

NOTES

MEASURING THE NATURAL RATE OF INTEREST

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Abstract—The natural rate of interest—the real interest rate consistent with output equaling its natural rate and stable inflation—plays a central role in macroeconomic theory and monetary policy. Estimation of the natural rate of interest, however, has received little attention. We apply the Kalman filter to estimate jointly time-varying natural rates of interest and output and trend growth. We find a close link between the natural rate of interest and the trend growth rate, as predicted by theory. Estimates of the natural rate of interest, however, are very imprecise and subject to considerable real-time measurement error.

I. Introduction

SINCE Wicksell (1936), the natural rate of interest—the real short-term interest rate consistent with output equaling its natural rate and constant inflation—has played a central role in macroeconomic and monetary theory. The natural or “equilibrium” real interest rate provides a benchmark for measuring the stance of monetary policy, with policy expansionary (contractionary) if the short-term real interest rate lies below (above) the natural rate. This role is illustrated clearly in monetary policy rules such as the Taylor (1993) rule, according to which the real interest rate exceeds the natural rate when inflation exceeds its target rate and vice versa, all else equal. Given the central position of the natural rate in the policy rule literature, the problem of real-time estimation of the natural rate of interest has received surprisingly little attention [exceptions include Rudebusch (2001) and Orphanides and Williams (2002)].¹ In this note, we address this issue by jointly estimating the natural rates of interest and output and the trend growth over the past 40 years of U.S. data, using the Kalman filter. A key finding is that estimates of the time-varying natural rate of interest, like those of the natural rate of unemployment and output (Staiger, Stock, & Watson, 1997; Orphanides & van Norden, 2002) are quite imprecise.

II. An Empirical Framework for Estimating the Natural Rate

Our basic approach to estimating the natural rate of interest is closely related to that of Staiger et al. (1997), Gordon (1998), and Laubach (2001), who apply the Kalman filter to estimate the natural

rate of unemployment (NAIRU).² We compare our Kalman filter estimates with those based on univariate methods designed to extract the lower-frequency component from a time series.

As is clear from its definition, the natural rate of interest is intimately related to the level of the natural rate of output, itself an unobserved variable. Moreover, economic theory implies that the natural rate of interest varies over time in response to shifts to preferences and the growth rate of output. Household intertemporal utility maximization yields the relationship between the real rate and growth:

$$r = \frac{1}{\sigma} g_c + \theta, \quad (1)$$

where r is the real interest rate, σ is the intertemporal elasticity of substitution in consumption, g_c is the growth rate of per capita consumption, and θ is the rate of time preference. Oliner and Sichel (2000) and Roberts (2001) provide evidence of shifts in the trend growth rate in the United States, suggesting one source of persistent movements in the natural rate of interest; changes in preferences and fiscal policy likely contribute as well to time variation in the natural rate. Based on the theoretical link between the natural rate of interest and the growth rate noted above, we assume that the law of motion for the natural rate of interest, denoted by r^* , is given by

$$r_t^* = c g_t + z_t, \quad (2)$$

where g_t is the trend growth rate of the natural rate of output and z_t captures other determinants of r^* , such as households' rate of time preference.

Given that the key determinants of the natural rate of interest are unobserved, we apply the Kalman filter to estimate jointly the natural rates of interest, output, and trend growth.³ Econometric identification of the natural rate of interest is achieved by specifying a simple reduced-form IS equation similar to that in Rudebusch and Svensson (1999), where the output gap (the percentage deviation of real GDP from the natural rate of output) is determined by its own lags, a moving average of the lagged real-funds-rate gap (the difference between the ex ante real funds rate and r^*), and a serially uncorrelated error:

$$\tilde{y}_t = a_{y,1} \tilde{y}_{t-1} + a_{y,2} \tilde{y}_{t-2} + \frac{a_r}{2} \sum_{j=1}^2 (r_{t-j} - r_{t-j}^*) + \epsilon_{1,t}, \quad (3)$$

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¹ See also Bomfim (1997), who estimates the natural rate of interest using the Federal Reserve Board's now decommissioned MPS model, and Kozicki and Tinsley (2001) and Bomfim (2001), who use forward rates derived from Treasury bonds and proxies of long-run inflation expectations to estimate investors' estimates of the long-run natural rate.

² Rotemberg and Woodford (1997) and Neiss and Nelson (2001) follow a complementary approach and compute the higher-frequency component of the natural rate—what Woodford (2000) terms the Wicksellian natural rate—from tightly structured macroeconomic models using detrended data. This latter approach yields the period-by-period movements in the real interest rate needed to attain constant inflation, abstracting from any lower-frequency drift in the natural rate of interest.

³ Our approach to estimating the level and growth rate of the natural rate of output builds on the work of Watson (1986), Clark (1987), and Kuttner (1994), with a key difference in that these authors do not include an interest-rate term in their models.

where $\bar{y}_t = 100 \times (y_t - y_t^*)$ denotes the output gap, y_t is the logarithm of real GDP, y_t^* is the logarithm of the unobserved natural rate of output, and r_t is the ex ante real federal funds rate.⁴ In constructing the ex ante real rate, we proxy inflation expectations with the forecast of the four-quarter-ahead percentage change in the price index for personal consumption expenditures excluding food and energy ("core PCE prices") generated from a univariate AR(3) of inflation estimated over the prior 40 quarters. Note that the output gap is related to the real short-term rate; an alternative specification replaces the real-funds-rate gap with a real long-term-bond-rate gap, as in Fuhrer (1997). However, because of limited availability of data on long-term inflation expectations, implementation of such an approach necessitates further identifying assumptions regarding long-run expectations formation that take us away from the simple VAR framework, and we leave this to further research.

The core PCE price inflation rate, π_t , is assumed to be determined by its own lags, the lagged output gap, two variables measuring relative price shocks—core import (excluding petroleum, computers, and semiconductors) price inflation, π^I , and lagged crude imported oil price inflation, π^O —and a serially uncorrelated error:

$$\pi_t = B_\pi(L)\pi_{t-1} + b_y\bar{y}_{t-1} + b_i(\pi_t^I - \pi_t) + b_o(\pi_{t-1}^O - \pi_{t-1}) + \epsilon_{2,t}. \quad (4)$$

We include eight lags of inflation in the equation and impose the restriction—not rejected in our sample—that the sum of the coefficients on lagged inflation must equal unity. This specification is similar to that used in the literature to estimate the natural rate of unemployment (Gordon, 1998; Brayton, Roberts, & Williams, 1999; Laubach, 2001). Equations (3) and (4) constitute the measurement equations of the basic version of our state-space model. We also consider an augmented model that includes an additional measurement equation that relates detrended private nonfarm employee hours, \tilde{h}_t , to the output gap, as in Roberts (2001):

$$\tilde{h}_t = f_1\bar{y}_t + f_2\bar{y}_{t-1} + f_3\tilde{h}_{t-1} + \epsilon_{6,t}, \quad (5)$$

where \tilde{h}_t denotes percentage deviations of private nonfarm employee hours h_t from a log linear trend h_t^* .⁵ The purpose of including this equation is to add more information that may improve the precision of the estimates of the output gap.

Turning to the transition equations of the model, we assume that the variable z defined in (2) follows an autoregressive process,

$$z_t = D_z(L)z_{t-1} + \epsilon_{3,t}. \quad (6)$$

We consider two specifications for z : (1) z is a stationary AR(2) process, and (2) z follows a random walk. We allow for shocks to both the level of the natural rate of output [the I(1) component] and its

trend growth rate [the I(2) component]. For reasons of parsimony, we specify a simple random-walk model for both components:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{4,t}, \quad (7)$$

$$g_t = g_{t-1} + \epsilon_{5,t}. \quad (8)$$

We assume $\epsilon_{3,t}$, $\epsilon_{4,t}$, and $\epsilon_{5,t}$ are serially uncorrelated and contemporaneously uncorrelated innovations. Equations (6)–(8) constitute the transition equations of our state-space model.

In applying the Kalman filter to this model, maximum likelihood estimates of the standard deviations of the innovations to z and the trend growth rate, σ_3 and σ_5 , are likely to be biased towards 0, owing to the so-called *pile-up problem* discussed in Stock (1994). We therefore use the Stock-Watson (1998) median unbiased estimator to obtain estimates of the ratio $\lambda_g \equiv \sigma_3/\sigma_4$, and, in cases where z is nonstationary, $\lambda_z \equiv (\sigma_3/\sigma_1)a_t/\sqrt{2}$. We impose these ratios when estimating the remaining model parameters (including σ_1 and σ_4) by maximum likelihood. Because the pile-up problem does not arise for stationary unobserved processes, in the case where we assume that z is stationary, we estimate the standard deviation of its innovation, σ_3 , by maximum likelihood simultaneously with the other model parameters.

Our estimation method proceeds in sequential steps. In the first step, we follow Kuttner (1994) and apply the Kalman filter to estimate the natural rate of output, omitting the real-rate gap term from equation (3) and assuming that the trend growth rate g is constant. From this preliminary estimate of the natural rate of output we compute the median unbiased estimate of λ_g . If the unobserved component of the natural rate of interest, z , is assumed to be a random walk, we impose the estimated value of λ_g from the first step and include the real-interest-rate gap in the output gap equation under the assumption that z is constant. We estimate the five model equations and obtain an estimate of λ_z .

In the final step, we impose the estimated values of λ_g from the first step and λ_z from the second step (under the assumption that z follows a random walk; otherwise we estimate σ_3 directly) and estimate the remaining model parameters by maximum likelihood as described by Harvey (1989).⁶ We compute estimated confidence intervals and corresponding standard errors for the estimates of the states using Hamilton's (1986) Monte Carlo procedure that allows for both filter and parameter uncertainty.⁷

III. Estimation Results

All state-space models are estimated using U.S. data from 1961q1 to 2002q2. For the model consisting of the measurement equations (3)–(4), the Stock-Watson median unbiased estimates indicate that the trend growth rate and the z component of the natural rate of interest each exhibit relatively modest variation over time. The estimated value of λ_g is 0.042, with the 90% confidence interval, computed by Monte Carlo simulations, ranging from 0 to 0.110.⁸ Assuming z

⁴ For the nominal funds rate, we use the quarterly average of the annualized rate; because the federal funds rate frequently fell below the discount rate prior to 1965, we use the Federal Reserve Bank of New York's discount rate prior to 1965.

⁵ In principle, h_t^* could be specified as an I(2) process analogous to y_t^* . However, tests for an intercept shift in the raw hours data over our sample show no evidence for such a shift. Moreover, when attempting to construct a preliminary estimate of trend hours by estimating a trend-cycle decomposition based on a constant drift in trend hours, trend hours are estimated to be a simple time trend. These results differ from those of Roberts (2001), who normalizes hours by population and finds this series to be I(2).

⁶ Estimates of the conditional expectation and covariance matrix of the initial state are computed using the GLS method discussed in Harvey (1989, p. 122), modified as discussed in Laubach (2001, p. 222).

⁷ Note that these estimated confidence intervals hold the imposed values of λ_g and λ_z fixed but take into account the uncertainty regarding σ_1 , σ_2 , σ_4 , σ_6 , and thus implicitly σ_3 and σ_5 .

⁸ To compute this confidence interval, we use parameter estimates based on the second-stage model with a time-varying trend growth and a constant natural rate of interest to construct 10,000 simulated series of $\{y_t^*, g_t\}$. For each pair of simulated series we compute the exponential Wald statistic for an intercept shift in the first difference of y_t^* .

TABLE 1.—PARAMETER ESTIMATES

Parameter	Stationary $z \sim \text{AR}(2)$	Random-Walk Model for z					
		Baseline	Low λ_z	High λ_z	Low λ_g	High λ_g	Hours
λ_g	.042	.042	.042	.042	.000	.110	.043
λ_z	.042 (.26)	.058	.006	.106	.012	.047	.022
Σa_y	.925	.945	.945	.945	.946	.948	.929
a_r	-.119 (3.45)	-.098 (3.81)	-.088 (3.79)	-.109 (3.78)	-.083 (3.77)	-.122 (3.71)	-.063 (2.82)
b_y	.057 (1.89)	.043 (1.63)	.060 (1.89)	.035 (1.53)	.058 (1.92)	.032 (1.32)	.089 (2.75)
c	1.008 (4.09)	1.068	1.011	1.010	—	1.043	.853
$\sigma_1(\bar{y})$.256	.387	.435	.348	.407	.282	.496
$\sigma_2(\pi)$.727	.731	.727	.733	.728	.735	.724
$\sigma_3(z)$.126	.323	.042	.480	.084	.153	.244
$\sigma_4(y^*)$.644	.605	.580	.619	.608	.642	.546
$\sigma_5(g)$.108	.102	.097	.104	—	.282	.094
$\sigma_{r^*} = \sqrt{c^2\sigma_5^2 + \sigma_3^2}$.167	.340	.107	.491	.084	.332	.256
Log likelihood	-373.5	-376.7	-375.6	-377.0	-375.8	-377.5	-485.0 ^a
S.E. (sample ave.)							
r^*	2.78	1.88	0.90	2.30	0.98	1.51	2.66
g	0.43	0.48	0.40	0.53	0.23	0.72	0.44
y^*	2.20	3.02	1.87	3.87	1.92	3.25	0.56
S.E. (final obs.)							
r^*	4.22	2.61	0.97	3.38	1.13	1.96	3.83
g	0.59	0.63	0.52	0.71	0.23	0.90	0.58
y^*	3.01	4.23	2.02	5.88	2.17	4.23	0.58

t-Statistics are indicated in parentheses; σ_5 is expressed at annual rate.
^a The log likelihood for the model augmented with the equation for employee hours is not comparable with the others.

follows a random walk and using the point estimate of λ_g , we obtain an estimate of λ_z of 0.058, with the 90% confidence interval covering 0.006 to 0.106.

Table 1 reports estimation results for the full set of model parameters for different model specifications and different assumed values of λ_g and λ_z . In all cases, the coefficient relating the output gap to the real rate gap, a_r , is negative and statistically significant. The first column reports results where we assume that z is a stationary AR(2) process. In this case, the coefficient relating the natural rate of interest to the trend growth rate is estimated to be unity and is significantly different from zero. The second column reports the baseline results when z is assumed to follow a random walk; the coefficient estimates are quite similar to those of the previous case. We note that in the cases where z is nonstationary, any inferences regarding the coefficient c based on the standard errors are invalid owing to the potential for spurious correlation (Granger and Newbold, 1974), and we do not report statistics for the estimates of the coefficient c in those cases.

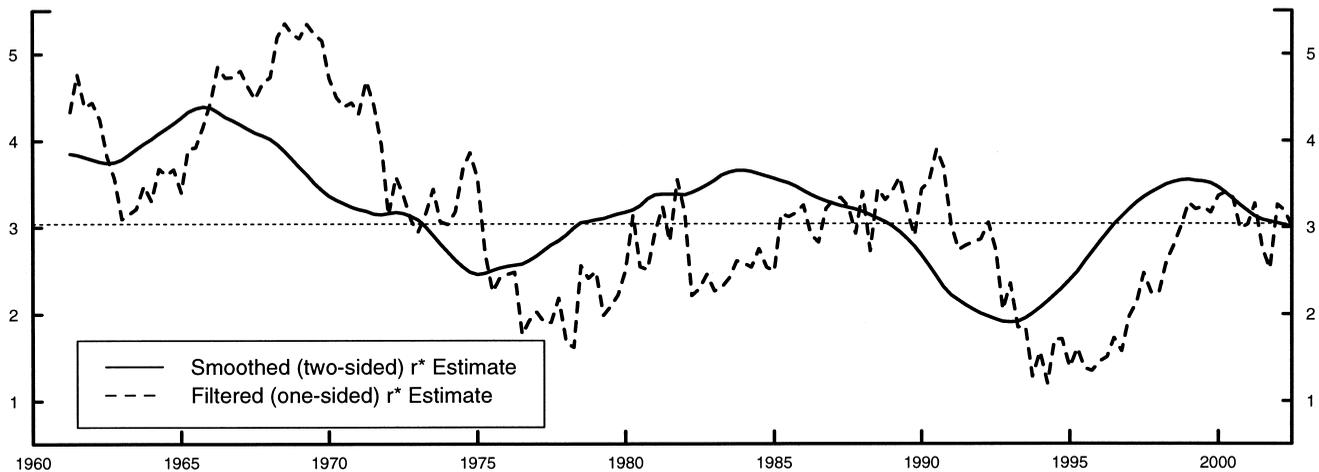
The coefficient estimates are robust to variation in the assumed values of λ_g and λ_z and the inclusion of the hours equation. The third through sixth columns examine the sensitivity of the random-walk specification to uncertainty regarding the values of λ_g and λ_z . For example, in the third column, labeled “Low λ_z ,” we replace the median unbiased estimate of λ_z with the value corresponding to the lower bound of the 90% confidence interval, as indicated in the second line of the table; the fourth column reports the results where λ_z is set equal to the upper bound of the 90% confidence interval. Note that when we modify the assumed value of λ_g , we reestimate λ_z according to the procedures outlined in the previous section. The coefficients related to output gap and inflation dynamics are relatively insensitive

to the assumptions regarding the values of λ_z and λ_g , and, based on the log likelihood values, the data are unable to distinguish clearly between the first six alternative specifications. The final column reports results for the baseline random walk model augmented with the equation for hours. The coefficients of this model differ somewhat from those of the other models, but the strong relationship between the trend growth and the natural rate of interest remains.

Even with hindsight, the natural rate of interest is estimated imprecisely, with the sample average standard error of between 0.90 and 2.78 percentage points, as shown in the section of table 1 labeled “S.E. (sample ave.)” Although the specification that includes the equation for employee hours, shown in the final column of the table, dramatically improves the precision of the estimate of the output gap, the natural rate of interest remains very imprecisely estimated. Overall, the imprecision of the natural rate estimates is larger the greater are the assumed values of λ_g and λ_z , that is, the greater the variation and uncertainty surrounding estimates of the trend growth rate and the unobserved process z . But, even in the cases where r^* is nearly constant, the standard error of nearly 1 percentage point is quite large and of similar magnitude to those associated with estimates of a time-varying natural rate of unemployment reported in Staiger et al. (1997).

The imprecision in estimates of the natural rate of interest is even greater when examining the one-sided estimates of r^* that correspond more closely to “real-time” estimates, in that only current and past observations are used in estimating the state. (Because the full sample is used in estimating the model parameters, the analogy, however, is not exact.) The final observation standard errors shown at the bottom of the table provide a measure of the imprecision of one-sided

FIGURE 1.—ONE- VS. TWO-SIDED ESTIMATES OF THE NATURAL RATE OF INTEREST (BASELINE MODEL)



The solid line shows the smoothed (two-sided) estimates of the natural rate of interest for the baseline specification in which z is assumed to follow a random walk. The dashed line shows the corresponding filter (one-sided) estimates.

estimates, assuming the true values of λ_g and λ_z are known. Figure 1 illustrates the differences between the one- and two-sided estimates for the baseline specification. The one-sided filter estimates of the natural rate of interest are shown by the dashed line, and the two-sided smoothed estimates are shown by the solid line. For comparison, the sample mean of the smoothed estimate—about 3%—is shown by the horizontal dashed line. The sample standard deviation of the difference between the one- and two-sided estimates is about 0.9 percentage point over the sample.

The standard errors reported above ignore uncertainty regarding λ_g and λ_z and model uncertainty and thus provide lower bounds on the “true” degree of uncertainty regarding these estimates. As seen in the top panel of figure 2, the estimates of the natural rate of interest are sensitive to the assumptions regarding z . This figure shows the smoothed (two-sided) estimates of the unobserved processes implied by our model estimates for the different assumptions regarding the process for z corresponding to the first four columns of table 1. In particular, the AR(2) specification for z yields estimates of r^* that are highly variable over time; in contrast, the random-walk specification with the lower bound value of λ_z yields r^* estimates that are nearly constant. Estimates of the trend growth rate and the output gap, however, are not sensitive to the specification of z , as seen in the middle and bottom panels of the figure.

The estimates of the natural rate of interest are also sensitive to the assumptions regarding estimation of the natural rate of output and its trend growth rate. Figure 3 shows the sensitivity of the smoothed estimates of the natural rate of interest, the trend growth rate, and of the output gap to changes in the specification for the processes for the natural rate of output and its trend growth rate, corresponding to the final three columns of table 1. The baseline estimates are also shown for comparison. Changes in the value of λ_g have a dramatic effect on the magnitude of time variation in the trend growth rate, as seen in the middle panel. These different estimates of the trend growth rate do not, however, lead to significantly different estimates of the output gap, shown in the bottom panel. The estimates of the natural rate of interest are affected by the degree of variation in the trend growth rate owing to the relationship between r^* and the growth rate. Relative to the baseline specification, the model augmented with the hours equation yields higher estimates of the natural rate of output during most of the first of the half of the sample and lower estimates during much

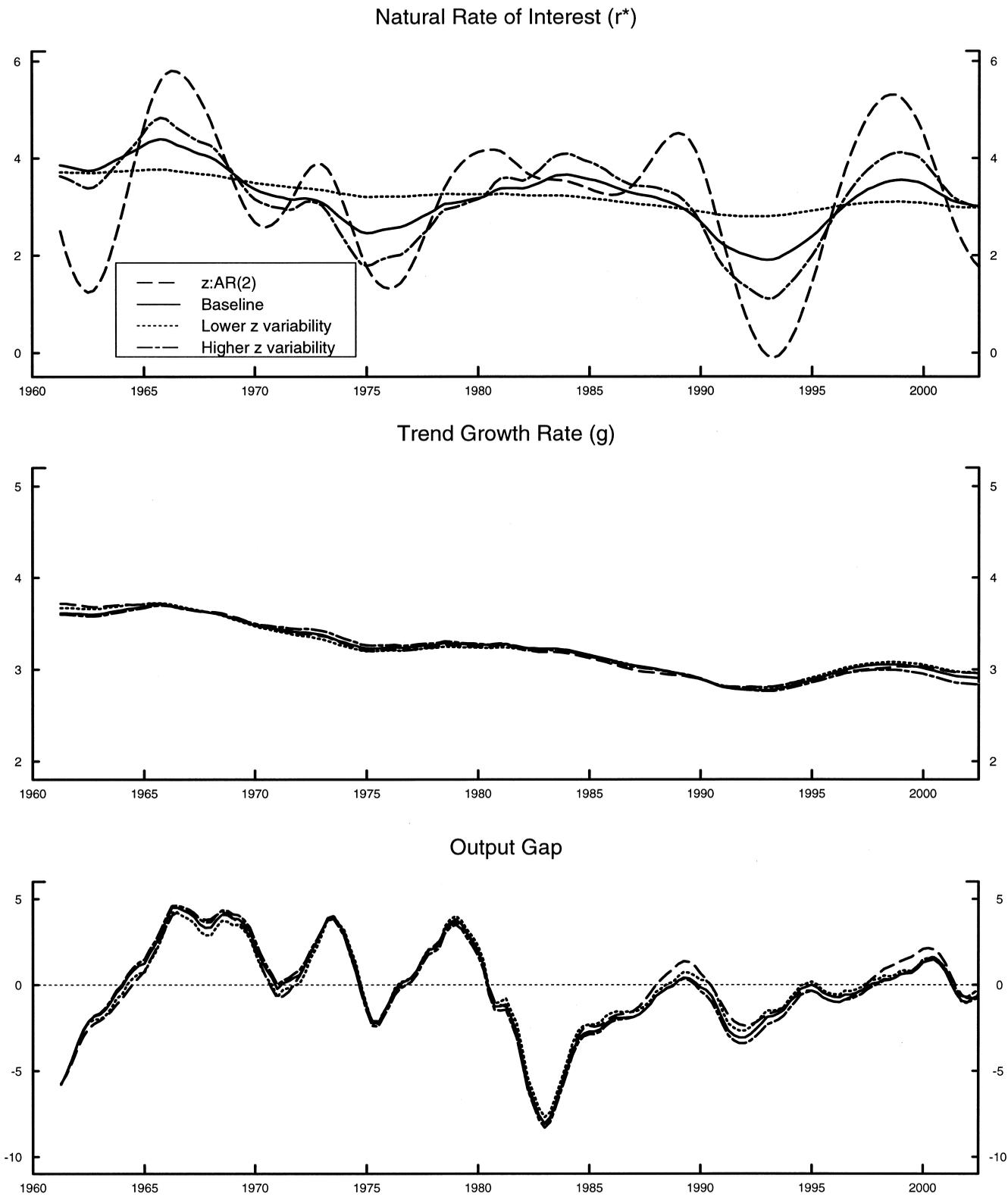
of the second half. As a result, the estimates of the natural rate of interest from the augmented model are lower than those from the baseline model during the earlier period and generally higher during the latter period. Taken together, these results cast considerable doubt on the ability to estimate the natural rate of interest with much precision in real time.

Given the imprecision of the Kalman filter natural-rate estimates, we now briefly examine the properties of other time series methods. Specifically, we compare our Kalman filter estimates of the natural rate of interest with estimates based on two popular univariate filters of the ex post real rate, defined to be the difference between the funds rate and the realized core PCE inflation rate. The first univariate method is the Hodrick-Prescott (1997) filter, for which we use a value of the smoothing parameter λ of 6400. The second method is the bandpass filter described by Baxter and King (1999), for which we use a 60-quarter window as in Staiger, Stock and Watson (2002).⁹ Figure 4 shows the baseline two-sided Kalman filter estimates and the two-sided estimates from the univariate filters. As seen in the figure, the estimates from the two univariate filter methods are very similar.

The univariate filter estimates arguably provide a misleading picture of the natural rate of interest because they ignore information from movements in inflation. By their nature as two-sided weighted averages, the univariate estimates imply that real rates were on average close to the natural rate during the 1960s and early 1970s, even though inflation rose steadily during that period. Similarly, the univariate filters inappropriately ascribe a large portion of the disinflationary policy action during the late 1970s and early 1980s to an upward shift in the natural rate. In contrast, the Kalman filter estimates show that policy was on average stimulative during the 1960s and early 1970s and that the natural rate moved very little during the later disinflationary period. For these reasons, the Kalman filter approach appears to do better at estimating lower-frequency movements in the natural rate of interest than univariate time-series methods. The same conclusion holds true for the one-sided Kalman and univariate filter

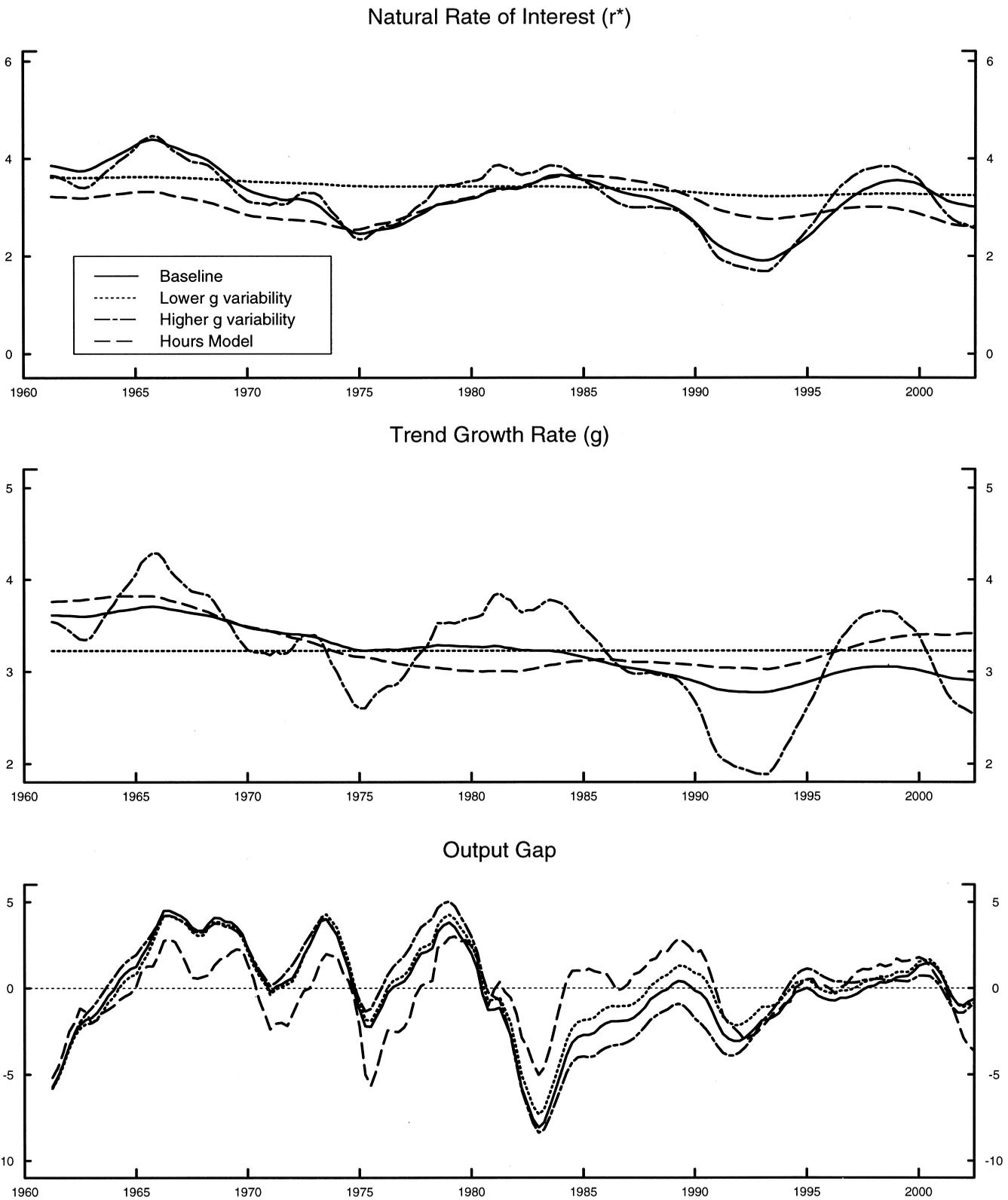
⁹ We chose these parameterizations of the filters so that the resulting estimates correspond to the lower-frequency component similar to that from the Kalman filter. Univariate filter estimates that impose less smoothing are qualitatively similar to those reported here, but are more sensitive to the actual data.

FIGURE 2.—SENSITIVITY OF SMOOTHED ESTIMATES TO ASSUMPTIONS REGARDING z



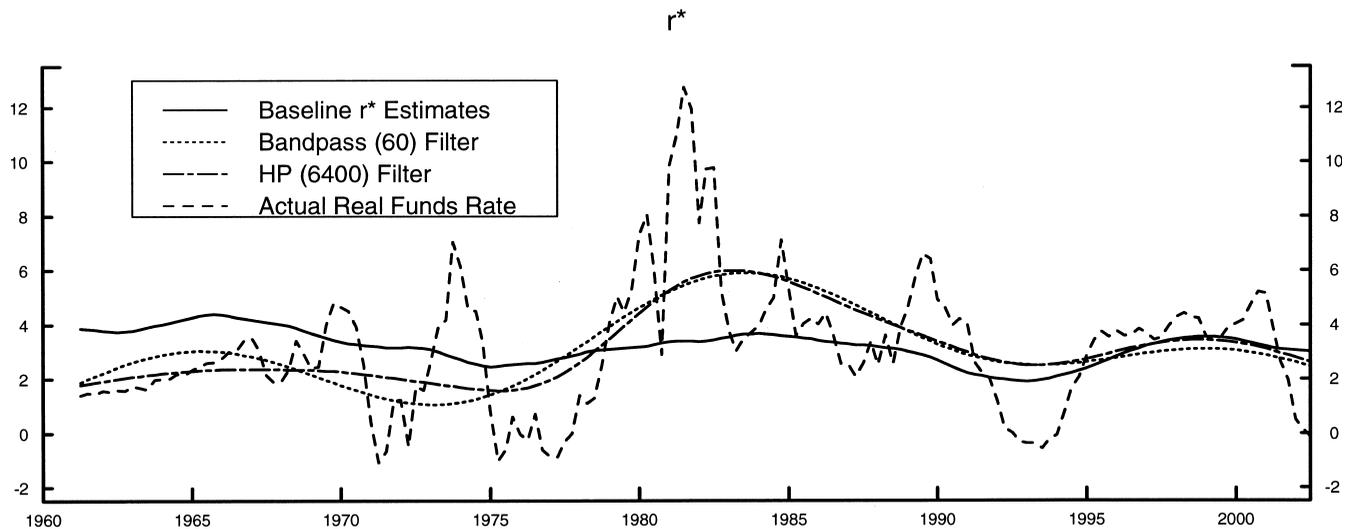
The dashed lines show the smoothed (two-sided) estimates of the unobserved states for the model in which z is assumed to follow an AR(2) process. The solid lines show the corresponding estimates for the baseline case where z is assumed to follow a random walk. The dotted lines show the estimates for the lower bound value of λ_z ; the dash-dot lines show the values for the upper bound value of λ_z .

FIGURE 3.—SENSITIVITY OF ESTIMATES TO ASSUMPTIONS REGARDING NATURAL RATE OF OUTPUT



The solid lines show the smoothed (two-sided) estimates of the unobserved states for the baseline case where z is assumed to follow a random walk. The dotted lines show the estimates for the lower bound value of λ_z ; the dash-dot lines show the values for the upper bound value of λ_z . The dashed line shows the estimates for the baseline model augmented with the hours equation.

FIGURE 4.—UNIVARIATE ESTIMATES OF THE NATURAL RATE OF INTEREST



The solid line shows the smoothed (two-sided) Kalman filter estimates of the natural rate of interest for the baseline case where z is assumed to follow a random walk. The dotted line shows the bandpass filter estimates using a 60-quarter window. The dash-dot line shows the HP filter estimates using a smoothing parameter of 6,400. The dashed line shows the actual ex post real funds rate.

estimates. Indeed, the latter are very sensitive to the actual data and thus especially poorly suited to estimating trends in real time, as discussed in Orphanides and van Norden (2002) and Orphanides and Williams (2002).

IV. Conclusion

In this paper, we have jointly estimated the natural rates of interest and output and the trend growth rate using the Kalman filter. The natural rate of interest shows significant variation over the past forty years in the United States, with variation in the trend growth rate an important determinant of changes in the natural rate, as predicted by theory. These results are robust to changes in specification. However, estimates of a time-varying natural rate of interest, like those of the natural rates of unemployment and output, are very imprecise and are subject to considerable real-time mismeasurement. These results suggest that this source of uncertainty needs to be taken account of in analyzing monetary policies that feature responses to the natural rate of interest.

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TESTING STEADY-STATE IMPLICATIONS FOR THE NAIRU

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Abstract—Estimates of the NAIRU are usually derived either from a Phillips curve or from a wage curve. This paper investigates the correspondence between the operational NAIRU concepts and the steady state of a dynamic wage-price model. We derive the parameter restrictions that secure that correspondence. The full set of restrictions can be tested by econometric analysis of the wage-price system, and this method is demonstrated for Norwegian data. A set of necessary conditions can be tested from estimated wage curves alone. Existing international evidence from empirical wage equations are reinterpreted in light of these conditions.

I. Introduction

GOVERNMENTS AND international organizations customarily refer to NAIRU calculations in their discussions of employment and inflation prospects,¹ and the existence of a NAIRU consistent with a vertical long-run Phillips curve is crucial to the framework of monetary policy.² An alternative to a Phillips-curve NAIRU is based on a negative relationship between the level of the real wage and the rate of unemployment, dubbed the *wage curve* by Blanchflower and Oswald (1994), coupled with firms’ price-setting schedule. As noted by Blanchard and Katz (1999), the wage curve has become the new consensus framework of the NAIRU in Europe, while a Phillips-curve version, dubbed “the triangle model of inflation” by Gordon (1983),

applies to the United States; see Gordon (1998) and Staiger, Stock, and Watson (2002) for recent contributions.³

In this paper we investigate the steady-state connotations of NAIRUs. Are the Phillips-curve or wage-curve NAIRUs good predictors of the true steady-state values of the rate of unemployment ground out by a dynamic model of the macro economy? To answer this question, we derive in section II the testable conditions for correspondence between a Phillips- or wage-curve NAIRU and the steady-state unemployment rate in a more general system. Section III contains an application of the implied testing procedure to Norwegian data. Finally, the framework is used to reinterpret existing evidence from wage equations of a larger number of countries. Section IV concludes.

II. An Encompassing Framework

The model is an extension of Kolsrud and Nymoen (1998). The variables are, in logarithms, the nominal wage w_t ; the producer price of domestic products, q_t ; the consumer price p_t ; the rate of unemployment, u_t ; and the productivity pr_t .⁴ Nominal wages and prices are integrated of order 1, denoted $I(1)$. The rate of unemployment may also be nonstationary—for example, its mean might change—but after removal of deterministic shifts, we assume that $u_t \sim I(0)$.

A. Long-Run Properties

A cointegrated long-run wage equation, consistent with the bargaining approach and the assumed temporal properties of the data, is thus

$$w_t - q_t - \beta_{pr} pr_t - \beta_{p-q}(p - q)_t = -\beta_u u_t + m_{wt} \sim I(0). \quad (1)$$

The quantity $(p - q)_t$ is the *wedge* between the consumer real wage and the producer real wage. The role of the wedge as a source of wage

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¹ See Elmeskov and MacFarlan (1993), Scarpetta (1996) and OECD (1997, Chapter 1) for examples.

² See the discussion in King (1999) for a central banker’s views.

³ Recently, the Phillips curve has enjoyed a revival in the theory of monetary policy (see Clarida, Gali, & Gertler, 1999), and it dominates in the theoretical literature on inflation targeting in particular (see for example Svensson, 2000), which of course increases its importance in Europe.

⁴ We abstract from the payroll tax rate. The rate of unemployment enters linearly in some U.S. studies; see for example Fuhrer (1995). However, for most other data sets, a concave transform improves the fit and the stability of the relationship; see for example Nickell (1987) and Johansen (1995).