the significance levels for this test are to be interpreted cautiously because a previous specification search has been conducted to test for the significance of the CDR term by Beaudry and Koop (1993). Rejecting linearities in the conditional mean, after accounting for outliers, conflicts with the results of Balke and Fomby (1994) and Scheinkman and LeBaron (1989). Rejection of linearities in the conditional mean, after accounting for conditional heteroskedasticity, again conflicts with the findings in the latter study. The positive point estimate for ω_1 indicates, as in Beaudry and Koop (1993) and Brannas and De Gooijer (1994), that negative shocks are less persistent than positive shocks.

IV. Switching autoregressions

To explore the robustness of our findings to alternate representations of the nonlinear conditional mean dynamics, we now turn to switching or threshold autoregressive models. The most general model we consider in this class is

$$\begin{aligned}&\Phi_1(L)(1-L)^d(\Delta y_t - \mu_1) = \epsilon_t,\\ &\quad \epsilon_t \vert I_{t-1} \sim z_t c_t, \qquad z_t \sim \text{i.i.d. } S_\alpha(0,1) \qquad \text{in regime 1}\end{aligned} \tag{2a}$$

$$\begin{aligned}&\Phi_2(L)(1-L)^d(\Delta y_t - \mu_2) = \epsilon_t,\\ &\quad \epsilon_t \vert I_{t-1} \sim z_t \gamma c_t, \qquad z_t \sim \text{i.i.d. } S_\alpha(0,1) \qquad \text{in regime 2}\end{aligned} \tag{2b}$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 \vert \epsilon_{t-1}/\gamma \vert^\alpha \tag{2c}$$

We estimate two versions of these models. In the first version, the model is in regime 1 when $\text{CDR}_{t-1} = 0$ and in regime 2 when $\text{CDR}_{t-1} > 0$. This type of switching regression model (but a Gaussian homoskedastic version with no fractional differencing) has been estimated by Beaudry and Koop (1993). In the second version, termed *self-exciting threshold autoregressions* (SETAR models), the model is in regime 1 when $\Delta y_{t-2} > 0$ and in regime 2 when $\Delta y_{t-2} \leq 0$.[8] A heteroskedastic version of such a model, without GARCH but with variances that differ across regimes but are otherwise constant within each regime and without fractional differencing, has been estimated by Potter (1995) and Brunner (1997). As in these latter two studies, we parameterize $\Phi_1(.)$ and $\Phi_2(.)$ to be of order 2.

A summary of the main results from estimating these models is reported in table 2. Panel 1 reports the summary statistics for the CDR switching model and panel 2 for the SETAR model. Most inferences on the null hypotheses of no GARCH, on a unit root and on normality remain qualitatively unchanged from the previous section. The sole exception is the test for $d = 0$ with the SETAR model for which the p-value is now 0.075. A test for linearity in the conditional mean dynamics can be formulated as a test for a single regime in threshold autoregressions.[9,10] Evidence against linear mean dynamics remains

[5] The data is taken from table 2A of the Survey of Current Business, May 1997. I thank Pok-sang Lam for bringing my attention to this revised data set and David Brasington for his help in collecting it.

[6] As an anonymous referee correctly points out, the p-values for this test and the preceding skewness and kurtosis tests are approximate because they fail to reflect the non-i.i.d. nature of the data.

[7] However, both the Goldfeld-Quandt test and the Box-Ljung test on squared residuals from models (iii) and (iv) fail to reject homoskedasticity at conventional significance levels.

[8] More generally, the switch between regimes in this class of models is determined by whether $\Delta y_{t-1} > s$, and l and s are estimated along with other parameters of the model.

[9] In general SETAR models (as in footnote 8) when the delay and threshold parameters are estimated, testing the null hypothesis of a single regime results in unidentified nuisance parameters, and standard asymp-

TABLE 2.—SWITCHING REGRESSION MODELS

$$(1 - \phi_{11}L - \phi_{12}L^2)(1 - L)^d(\Delta y_t - \mu_1) = \epsilon_t, \quad \epsilon_t|I_{t-1} \sim z_t c_t, \quad z_t \sim \text{i.i.d. } S_\alpha(0,1) \text{ in regime 1}$$

$$(1 - \phi_{21}L - \phi_{22}L^2)(1 - L)^d(\Delta y_t - \mu_2) = \epsilon_t, \quad \epsilon_t|I_{t-1} \sim z_t \gamma c_t, \quad z_t \sim \text{i.i.d. } S_\alpha(0,1) \text{ in regime 2}$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3|\epsilon_{t-1}/\gamma|^\alpha$$

	Model (iv)	Model (iii)	Model (ii)	Model (i)
Panel 1 − CDR switching				
Number of parameters	8	9	12	13
Log$L$	−269.10	−265.92	−256.07	−256.05
LR (one regime)	18.08 (0.001)	16.85 (0.002)	9.64 (0.047)	9.10 (0.059)
LR ($\alpha = 2$)		6.38 (<0.003)	3.60 (<0.011)	3.02 (<0.013)
LR (no GARCH)			19.69 (1.2e-4)	
LR ($d = 0$)				0.05 (0.819)
Panel 2 − SETAR model				
Number of parameters	8	9	12	13
Log$L$	−273.24	−266.38	−258.36	−256.77
LR (one regime)	9.80 (0.044)	15.92 (0.003)	5.07 (0.280)	7.65 (0.105)
LR ($\alpha = 2$)		13.72 (<0.001)	6.46 (<0.003)	7.44 (<0.002)
LR (no GARCH)			16.05 (0.001)	
LR ($d = 0$)				3.18 (0.075)

Notes:
1. As in table 1.
2. LR (one regime) is a test for linear conditional mean dynamics. The joint null hypothesis is $\mu_1 = \mu_2$, $\phi_{11} = \phi_{21}$, $\phi_{12} = \phi_{22}$, and $\gamma = 1$. The χ^2_4 p-values are given in parentheses.

strong, with the p-value for the LR test being better than 0.06 in all cases with two exceptions: SETAR model (i) fails to reject linear mean dynamics marginally at the 0.10 level, and the SETAR model (ii) fails to reject at even the 0.20 level. A more extensive specification search with SETAR models, incorporating GARCH and fractional differencing, could reconcile this discrepancy.

V. Conclusions

Our results indicate that nonlinearities in the conditional mean of real GNP reported in earlier studies are robust to the possible presence of outliers, conditional heteroskedasticity, and long memory in the series. Nonlinearities and nonnormalities beyond what can be accounted for by conditionally heteroskedastic GARCH models with Gaussian innovations are important. Measures of persistence of shocks to GNP based on linear models are likely to be biased. Given the findings in this paper, real business-cycle theorists may need to do more than merely match the first two moments and the autocorrelations and cross-correlations of different macroeconomic variables in order to validate their theories.

totic distribution theory does not go through (Hansen (1996)). Here, because we do not estimate these parameters but instead set $l = 2$ and $s = 0$ in accordance with the findings in previous studies, our tests do not suffer from this problem. In any case, Monte Carlo simulations with the SETAR model (iii) suggest that our inferences hold qualitatively even with these small-sample critical values.

[10] The null hypothesis for this test is the joint hypothesis $\mu_1 = \mu_2$, the corresponding coefficients in the lag polynomials $\Phi_1(.)$ and $\Phi_2(.)$ are equal, and $\gamma = 1$. In the class of models considered under the alternative hypothesis where regime switching is governed by the CDR term, this test is still standard (cf. with footnote 4). See Hess and Iwata (1997), footnote 4, for the validity of this test.

REFERENCES

Baillie, R. T., ‘‘Long Memory Processes and Fractional Integration in Econometrics,’’ *Journal of Econometrics (Annals)* 73(1) (1996), 5–59.

Baillie, R. T., C. F. Chung, and M. A. Tieslau, ‘‘Analysing Inflation by the Fractionally Integrated ARFIMA-GARCH Model,’’ *Journal of Applied Econometrics* 11 (1996), 23–40.

Balke, N. S., and T. B. Fomby, ‘‘Large Shocks, Small Shocks, and Economic Fluctuations: Outliers in Macroeconomic Time Series,’’ *Journal of Applied Econometrics* 9 (1994), 181–200.

Beaudry, P., and G. Koop, ‘‘Do Recessions Permanently Change Output?’’ *Journal of Monetary Economics* 31 (1993), 149–63.

Blanchard, O. J., and M. W. Watson, ‘‘Are Business Cycles All Alike?’’ in R. J. Gordon (Ed.), *The American Business Cycle: Continuity and Change* (University of Chicago Press, 1986), 123–79.

Box, G. E. P., and G. M. Jenkins, *Time Series Analysis, Forecasting and Control* (San Francisco: Holden-Day, 1976).

Brannas, K., and J. G. De Gooijer, ‘‘Autoregressive-Asymmetric Moving Average Models for Business Cycle Data,’’ Journal of Forecasting 13 (1994), 529–544.

Brockwell, P. J., and R. A. Davis, *Time Series: Theory and Methods* (New York: Springer-Verlag, 1991).

Brunner, A. D., ‘‘Conditional Asymmetries in Real GNP: A Seminonparametric Approach,’’ Journal of Business and Economic Statistics 10(1) (1992), 65–72.

———, ‘‘On the Dynamic Properties of Asymmetric Models of Real GNP,’’ this REVIEW 79 (1997), 321–326.

DeLong, J. B., and L. H. Summers, ‘‘Are Business Cycles Symmetrical?’’ in R. J. Gordon (Ed.), *The American Business Cycle: Continuity and Change* (University of Chicago Press, 1986), 166–179.

Diebold, F. X., and G. D. Rudebusch, ‘‘A Nonparametric Investigation of Duration Dependence in The American Business Cycle,’’ *Journal of Political Economy* 98 (1990), 596–616.

Falk, B., ‘‘Further Evidence on the Asymmetric Behavior of Economic Time Series over the Business Cycle,’’ *Journal of Political Economy* 94(5) (1986), 1096–1109.

French, M. W., and D. E. Sichel, ‘‘Cyclical Patterns in the Variance of Economic Activity,’’ *Journal of Business and Economic Statistics* 11(1) (1993), 113–119.

Hansen, B. E., ‘‘Inference When a Nuisance Parameter is not Identified under the Null Hypothesis,’’ *Econometrica* 64(2) (1996), 413–430.

Hess, G. D., and S. Iwata, ‘‘Asymmetric Persistence in GDP? A deeper Look at Depth,’’ *Journal of Monetary Economics* 40 (1997), 535–554.

Hosking, J. R. M., ‘‘Modeling Persistence in Hydrological Time Series Using Fractional Differencing,’’ *Water Resources Research* 20(12) (1984), 1898–1908.

Kokoszka, P. S., and M. S. Taqqu, ''Fractional ARIMA with Stable Innovations,'' *Stochastic Processes and their Applications* 60 (1995), 19–47.
Li, W. K., and A. I. McLeod, ''Fractional Time Series Modeling,'' *Biometrika* 73(1) (1986), 217–221.
McCulloch, J. H., ''Numerical Approximation of the Symmetric Stable Distribution and Density,'' in R. Adler, R. Feldman, and M. S. Taqqu (Eds.), *A Practical Guide to Heavy Tails: Statistical Techniques for Analyzing Heavy Tailed Distributions* (Boston: Birkhauser, 1996).
Mikosch, T., T. Gadrich, C. Kluppelberg, and R. J. Adler, ''Parameter Estimation for ARMA Models with Infinite Variance Innovations,'' *Annals of Statistics* 23 (1995), 305–326.
Neftci, S. N., ''Are Economic Time Series Asymmetric over the Business Cycle?'' *Journal of Political Economy* 92(2) (1984), 307–328.
Potter, S. M., ''A Non-Linear Approach to U.S. GNP,'' *Journal of Applied Econometrics* 10 (1995), 109–125.
Ramsey, J. B., and P. Rothman, "Time Irreversibility and Business Cycle Asymmetry," *Journal of Money, Credit, and Banking* 28 (1996), 1–21.
Scheinkman, J. A., and B. LeBaron, ''Non-Linear Dynamics and GNP Data,'' in W. A. Barnett et al. (Eds.), *Economic Complexity: Chaos, Sunspots, Bubbles, and Non-linearity* (Cambridge: Cambridge University Press, 1989), 213–227.
Sichel, D. E., ''Are Business Cycles Asymmetric? A Correction,'' *Journal of Political Economy* 97(5) (1989), 1255–1260.
Sowell, F., ''Modeling Long-Run Behavior with the Fractional ARIMA Model,'' *Journal of Monetary Economics* 29 (1992a), 277–302.
———, ''Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models,'' Journal of Econometrics 53 (1992b), 165–188.
Tsay, R. S., ''Non-Linear Time Series Analysis of Blowfly Population,'' *Journal of Time Series Analysis* 9(3) (1988), 247–263.
Zolotarev, V. M., One-Dimensional Stable Laws (American Mathematical Society, 1986 translation of *Odnomernye Ustoichivye Raspredeleniia,* NAUKA, Moscow, 1983).

INFLATION AND ASYMMETRIC PRICE ADJUSTMENT

Robert A. Buckle and John A. Carlson*

Abstract—Using a unique micro data set, we find pervasive evidence of price asymmetry that is systematically related to inflation. An ordered probit model of pricing by manufacturing, building and merchandising firms shows that inflation: (i) increases the probability of a price increase in response to cost increases and (ii) decreases the probability of a price decrease in response to decreases in demand. Predicted inflation-induced asymmetries also show up for price responses to cost decreases and demand increases but not as overwhelmingly. Similar asymmetries are evident in firm's expectations of price changes, with a slight optimistic bias relative to actual changes.

I. Introduction

The nature of nominal price rigidity has a crucial influence on the real effects of monetary policy and the characteristics of business cycles. Some economists have argued that these price rigidities are likely to be asymmetric (for example, Tobin (1972)) and some textbooks have captured this idea with a convex aggregate supply curve (for instance, Lipsey (1983, chapter 41)) while Ball and Mankiw (1994) have recently suggested a microtheoretical foundation for price asymmetry in which price asymmetry depends on inflation. However, to date there has been little empirical research evaluating price asymmetry, and there has been no previous evidence that price asymmetries depend on inflation. The purpose of this paper is to fill that gap.

We make use of a unique micro data set to test for price asymmetries at the firm level and to evaluate the Ball/Mankiw proposition that price asymmetry depends on general inflation. In the conclusion to their paper, Ball and Mankiw comment that ''an aspect of our model that might be examined in future empirical work is the relation between price adjustment and inflation'' (p. 261). This is the principal objective of this paper.

The data are obtained from a survey of New Zealand firms that provide information about changes to their selling prices, costs, and demand. Each firm's response to each question can be identified, so we can readily identify those firms that report increases and those that report decreases in prices, costs, and demand in order to compare the relative sensitivity of prices to increases and decreases in costs and demand across different inflation environments. Furthermore, firms can be identified by industry type, and the survey provides information about expected changes to prices, cost, and demand, as well as actual changes to these variables.

The remainder of the paper is structured as follows. Section II describes in more detail the Ball/Mankiw pricing model and how the interaction of menu costs and inflation can induce an asymmetric response to shocks to desired price. Section III explains the survey data used to evaluate these ideas empirically. Section IV describes the way we have arranged the survey data, the estimation procedure, and the results. Section V concludes.

II. Explanations for Price Asymmetries

Some of the most well developed ideas providing microfoundations for nominal price rigidities have followed the ''menu cost'' approach, as reflected in the collection of papers in Sheshinski and Weiss (1993). A feature of this approach is that, if it is costly to change price, firms will delay changes until the private benefits outweigh the private costs. If there is general inflation, a firm's real price will automatically fall—thereby possibly offsetting a need to lower its nominal price.

Ball and Mankiw use these ideas to argue that nominal price adjustments are asymmetric. They consider a model, based on the monopolistic competition models of Blanchard and Kiyotaki (1987) and Ball and Romer (1989), which combines elements of time-contingent pricing, in which a firm adjusts prices on a regular time schedule, and state-contingent pricing, in which a firm has the option of changing prices whenever economic circumstances warrant a change. If midway between regular price changes shocks are large enough, the firm will pay a menu cost and make an additional price change. This set-up enables Ball and Mankiw to avoid the complications created by cumulative shocks over several periods and to concentrate on whether or not a firm should change price in response to a single shock.

Formally, let θ be an exogenous shock to a firm's desired price in the absence of any menu costs, π the general rate of inflation, and C the

Received for publication June 2, 1997. Revision accepted for publication March 2, 1999.

* Victoria University of Wellington and Purdue University, respectively.
We would like to thank two anonymous referees for their helpful comments.