BUDGET-CONSTRAINED FRONTIER MEASURES OF FISCAL EQUALITY AND EFFICIENCY IN SCHOOLING

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Abstract — Equality and efficiency are key issues in educational reform. Here the authors analyze the efficiency and equality consequences of various school finance reforms using a cost-indirect output distance function. This function readily models multiple-output production under conditions of budgetary constraint, and provides a natural measure of performance that is closely related to Farrell-type measures of efficiency. The analysis suggests that despite school district inefficiency, finance reforms can affect student achievement. However, any potential gains in output from redistribution are dwarfed by the potential gains from increased efficiency. More strikingly, the analysis demonstrates that budgetary reforms designed to equalize expenditures could actually increase the inequality of student achievement.

I. Introduction

A CONSIDERABLE body of economic research suggests that public schools in the United States are inefficient. Eric Hanushek’s (1986) survey of the literature on educational production functions found no systematic, positive relationship between school district expenditures and student academic achievement. Studies of educational cost functions and most subsequent analyses of educational production functions have reached similar conclusions. (For examples, see Berger and Toma (1994), Callan and Santerre (1990), Wahlberg and Fowler (1987), and Eberts and Stone (1986).) Chubb and Moe (1990) found that the organization of schools and the allocation of resources within schools were more important determinants of student academic achievement than the level of school spending.

However, despite the accumulating evidence against “throwing money at schools,” judicial authorities continue to require fiscal reforms designed to make expenditures more equal. For example, the Supreme Court of Texas recently invalidated the state’s school finance system, ruling that it was incompatible with requirements in the Texas Constitution. In the court’s opinion, an efficient school system requires that “the funds available for education be distributed equitably and evenly.” The court directed the state legislature to devise such a system.

In this analysis, we explore the likely impact of legal initiatives designed to produce expenditures equality, given the current distribution of school inefficiency. We use a multioutput model of the educational production process to simulate the effect of budgetary changes on the output level of each school. For each budgetary reform considered, we generate measures of fiscal equality. Our fiscal equality measure compares the observed level of output to the level of output that each school district could produce if its degree of inefficiency were unchanged but it faced an equalized budget. We measure each school district’s degree of inefficiency by comparing the observed level of output for a school district with the level of output the school district could be expected to produce if it were using its current budget efficiently. We find that given the disparate distribution of student characteristics and the inefficiencies in the current system, the pursuit of expenditures equality may actually reduce achievement equality.

In section II we review recent work on the empirical application of production functions and efficiency measurement in education. Section III reviews the distance function approach that we use to model multioutput production technologies. It also presents our measures of efficiency and fiscal equality in schooling. Section IV describes the data employed to derive our output measures for Texas school districts. In section V we provide empirical results of the model presented in section III and simulate various redistributive reforms. Section VI concludes.

II. Models of School Production and Efficiency

Over the years, researchers have used a variety of techniques to evaluate school performance. Most researchers have focused on estimating single-output, average production functions for schooling. Although a few recent studies have examined monetary returns to schooling (Betts (1993) and Card and Krueger (1992a, b)), the most common measures of educational outputs have been test scores. (For example, see Berger and Toma (1994), Eberts and Stone (1987), Wahlberg and Fowler (1987), and the literature surveyed in Hanushek (1986).) In general, researchers assume that schools produce these educational outputs using inputs related to school personnel, per-pupil expenditures, and family background.1

The production functions yield estimates (based on average performance) of the marginal products of the inputs, and allow researchers to infer which inputs would have the greatest marginal impact on achievement.2 Most researchers using this approach have found that inputs within school district control (such as expenditures or class sizes) have little or no marginal impact on test scores (Hanushek (1986)). Card and Krueger (1992a, b) find evidence that school inputs have a positive effect on the monetary returns to schooling, but their analysis is based on state-level data

1 See Cohn and Geske (1990) for a thorough review of the output and input measures employed in these types of studies.

2 See Levin (1974) and Hanushek (1979) for critical reviews of the production function approach.
about school characteristics and may be subject to aggregation bias (see Hanushek et al. 1996). Using less aggregate data, Betts (1995) finds no evidence of marginal effects.

Recently, some researchers have modified production function analysis to incorporate scale, technical and allocative inefficiencies, and multiple measures of educational output. Most of the researchers using the generalized approach have relied on nonstochastic techniques such as data envelopment analysis (e.g., Bessent and Bessent 1980, Bessent et al. 1982, 1984, Färe et al. 1989, and Grosskopf et al. 1994)). However, a few researchers have used stochastic techniques. Deller and Rudnicki (1993) assume that school inefficiency has a half-normal distribution and use maximum-likelihood techniques to estimate a single-output frontier production function. McCarty and Yaisawarng (1993) and Ray (1991) combine DEA and regression analysis in a partially stochastic two-step procedure that incorporates multiple outputs. 3 Like the production function analyses, these studies of the production possibilities frontier find evidence of substantial school inefficiency.

Analyses of educational cost functions yield similar results. Barrow (1991) estimated a cost function frontier for schools in England and found that actual costs were 4% to 16% above the minimum estimated cost for the schools in his sample. Callan and Santerre (1990) found evidence that school districts in Connecticut produce primary and secondary education using inefficiently large quantities of capital and transportation services. Jimenez (1986) concluded that schools in Bolivia and Paraguay used excessive amounts of capital and that many of the schools in Bolivia exhibited diseconomies of scale. Eberts and Stone (1986) found that rent extraction in the form of higher teacher salaries adds between 7% and 15% to educational costs in unionized school districts in the United States.

Rather than adopt one of these familiar research strategies, we use an indirect output distance function to evaluate the impact of budgetary reform on school district performance. The indirect output distance function is particularly well suited to modeling the technology of public enterprises that produce multiple outputs under conditions of budgetary constraint. While budget determination is a complex process guided by public sector performance and citizen-voter satisfaction, court mandates make it useful to think of school officials as facing exogenous budget constraints.

The indirect output distance function lends itself to fully stochastic frontier estimation without sacrificing the ability to evaluate multiple outputs. Although the cost function is also capable of stochastically modeling a multivariate production technology, the indirect output distance function is more appropriate for evaluating firms that are cost-constrained because it takes cost as exogenous. 4 In addition, the indirect output distance function makes no a priori behavioral assumption while the cost function implies cost minimizing behavior on the part of the economic agent. 5 Thus, the indirect output distance function is a convenient tool for analyzing potentially inefficient public enterprises.

### III. The Indirect Output Distance Function

In this section, we review the properties of the indirect output distance function as presented by Färe and Primont (1990) and provide a functional form that can be employed to estimate it. 6 The following notation is used throughout the paper:

- \( x = (x_1, \ldots, x_n) \), vector of variable input quantities
- \( p = (p_1, \ldots, p_n) \), vector of variable input prices
- \( u = (u_1, \ldots, u_m) \), vector of output quantities
- \( z = (z_1, \ldots, z_l) \), vector of fixed input quantities
- \( c = (c) \), scalar cost or budget.

Figure 1 illustrates the construction of the indirect output distance function for a typical school district that produces two outputs. The set \( G(p/c, z) \) gives all the possible combinations of two outputs that can be produced given the budget.

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3 In the first step, they construct efficiency measures for schools by applying DEA to data on multiple educational outcomes and discretionary inputs (such as teachers and administrators). In the second step, they regress the efficiency measures on a set of nondiscretionary inputs (such as student body characteristics).

4 Both the indirect output distance function and the cost function hold outputs and input prices exogenous. However, the indirect output distance function includes the budget by which the decision maker is constrained, thereby holding total cost exogenous as well.

5 While the cost function assumes cost-minimizing behavior, inefficiency can be allowed for in the cost function using techniques outlined by Schmidt and Sickles (1984). Furthermore, if the firm does minimize costs and technology is homogeneous, the cost function and the indirect output distance function are equivalent.

6 For a further discussion of the indirect output distance function, see Färe and Grosskopf (1994) and Färe et al. (1994, 1988).
constraint faced by the school district. Formally,

$$G(p/c, z) = \min \{u: u \in P(x, z) \text{and } p'x \leq c\}$$ \hspace{1cm} (1)$$

where \(P(x, z)\) is the production possibility set for a given \((x, z)\) and \(G(p/c, z)\) is the largest production possibility set allowing \(x\) to vary, but requiring that \(x\) satisfy the budget constraint. In other words, \(G(p/c, z)\) represents the envelope of all \(P(x, z)\) that satisfy \(p'x \leq c\).

The school district produces a combination of outputs represented by point \(U\) in figure 1. The corresponding (short run)\(^7\) indirect output distance function can be defined as

$$ID_o(p/c, z, u) = \min_{0, s} \{u: u \theta \in G(p/c, z)\}. \hspace{1cm} (2)$$

The ratio \(0U/0A\) gives the value of the indirect output distance function, and is the measure we will use to judge the efficiency of individual school districts (which we call EFF). The reciprocal of the indirect output distance function gives the factor by which all outputs could be expanded proportionally if the school district was operating efficiently. It follows that when the school district is producing efficiently (on the frontier of \(G(p/c, z)\)), the value of \(ID_o(\cdot)\) equals 1.

Figure 1 also illustrates the effect of a finance reform that expands the school district’s budget set from \((p/c)\) to \((p/c)\)\(^*\). The set \(G((p/c)\)\(^*\), \(z\)\) gives the maximum production possibility set that could be attained if the school district faced a new, higher budget with its original fixed inputs \((z)\). The ratio \(0A/0B\) is our measure of fiscal equalization (which we call FE for short) for the typical school district. It measures the amount that the school district could expand all outputs proportionally if they were originally operating efficiently (on their own \(G(p/c, z)\) frontier) and then were faced with a new (larger) budget set. The reciprocal of this ratio \((0A/0B)\) equals the value of the indirect output distance function for the new production possibilities frontier \(G((p/c)\)\(^*\), \(z)\), assuming that the school was observed at point \(A\) on the original \(G(p/c, z)\) frontier. Because the indirect output distance function is homogeneous in outputs, the fiscal equality measure is also the proportion by which the school district could expand all outputs if its degree of inefficiency were unchanged but it faced a revised budget constraint.

In order to ultimately estimate the indirect output distance function, we exploit several of its properties. Färe and Primont (1990) demonstrate that

$$ID_o(p/c, z, u) = \min_{x} \{D_o(x, z, u): (p/c)'x \leq 1\} \hspace{1cm} (3)$$

where \(D_o(x, z, u) = \min\{u: u \theta \in P(x, z)\}\). \(^9\) Since \(x\) in (3) is chosen to minimize \(D_o(x, z, u)\), we can invoke the envelope theorem to yield

$$\partial ID_o(\cdot)/\partial (p/c) = x(p/c, z, u) \hspace{1cm} (4)$$

where \(x(\cdot)\) is the input demand function for normalized price vector \(p/c\), fixed inputs \(z\), and multioutput level \(u\). \(^10\) Furthermore, if we take the logarithm of the indirect output distance function and differentiate it with respect to log-normalized prices, we obtain the budget share equations for the inputs

$$\partial \ln ID_o(p/c, z, u)/\partial \ln (p/c) = w(p/c, z, u) = px/c. \hspace{1cm} (5)$$

Finally, by definition the indirect output distance function is homogeneous of degree +1 in outputs. That is,

$$\lambda ID_o(p/c, z, u) = ID_o(p/c, z, \lambda u). \hspace{1cm} (6)$$

Because \(ID_o(p/c, z, u)\) on the left-hand side of (6) is not observable, we specify the relationship between observed and potential values of the indirect distance function as

$$\lambda_s = ID_o(p/s/c, z, \lambda u_s; B) \cdot e_s, \hspace{1cm} s = 1, \ldots, S \hspace{1cm} (7)$$

where \(s = 1, \ldots, S\) is an index of observations and \(B\) is a vector of coefficient estimates. \(^{11}\) We will use \(ID_o(p/s/c, z, \lambda u_s; B)\) (or equivalently \((\hat{e}_s)^{-1}\)) as our estimate of the distance function for each observation \(s\) by definition, the distance function is less than or equal to 1. Therefore, \(e_s \geq 1\).

Three issues arise in estimating (7): we need an observable variable for \(\lambda_s\), the functional form of (7) is unknown, and the error structure is one-sided by assumption.

The efficiency framework developed by Farrell (1957) suggests an appropriate variable for \(\lambda_s\). For each school district \(s\), the Euclidean norm of the output measures \((\parallel u_s \parallel = u_{s1}^2 + u_{s2}^2 + \cdots + u_{sm}^2)^{1/2}\) can be interpreted as a scalar measure of the “size” of the output vector or its distance from the origin. We let \(\lambda_s = 1/\parallel u_s \parallel\) and transform

\(^9\) \(D_o(x, z, u)\) is the direct output distance function. (See Färe and Primont (1990, p. 883) or Färe and Grosskopf (1994)).

\(^10\) This is true when technology is homogeneous.

\(^11\) Because \(D_o(p/s/c, z, u) \leq 1\), the error term in (7) will be \(e_s = \epsilon\) \(ID_o(p/s/c, z, u) \geq 1\), where \(\epsilon\) captures a normal error. As Schmidt and Sickles (1984) demonstrate, one could use panel data to consistently estimate \(\epsilon\) and its components. Unfortunately, we have cross-sectional data. Hence, we use the “corrected ordinary least-squares” approach suggested by Greene (1980). Alternatively, one could apply a stochastic frontier approach and assume distributions for the two components of the error term, as in Deller and Rudnicki (1993) or Grosskopf and Hayes (1993).
equation (7) into

\[
\frac{1}{\|u_s\|} = ID_\delta \left( p_s / c_s, z_s, \frac{u_s}{\|u_s\|} ; B \right) \cdot \epsilon_s. 
\] (7')

Note that the transformation yields normalized output vectors \( u_s / \|u_s\| \) on the right-hand side. These \( u_s / \|u_s\| \) are measures of output mix and can be considered exogenous in the Farrell (1957) framework because output expands in fixed proportions (see figure 1).

Because we have no a priori expectations about the functional form of the indirect output distance function, we estimate it using the translog form,

\[
\ln \left( \frac{1}{\|u_s\|} \right) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln \left( \frac{p_i}{c_i} \right) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \ln \left( \frac{p_i}{c_i} \right) \ln \left( \frac{p_j}{c_j} \right) \\
+ \sum_{k=1}^{m} \beta_k \ln \left( \frac{u_{ks}}{\|u_s\|} \right) \\
+ \sum_{i=1}^{n} \sum_{k=1}^{m} \beta_{ik} \ln \left( \frac{p_i}{c_i} \right) \ln \left( \frac{u_{ks}}{\|u_s\|} \right) \\
+ \sum_{r=1}^{t} \gamma_r \ln z_{rs} + \sum_{i=1}^{n} \sum_{r=1}^{t} \gamma_{ir} \ln \left( \frac{p_i}{c_i} \right) \ln z_{rs}. 
\] (8)

Estimating the budget share equations and the distance function in a system of simultaneous equations would improve the efficiency of the estimated parameters. As equations (4) and (5) show, differentiating the above equation with respect to \( \ln \left( \frac{p_j}{c} \right) \) yields the budget shares for the \( i = 1, \ldots, n \) variable inputs:\(^{12}\)

\[
w_{is} = p_{is} x_{is} / c_s \\
= \alpha_i + \sum_j \alpha_{ij} \ln \left( \frac{p_j}{c_i} \right) \\
+ \sum_k \beta_{ik} \ln \left( \frac{u_{ks}}{\|u_s\|} \right) + \sum_r \gamma_{ir} \ln z_{rs}, \hspace{1em} i = 1, \ldots, n. 
\] (9)

We follow Greene (1980) in adjusting OLS parameter estimates for the nonnormality assumption concerning \( \epsilon \). Specifically we adjust the OLS predicted values by scaling our estimate of the distance function,

\[
\frac{1}{\|u_s\|} = \frac{\hat{ID}_\delta \left( p_s / c_s, z_s, \frac{u_s}{\|u_s\|} ; B \right)}{\|\hat{u}_s\|} = \frac{1}{\|\hat{u}_s\|}.
\]

by the smallest OLS residual (\( \min \{ \epsilon_i \} \)). The scaling yields values of the estimated distance function between 0 and 1, as required by the definition of the distance function. These scaled values of the estimated distance function are our measures of efficiency (\( EFF_s \)),\(^{13}\)

\[
EFF_s = \frac{\|u_s\| \cdot \min \{ \epsilon_i \} }{\|\hat{u}_s\|}. 
\] (10)

To measure fiscal equality for each school district (\( FE_s \)), we forecast what would happen to the value of \( ID_\delta (\cdot) \) if the \( s \)th school district had access to a different budget set, either an altered expenditure level (\( c^* \)) or a different vector of normalized prices (\( \frac{p}{c^*} \)). Because \( FE \) is measured relative to the initial frontier, we inflate each output vector by the

\[
\begin{align*}
\sum_{i=1}^{n} \alpha_i &= 1 \\
\sum_{i=1}^{n} \alpha_{ij} &= 0, \hspace{1em} j = 1, \ldots, n \\
\sum_{r=1}^{t} \gamma_r &= 0, \hspace{1em} r = 1, \ldots, t \\
\sum_{k=1}^{m} \beta_{ik} &= 0, \hspace{1em} k = 1, \ldots, m.
\end{align*}
\]

By Young’s theorem, the twice continuously differentiable indirect output distance function is also symmetric in normalized input prices, leading to the restriction \( \alpha_{ij} = \alpha_{ji} \) for \( i \neq j \). Finally, one budget share equation must be dropped to avoid exact linear dependence of the error terms. The analysis is not sensitive to which equation is dropped.

\(^{13}\) The indirect output distance function yields a measure of performance or efficiency that captures technical efficiency in terms of outputs (i.e., no account is taken of relative output prices). Note, however, that by definition, the indirect output distance function allows for reallocation of inputs as long as the budget constraint is satisfied. This implies that observations that are interior to the frontier of \( G(\cdot) \) may be inside the frontier because they have chosen an input bundle that is allocatively inefficient.
efficiency score as calculated by (10), \( u_s / EFF_s \), and then forecast

\[
\ln \tilde{D}_o \left( (p_s / c_s)^* \left( \frac{u_s}{\|u_s\|} \right) \left/ EFF_s, z_s; B \right. \right) \\
= \hat{\alpha}_0 + \sum_i \hat{\alpha}_i \ln [(p_s / c_s)^*]
\]

\[
+ \frac{1}{2} \sum_i \hat{\alpha}_i \ln [(p_s / c_s)^*] \ln [(p_s / c_s)^*]
\]

\[
+ \sum_k \hat{\beta}_k \ln \left( \frac{u_k}{\|u_k\|} \right) \left/ EFF_s \right.
\]

\[
+ \sum_i \sum_k \hat{\beta}_{ik} \ln [(p_s / c_s)^*] \ln \left( \frac{u_k}{\|u_k\|} \right) \left/ EFF_s \right.
\]

\[
+ \sum_{r=1}^t \hat{\gamma}_r \ln z_r + \sum_{i=1}^n \sum_{r=1}^t \hat{\gamma}_{ir} \ln [(p_s / c_s)^*] \ln z_r.
\]

We should note that in estimating the indirect output distance function and calculating our measures of efficiency and fiscal equality, we treat school district budgets as exogenous. In reality, schools have control over their budgets to the extent that they have control over the production processes and to the extent that parents, homeowners, and businesses self-select into school districts that best match their own preferences. Furthermore, most educational reforms incorporate some local variation in expenditures. Although beyond the scope of this paper, school inefficiency and fiscal equality might further be modeled by combining the indirect output distance function with a median voter model, so that the production process and voting outcome simultaneously determine the budgets that school administrators receive.

\[ FE_s = \frac{1}{\ln \tilde{D}_o \left( (p_s / c_s)^*, z_s, \frac{u_s}{\|u_s\|} \right) \left/ EFF_s; B \right.}. \]  

IV. Data

To implement the model described in the previous section, we employ data from 310 Texas school districts with enrollments of between 1000 and 5000 students. Our variable inputs consist of various categories of employment representing more than 80% of current operating expenses. We include expenditures on maintenance and operations as a proxy for fixed capital inputs. We also construct a set of variables that represent fixed, home-produced inputs, which is explained in more detail below.

The literature on measuring school effects has reached a broad consensus that the most appropriate measure of schooling product is the marginal effect of the school on educational outcomes. (See, for example, Hanushek (1986), Hanushek and Taylor (1990), Aitkin and Longford (1986), Meyer (1993), and Boardman and Murnane (1979).) The Texas Educational Assessment of Minimum Skills (TEAMS) test provides information on student achievement in reading, writing, and mathematics. We extract the marginal effect of schools by following the value-added residuals techniques described in Hanushek and Taylor (1990) and Aitkin and Longford (1986).\(^{14}\) That is, for each of four grade levels (3, 5, 9, and 11) we estimate the following equation:

\[
\text{TEAMS}^{89}_{s,g} = \delta_g + \sum_{i=3}^{11} \delta_{i,g}\text{ETHNICITY}_{i,s} + \delta_{4,g}\text{SES}_s + \delta_{g,\text{XCOHORT}}_{s,g} + \sum_{j=6}^{g} \delta_{j,g}\text{TEAMS}^{87}_{j,s,g-2} + \epsilon_{s,g},
\]

where \( \text{TEAMS}^{89}_{s,g} \) is the average total TEAMS scores for school district \( s \) for grade level \( g \) in 1989, \( \text{TEAMS}^{87}_{j,s,g-2} \) is the average TEAMS score in subject \( j \) (reading, writing, and mathematics) for the same cohort two years previously,\(^{15}\) and \( \text{ETHNICITY}_{i,s} \) is the fraction of the student body of school district \( s \) that is Asian, Black, or Hispanic. \( \text{SES}_s \) is the fraction of the student body of school district \( s \) that is receiving free or reduced-price lunches (the best available proxy for socioeconomic status), and \( (1 - \text{XCOHORT}_{s,g}) \) is the percentage change in the size of the grade \( g \) cohort between 1987 and 1989 (this controls for schools that try to improve scores by shedding students). The estimates residual \( (\epsilon_{s,g}) \) represents the average value added in school district \( s \). Because the four value-added equations share common regressors, we estimate the system simultaneously using the SAS package for seemingly unrelated regressions.

\(^{14}\) We note this general technique was also employed by Callan and Santerre (1990) to arrive at a measure of educational quality. However, Callan and Santerre did not have access to pretest information and, therefore, were unable to derive a value-added quality measure.

\(^{15}\) Texas administered the TEAMS test annually to odd-numbered grades.
Estimating school district outputs as equation residuals generates output measures that represent deviations from the state average. School districts that add less value than the state average have negative output measures. Since the distance function methodology cannot handle negative outputs, we transform the value-added residuals into tractable distance function methodology cannot handle negative outputs, we transform the value-added residuals into tractable distance function methodology cannot handle negative outputs, we transform the value-added residuals into tractable...
V. Empirical Results

We estimated the value of the indirect output distance function for each of the 310 school districts in our sample using seemingly unrelated regression on the system of equations in (8) and (9). 19 Because the school outputs and fixed inputs are estimated, we follow Murphy and Topel (1985) to correct the standard errors. 20 The parameter estimates, SUR standard errors, and corrected standard errors are reported in table 3.

Predicted values for the output distance function yield our measure of school district efficiency $E_F$, (see equation (10)). The mean value of $E_F$ for the 310 Texas school districts in our sample is 0.708. If each school district allocated its given budget according to the best practice in the sample, then we would expect outputs to increase by more than 40% (1/0.708 = 1.41) on average. In a constant returns to scale world, such an increase in outputs is equivalent to current production with 29% lower cost (0.708·c). This result suggests that there are considerable gains to be made by improving the efficiency with which current resources are allocated. 21

To assess the potential effects of fiscal equalization, we examine two potential reform proposals that equalize the size of the budget directly, and a third reform proposal that equalizes purchasing power by changing normalized input prices. First, we consider the option of “levelling up” school expenditures by increasing the budgets of school districts with below-average spending. The 137 school districts in our sample that already have above-average budgets would be unaffected, while the budget sets would expand to

\[ G(p/c, z) \] for the remaining 173 schools in our sample. Second, we consider equalizing total per-pupil expenditures across all school districts at the sample mean. This reform would be analogous to giving each school district a budget set of \( G(p/c, \text{mean}) \), where \( \text{mean} \) is the sample mean expenditure per pupil. The remaining reform adjusts both input prices and total expenditures so that all school districts face the mean observed normalized price for each input \( G(p/c, \text{mean}) \). The first equalization proposal requires additional resources for education, while the second two proposals redistribute the existing level of resources.

It is worth noting that, under reforms 2 and 3, the production possibilities frontier for some of the observations will be reduced due to decreases in \( c \) or increases in \( p/c \). For example, when equalizing budgets at the mean level \( G(p/c, \text{mean}) \), \( G(p/c, z) \) will be smaller for some observations and larger for others. When \( G(p/c, z) \) expands, \( FE \) is greater than 1, and when it contracts, \( FE \) is less than 1.

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19 Although \( u_i/||u_i|| \) in equation (8) can be considered exogenous in the Farrell framework, we were concerned that we had induced a correlation between the regressors and the residuals by exploiting the homogeneity property of the indirect output distance function. If so, the parameters of (8) are unbiased and efficient estimators only when estimated with instrumental variables. We used Hausman’s $m$-statistic to test for the exogeneity of the regressors and could not reject the hypothesis that the parameter vector of the model specified in (8) is consistently and efficiently estimated using seemingly unrelated related regression. Hausman’s $m$-statistic is

\[
m = N(\hat{\beta}_1 - \hat{\beta}_2)/\{V(\hat{\beta}_1) - V(\hat{\beta}_2)\}^{-1/2}\}
\]

where $\hat{\beta}_1$ and $\hat{\beta}_2$ are the parameter estimates under SUR and 3SLS, respectively, $N$ is the number of observations, and $V(\hat{\beta}_1)$ and $V(\hat{\beta}_2)$ correspond to the inverse of the information matrix under each estimation method. Hausman (1978) has shown that the $m$-statistic is distributed as $\chi^2_K$, where $K$ is the number of parameters estimated. In the indirect distance function given by (8) there are 66 parameters with 24 restrictions for a total of 42 free parameters to be estimated. We used second-order and cross terms as instrumental variables for the output vector in the three-stage least-squares estimation. The value of the test statistic is $m = 14.29$. The critical chi-square with 1% significance and 42 degrees of freedom is 66.206. We would like to thank Susan Porter-Hudak for her advice on this issue.

20 The derivation of the standard errors may be obtained from the authors, who thank the referee for recommending this approach.

21 While the average level of technical inefficiency is rather high, other studies of efficiency for firms operating in the private sector have found inefficiency near the same level. For one example, see the study of air carriers by Schmidt and Sickles (1984).
outputs if its degree of inefficiency were unchanged but it faced a revised budget constraint. Using FE, we forecast school district output under reform as \( \hat{y}_s = y_s \cdot FE_s \). We then examine the distribution of schooling outcomes by comparing coefficients of variation (CV). In calculating the CVs, each observation is weighted by the corresponding school district’s share of total sample enrollment.

Table 5 presents the CVs for educational outcomes under the status quo and under each of the reform proposals. The value-added CVs measure the distribution of school outputs for each grade level (\( \hat{u}_{s,g} \)). The test-score CVs measure the distribution of student achievement for each grade level (\( \hat{u}_{s,g} + \text{STUINPUT}_{s,g} \)).

Each of the reform proposals generates a distribution of student achievement and school outputs that is dramatically less equal than the status quo. Holding the distribution of inefficiency constant, equalizing expenditures at the mean would double the CVs for school outputs and nearly double the CV for student achievement. Equalizing both input prices and total expenditures would produce a slightly more even distribution of outcomes than would equalizing only expenditures, but it would still reduce equality. Leveling up would also reduce equality.

Efforts to increase fiscal equality fail to produce outcome equality because they fail to consider differences in fixed inputs. Judging by CVs, there is substantially more variation in the fixed inputs than there is in school outputs prior to reform. Variations in the capital stock are particularly pronounced and more than twice as great as variations in personnel expenditures. Furthermore, the pattern of school

For each of the three reforms in this study, table 4 reports the mean values for our efficiency measure given by (10) and our equity measure given by (12). The FE measures indicate that reallocating resources among school districts could produce modest increases in output. For example, consider the second reform proposal, in which funds are redistributed so that all school districts in the sample can spend the same amount per pupil. The 173 school districts with below-average expenditures would experience an increase in their budgets (\( FE > 1 \)) while the remaining 137 school districts would experience a decrease in their budgets (\( FE < 1 \)). On average, \( FE = 1.011 \), suggesting that such redistribution would increase the state’s total educational output by approximately 1%, even if the school districts became no more efficient.

The other two reform proposals would have similar effects on output, assuming no change in efficiency. The average gains in output from equalizing purchasing power (reform 3) would be nearly 1% (\( FE = 1.007 \)). The average gains in output from leveling expenditures up to the mean (reform 1) would be substantially greater (average \( FE = 1.040 \)), but this option would also require a substantial increase in expenditures.

More strikingly, the analysis suggests that finance reforms could actually increase the inequality of schooling outcomes. For each of the reform proposals, FE represents the proportion by which the school district could expand all

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Level budgets below mean up to mean ( p(c_{\text{mean}}) )</td>
<td>All districts:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EFF</td>
<td>310</td>
<td>0.708</td>
<td>0.058</td>
<td>0.567</td>
<td>1.000</td>
</tr>
<tr>
<td>FE</td>
<td>310</td>
<td>1.040</td>
<td>0.061</td>
<td>1.000</td>
<td>1.379</td>
</tr>
<tr>
<td>Cost/pupil</td>
<td>310</td>
<td>85.50</td>
<td>114.68</td>
<td>0</td>
<td>566.46</td>
</tr>
</tbody>
</table>

| Districts below mean c: | | | | | |
| EFF | 173 | 0.726 | 0.056 | 0.595 | 1.000 |
| FE | 173 | 1.071 | 0.066 | 1.001 | 1.379 |
| Cost/pupil | 173 | 153.20 | 114.87 | 0 | 566.46 |

| Districts above mean c: | | | | | |
| EFF | 137 | 0.685 | 0.053 | 0.567 | 0.810 |
| FE | 137 | 1.000 | 0 | 1.000 | 1.000 |
| Cost/pupil | 130 | 0 | 0 | 0 | 0 |

| 2. Equalize budgets to mean \( p(c_{\text{mean}}) \) | | | | | |
| EFF | 310 | 0.708 | 0.058 | 0.567 | 1.000 |
| FE | 310 | 1.011 | 0.091 | 0.733 | 1.379 |
| Cost/pupil | 310 | 11.95 | 253.87 | -1389.37 | 566.46 |

| 3. Equalize purchasing power at mean \( p_{\text{mean}}(c_{\text{mean}}) \) | | | | | |
| EFF | 310 | 0.708 | 0.058 | 0.0567 | 1.000 |
| FE | 310 | 1.007 | 0.075 | 0.850 | 1.409 |
| Cost/pupil | 310 | 1.46 | 15.80 | -57.83 | 41.67 |

22 Equalizing input price vectors has slightly less effect on output than equalizing total budgets because relative input prices are changing in the former case and not in the latter.
district inefficiency seems to mitigate some of the variations in fixed inputs. We calculate that the CVs for reform proposal 3 (identical budget sets) would be between 20% and 25% greater if all school districts were operating on the production possibilities frontier.

VI. Conclusions

This paper has employed a new methodology for examining questions relating to efficiency and fiscal equalization in schooling. The method allowed a multiple-output production technology to be specified for a public enterprise that is restricted by a budget constraint and has no a priori behavioral objective. The analysis suggests that school finance reforms designed to equalize expenditures can affect student achievement, but that any potential gains in output from redistribution are small compared with the potential gains from increased efficiency. More strikingly, the analysis clearly demonstrates that legislative initiatives equalizing expenditures could actually increase the inequality of school products and educational outcomes.

REFERENCES


Färe, Rolf, and Shawna Grosskopf, Cost and Revenue Constrained Production (New York: Springer-Verlag, Bilkent University Lectures Ser., 1994).


