MONETARY POLICY WHEN INTEREST RATES ARE BOUNDED AT ZERO

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Abstract—This paper assesses the importance of the zero lower bound on nominal interest rates for the interest-rate channel of monetary policy. We simulate several interest-rate setting policy rules with either high or low inflation targets. We determine the extent to which the zero bound prevents real rates from falling, thus cushioning aggregate output in response to negative spending shocks. For small temporary and large permanent shocks, the output path with zero inflation lies modestly below that for higher inflation. For large shocks persisting a few quarters, differences in output paths across high- and low-inflation scenarios can be larger.

I. Introduction

This paper assesses the importance of the zero lower bound on nominal interest rates for the conduct of monetary policy. In the context of arguing that the optimal rate of inflation is positive, Summers (1991) stated that a possible drawback of aggregate price stability is that the central bank would be constrained in its ability to offset adverse spending shocks because nominal interest rates cannot turn negative. Cushioning output appreciably in the face of a negative demand shock may require moving long-term real rates down significantly. If short-term nominal interest rates were already low before the shock because inflation was low, the central bank may not be able to reduce short-term real rates much. The argument assumes implicitly that the inability to lower short-term real rates significantly impedes the downward adjustment of long rates.

In this paper we assess this argument using a small forward-looking model. The model was estimated by Fuhrer and Moore (1995b). It incorporates multiperiod pricing contracts, a standard IS curve that depends on long-term real interest rates, and a forward-looking bond market in which real long-term rates are set consistent with market participants’ expectations of future short-term real rates. This model and its characteristics are described in some detail in the next section.

We examine solution paths for the model under higher and lower rates of inflation and a variety of monetary policy reaction functions. We take the higher rate of inflation to be 4% and the low rate to be zero. (We have ignored biases in price indexes that may cause the desired measured rate of inflation to be positive rather than zero.) We assess differences in the high- and low-inflation scenarios by examining relative deviations of output from baseline.

We note that focusing on the differences in output gaps is only an approximation to welfare analysis, in two respects. First, a zero aggregate output gap may still imply nonzero deviations from optimal output for individual firms. Second, this approach ignores the possible welfare costs of positive and/or variable inflation. We abstract from these considerations because the model that we use has no means of reflecting either the deadweight loss to holders of non-interest-bearing money or the welfare loss associated with variable (and presumably unpredictable) inflation. However, we feel that it is unlikely that the low inflation rates studied in this paper correspond to those associated with high inflation variability (and low predictability), as documented in the empirical work of Logue and Willett (1976) and Engle (1983).

The monetary policy reaction function specifies the response of the monetary policy instrument—nominal short-term interest rates—to deviations of nominal income or nominal income growth from target. Thus we do not study the implications of the zero lower bound on interest rates under “optimal” monetary policy. Rather, we study the implications of the zero lower bound for various interest-rate rules of the type often used to characterize recent Federal Reserve behavior (see, e.g., Taylor (1993)).

One limitation of this approach is that it focuses solely on the interest-rate channel for monetary policy. Lebow (1993) and others have suggested that the Fed might circumvent the zero constraint by flooding the economy with reserves when nominal interest rates reach zero. The presumption is that this would raise inflation and lower real interest rates, providing the desired stimulus. We do not explore this possibility in this paper for two reasons. First, in the model we use, inflation rises only when output exceeds potential. Thus reserves expansion will not increase inflation when nominal rates and inflation are zero. Second, we find no empirical support in U.S. data for the kind of “real balance” effect that would admit a direct influence of reserves on aggregate demand (and thus on inflation).

We consider two types of demand shocks:

- Permanent unanticipated
- Temporary unanticipated

Simulations for anticipated shocks were also carried out, but shed little light on the issue addressed in this paper.

In the case of permanent shocks, we reduce the natural real rate by 50 basis points from the model’s estimated

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1 Lebow (1993) examines this as well as other arguments relating to the zero bound on nominal interest rates.

2 See Fuhrer (1994, pp. 288–291) for documentation of this empirical finding.

3 In the context of the current model, long real rates fall sharply in anticipation of a permanent expected shock. Output moves well above its baseline initially, and short rates rise to restrain it. Output then falls upon realization of the shock, but does not move far below its baseline. Consequently, there is little need for policy to move short rates down aggressively, and the zero bound does not come into play.

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long-term real rate of 2.1%. (Given the estimated interest sensitivity of spending in this model, this is equivalent to a drop in aggregate demand of 0.4%). This sort of event may be comparable to the presumably permanent cutback in federal defense expenditures that has recently occurred in the United States. In the case of temporary shocks, the shock is 0.4% of nominal income, for comparability with the situation of permanent shocks, and lasts for one quarter. Lags in the model, of course, extend the impact of the shock over time. Given the extreme simplicity of the model, there is no distinction between lowering government spending and reducing the natural rate of interest.

We can trust the conclusions drawn from our simulations to the extent that (1) the model is well specified, and (2) the shocks to which we subject the model are drawn from the same distribution as the shocks that we identify in estimating the model. Evidence supporting (1) is provided in Fuhrer and Moore (1995b). As for the shocks, the estimated model assumes a linear trend for potential output, so that permanent shocks to output are not identified. Thus we do not know if the permanent shocks entertained in this paper are consistent with the shocks underlying the estimated model. The temporary output shocks that we simulate, however, fall well within the estimated distribution of shocks to the output process.4

We enforce the zero bound on nominal interest rates through three alternative techniques involving the monetary policy reaction function, rather than through a nonlinear money demand equation.5 (In fact, the model includes no money demand equation or variable measuring the quantity of money.) The techniques are:

- The left-hand side of the reaction function is specified in terms of log differences of the short-term nominal rate.
- The left-hand side of the reaction function is specified instead in terms of levels of short-term nominal rates, but the response to nominal income is adjusted to keep nominal rates from becoming negative.
- The left-hand side of the reaction function is specified in terms of levels of short-term nominal rates, but the response to nominal income is not adjusted as above. Instead, nominal rates are allowed to drop to the zero lower bound, and the model is solved conditional on nominal rates being at their bound.

An advantage of the first technique is that such a policy rule can be specified fully in advance; no negative nominal rate is arithmetically possible. Thus a reaction function with given weights on deviations of the targeted variable can be employed in high- and low-inflation scenarios.

The second set of methods specifies the reaction function in levels of interest rates but adjusts the parameters of the reaction functions on an ad hoc basis to prevent nominal interest rates in both inflation scenarios from becoming negative. This technique helps assess how much more aggressive monetary policy can be in fighting recession in terms of the numerical strength of the response to deviations from target.

One advantage of the final method is that it allows the nominal rate to drop quickly to its lower bound, thus representing the most aggressive interest-rate response available in the face of zero inflation. A disadvantage of this method is that it probably implies more aggressive interest-rate responses than are realistic in a policy environment that is complicated by uncertainty.

Most of the simulations were obtained using a technique dubbed “resolver,” which was developed by Madigan (1996). Resolver is a general method for solving nonlinear forward-looking models. (The nonlinearities in the model employed in this paper are in the forward-looking consol equation, which links current short- and long-term rates to future long-term rates, and in the log-difference reaction functions.) Resolver essentially combines the AIM technique of Anderson and Moore (1985) with a Newton–Raphson procedure. The technique is comparable in some respects to relaxation methods for obtaining numerical solutions to differential equations. Simulations of the purely linear models were obtained using AIM.

II. The Model

The simple structural model that we use comprises three sectors: an IS curve that relates output to the ex ante long-term real interest rate, a monetary policy reaction function that moves the short-term nominal interest rate in response to deviations of target variables from desired values, and a price contracting specification in which nominal price contracts are negotiated in real terms. The model has been estimated on postwar quarterly data for the 3-month Treasury bill, the deflator for nonfarm business output, and a measure of the output gap for nonfarm business output, defined as the residual from a regression of log per-capita nonfarm output on a constant and a linear time trend. Maximum-likelihood estimation yields significant estimates of all the structural parameters. The dynamics implied by the model, as summarized by the vector autoregression, is that the dynamics from an unrestricted vector autoregression very well. At the estimated parameter values, the model implies a sensible sacrifice ratio, about in line with the estimates in Gordon (1985). Overall, the model behaves similarly to a conventional macroeconometric model.

4 A third concern is the stability of the estimated parameters across different monetary policy regimes—a potential implication of the “Lucas critique.” Fuhrer (1997) tests the stability of the contracting and IS parameters across three different historical monetary regimes, and cannot reject the hypothesis that the contracting specification and the IS interest elasticity are stable across regimes. The lag coefficients in the IS curve appear to shift slightly after 1982.

5 That is, a possible alternative procedure would involve a demand for central bank money that went to infinity as nominal short rates asymptotically approached zero and a reaction function specified in terms of money rather than in terms of short-term interest rates.
such as the MPS model, despite its forward-looking asset and price sectors. Fuhrer and Moore (1995b) present a more extensive discussion of the model and its properties.

A. The IS Curve

Let $R_t$ be the yield to maturity on a coupon bond selling at par, and let $M$ be the maturity of the bond at the end of quarter $t$. Then the duration of the bond is given by

$$D_t = \frac{1 - e^{-R_M}}{R_t}$$

(1)

and the holding-period yield on the bond from quarter $t$ to quarter $t + 1$ is

$$R_t - D_t(R_{t+1} - R_t).$$

(2)

Here we consider a real consol bond with yield to maturity $\rho_t$. Maturity is infinite, so duration simplifies to $1/\rho_t$. Thus the intertemporal arbitrage condition that equalizes the expected holding-period yields (up to a term premium) on real Treasury bills and the real consol bond is

$$\rho_t - \frac{1}{\rho_{t+1}}[\rho_{t+1} - \rho_t] = i_t - \pi_{t+1},$$

(3)

where $i_t$ denotes the Treasury bill rate, $\pi_t$ denotes the one-period rate of inflation, defined as $\log(P_1) - \log(P_{t-1})$, and $P_t$ denotes the current price level. The abstraction from a term premium could be important in the present context. Equation (3) implies that in the steady state, the short nominal rate will settle at $\rho^*$, the equilibrium long real rate, plus $\pi^*$, the target inflation rate. If part of $\rho^*$ is a term premium $\tau$, then $i_t$ should settle at $\rho^* + \pi^* - \tau$. Beginning from a steady state with low inflation rates and a modest term premium, this implies even less room to lower the nominal rate. In the simulations conducted below, we generally ignore the possibility of a term premium on the real long bond. The implications for this issue of a term premium are discussed in Section IV A.

Because monetary policy in effect targets the inflation rate in the long run, both the short-term real nominal rate and the inflation rate are stationary in this model. This implies that at long horizons, nominal rates and inflation are forecast to be at their means. As a result, the long real rate that is constructed assuming a long duration will not exhibit much volatility (compared with, say, estimated ex post long-term real rates). This observation has been made in the substantial “variance bounds” literature. While the correlation of long real rates with the output gap is unaffected by the scale of long rate volatility, one should recognize that the volatility of long rates depends on the particular monetary policy rule, as well as the parameters of the model. Thus the volatility of long rates observed in this paper’s simulations may not correspond closely with that observed historically.

Given the definition of the expected long real rate, the real economy is represented as a simple IS curve that relates the output gap $\hat{y}_t$ (the deviation of the log of output from the log of potential output) to its own lagged values and one lag of the long-term real interest rate, $\rho_{t-1}$,

$$\hat{y}_t = 0.017 + 1.254\hat{y}_{t-1} - 0.415\hat{y}_{t-2} - 0.798\rho_{t-1} + \epsilon_{yt},$$

(4)

where the parameters are taken from Fuhrer and Moore (1995b) and $\rho_t$ is the rate on consols defined previously. The demand shock $\epsilon_{yt}$ will be an unanticipated temporary or permanent shock in the simulations below. In the steady state, $y = 0$, so the IS curve defines the equilibrium or “natural” real rate of interest $\rho^*$ to be 2.1% (0.017/0.798). Note that the equilibrium real rate includes any real term premium built into the long rate.

One potential concern over using such a simple IS curve when inflation and nominal rates are near zero is that the linear representation will not capture an important nonlinear response of spending to interest rates when nominal rates are near zero. However, the period of estimation for the IS curve includes 1975–1979, during which short-term real rates varied from −6% to 0% and the long-term real rate implied by the model dropped well below its equilibrium. The IS curve shows no sign of misbehaving during this period, suggesting that if a nonlinear response exists, it is not of primary importance for total spending.

B. Contracting Specification

Agents negotiate nominal price contracts that remain in effect for four quarters. The aggregate log price index in quarter $t$, $p_t$, is a weighted average of the log contract prices $x_{t-i}$ that were negotiated in the current and the previous three quarters and are still in effect. The weights $f_i$ are the proportions of the outstanding contracts that were negotiated in quarters $t - i$.

$$p_t = \sum_{i=0}^{3} f_i x_{t-i}$$

(5)

where $f_i \geq 0$ and $\Sigma f_i = 1$. We characterize the distribution of contract prices with a downward-sloping linear function of contract length,

$$f_i = 0.25 + (1.5 - i)s, \quad 0 < s \leq \frac{1}{6},$$

(6)

$$i = 0, \ldots, 3.$$

These parameters are consistent, but inefficient, partial-information estimates. However, they differ insignificantly from the full-information estimates presented in Fuhrer and Moore (1995b). The full-information estimate of the interest elasticity parameter, for example, is $-0.746$, with a standard error of 0.25.
This distribution depends on a single slope parameter $s$ and it is invertible. When $s = 0$, it is the rectangular distribution of Taylor, and when $s = \frac{1}{2}$ it is the triangular distribution.

Let $v_t$ be the index of real contract prices that were negotiated on the contracts currently in effect.\(^7\)

$$v_t = \sum_{i=0}^{3} f_i(x_{t-i} - p_{t-i}).$$  \hfill (7)

Agents set nominal contract prices so that the current real contract price equals the average real contract price index expected to prevail over the life of the contract, adjusted for excess demand conditions,

$$x_t - p_t = \sum_{i=0}^{3} f_iE_i(v_{t+i} + \gamma y^i_{t+i}).$$ \hfill (8)

Substituting equation (7) into equation (8) yields the real version of Taylor’s contracting equation,\(^8\)

$$x_t - p_t = \sum_{i=1}^{3} \beta_i(x_{t-i} - p_{t-i}) + \sum_{i=1}^{3} \beta_iE_i(x_{t+i} - p_{t+i})$$
$$+ \gamma^* \sum_{i=0}^{3} f_iE_i(\tilde{y}^i_{t+i}).$$ \hfill (9)

In their contract price decisions, agents compare the current real contract price with an average of the real contract prices that were negotiated in the recent past and those that are expected to be negotiated in the near future. The weights in the average measure the extent to which the past and future contracts overlap the current one. When output is expected to be high, the current real contract price is high relative to the real contract prices on overlapping contracts.

The contracting specification is parameterized by $s$, the slope of the contract distribution, and by $\gamma$, the response of real contract prices to expected excess demand conditions. The maximum-likelihood estimates of these parameters and their standard errors are given in Table 1.\(^9\)

\(^7\) This is a convenient simplification from the theoretically preferable specification that defines the real contract price as the difference between the nominal contract price and the weighted average of price indexes that are expected to prevail over the life of the contract. The simplification yields an algebraically more straightforward model. The effects of the simplification on the empirical properties of the model are relatively small. See Fuhrer and Moore (1995a) for details on the alternative specification and associated empirical results.

\(^8\) Compare equation (9) with Taylor (1980, eq. (1)). The coefficients in equation (9) are $\beta_i = \Sigma f_i f_{t+i}/(1 - \Sigma f^2_i)$ and $\gamma^* = \gamma(1 - \Sigma f^2_i)$.

\(^9\) In a series of recent working papers, Roberts (1994) finds that the original sticky price specifications of Taylor and others can be reconciled with the persistent inflation data by using survey expectations, rather than rational expectations, to close the model. With survey expectations, the additional persistence imparted by the real contracting specification discussed in this section is not necessary to reconcile the model with the data. Under rational expectations, the maintained assumption in this paper, the early sticky price specifications imply far less inflation persistence than is evidenced in the data.

### Table 1.—Parameter Estimates and Standard Errors

<table>
<thead>
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<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-Statistic</th>
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</thead>
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<tr>
<td>$s$</td>
<td>0.0797</td>
<td>0.0116</td>
<td>6.9</td>
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<tr>
<td>$\gamma$</td>
<td>0.0045</td>
<td>0.0018</td>
<td>2.6</td>
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<tr>
<td>Ljung–Box Q(12) statistic</td>
<td>27.2</td>
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### C. Reaction Function

Monetary policy is represented as a policy reaction function that moves the short-term nominal rate in response to deviations of its ultimate targets from their desired values. In this paper, while the operating instrument is always the short nominal rate, we consider two basic variants of the reaction function:

$$\log (i_t) - \log (i_{t-1}) = \lambda(z_{t-1} - \bar{z}^*)$$ \hfill (10)

$$i_t = \rho^* + \pi^* + \lambda(z_{t-1} - \bar{z}^*).$$ \hfill (11)

In both cases $z$ is the ultimate target of monetary policy, and a larger $\lambda$ implies a more aggressive policy response to deviations of $z$ from its desired or target value. The first variant imposes the nonnegativity constraint on nominal rates and incorporates the incentive to smooth interest rates. In the second variant, the response to nominal income $\lambda$ is adjusted on an ad hoc basis to keep nominal rates from becoming negative. This technique effectively does away with interest-rate smoothing, and in a stronger sense than the first variant implies that the monetary authority knows the equilibrium long-term real rate. The ultimate target is either nominal income or nominal income growth.

### III. Simulations

#### A. Permanent Unanticipated Shocks

This section discusses simulations conducted under a permanent unanticipated shock. The shock increases the output gap initially by 0.4% by reducing the natural rate of interest by 50 basis points. In the post-shock steady state, real long rates will be 1.6% (2.1% minus 50 basis points); short nominal rates will be 5.6% with 4% inflation and 1.6% with zero inflation.

**Operating Instrument—Log-DifferenceNominal Rates:** Figure 1 illustrates a simulation using a log-difference reaction function and a nominal income target. The response of interest rate differences to deviations of the level of nominal income from target—-$\lambda$—is set to a value of 60. This value is the maximum at which a simulation could be obtained. As shown by the solid line in the upper panel, the nominal short rate in the zero-inflation case adjusts down over a period of about a year by a total of nearly 1 1/4 percentage points. By contrast, the nominal rate in the high-inflation scenario (shown by the dashed line) falls about 3 percentage points. As would be expected with the log-difference reaction
function, the percent reduction in nominal interest rates is similar in the two cases. The small difference reflects the slightly stronger nominal income in the high-inflation case and the feedback through the reaction function to the nominal rate.

The middle panel of figure 1 shows that the long real rate overshoots in both cases—it initially falls by more than the 50 basis point decline in the natural real rate, as markets bring forward in time the lower short real rates in the future that will result from lower nominal short rates combined with only sluggishly declining inflation. In the zero-inflation case, the long rate falls 56 basis points right away and then trends up gradually. In the high-inflation scenario, the long rate initially drops a bit more—62 basis points. The long real rate then rises more steeply than in the low-inflation case, reflecting the anticipated need for monetary policy to lean more heavily against the stronger upward burst of output shown in the lower panel. The drop in real rates is obviously similar across the two scenarios.

The drop in output in the first quarter is identical in the two cases—0.4%. The model incorporates a one-quarter lag in the response of demand to interest rates, so the drop in output in the first quarter represents solely the exogenous decline in demand, and hence is identical in the two scenarios. The slightly lower initial real rates of the high-inflation scenario cushion output in the second quarter, essentially preventing it from falling further as it does in the zero-inflation case. The difference, however, is slight—less than 0.1% of the level of output. Output subsequently recovers a little more steeply in the high-inflation case. The end of the recovery—defined as the point at which output “recovers” its prerecession level—comes about a quarter earlier. The subsequent cycles are of greater amplitude in this case.10

10 The thinner lines in the bottom two panels of figure 1 indicate 90% confidence intervals. These confidence intervals reflect the uncertainty in the simulated paths arising from the sampling error in estimating the model’s coefficients. The intervals depicted are the fiftieth highest and
Figure 1 provides 90% confidence intervals for the simulations (Computational details are provided in footnote 10.) As the figure indicates, the time paths of the simulated real rate and output gap are significantly different from zero for the early years of the simulation, especially for the 0% simulations. The long-term real rate falls significantly below its long-run equilibrium, and the output gap turns significantly negative in the first two years. In the terminal years of the simulation, of course, the confidence intervals suggest that the small deviations from the model equilibrium are not significantly different from zero. Because the amount of uncertainty arising from sampling error is quite small for this model, we do not depict confidence intervals in the following charts.

We compute a second simulation somewhat similar to that of figure 1, except that the policy reacts to nominal income growth rather than nominal income levels. Under nominal income growth targeting, the price level will be lower in the post-shock steady state than under its baseline rate, while the inflation rate will return to its baseline rate, which is equal to the targeted growth rate of nominal income. By contrast, under nominal income targeting, both the price level and the inflation rate ultimately return to baseline after a shock. The requirement that the price level return to baseline in the nominal income targeting case induces additional cycles in the solution relative to the nominal income growth case, which only requires the inflation rate to return to its targeted level.

With policy reacting only to income growth rather than income levels, the short rate is reduced by less and is brought up to the vicinity of its new equilibrium sooner. Real rates consequently drop a little less. In the low-inflation case, the real long rate drops immediately to, but not below, the new natural rate, while in the high-inflation case the long rate overshoots. The lower real rates of the high-inflation case bring the output gap to zero appreciably more quickly than in the zero-inflation scenario.

Operating Instrument—Nominal Short-Rate Levels: In the preceding section the 0% floor on nominal interest rates was enforced by considering reaction functions in log-difference form. In this section the operating instrument is considered to be levels of short-term nominal rates, as in equation (11). The policy responsiveness coefficient $\lambda$, is adjusted on a case-by-case basis in such a way as to allow the nominal short rate to fall to, but not below, zero.

Such a reaction function, that is, one specified in terms of levels, seems most consistent with a view that the central bank can determine the true level of the natural rate of interest with a high degree of precision. In such a situation, the central bank presumably would wish to move interest rates promptly to appropriate levels. By contrast, the previous section’s log-difference reaction function, which embodies interest-rate smoothing, might better characterize policy as actually practiced, as monetary policy makers take into account uncertainty, any costs of interest-rate variability, and perhaps an aversion to frequent reversals of course.

As shown by the dashed line in figure 2, in the 4% inflation case, policy lowers the short rate to zero in the second period, after nominal income began to fall significantly below target in the first period. That is, nominal short rates fall 6.1 percentage points almost immediately. This responsiveness corresponds to a $\lambda$ equal to 14. By contrast, the lower level of nominal rates in the low-inflation case permits a much less aggressive policy response: short rates can fall only 2.1 percentage points, corresponding to a $\lambda$ of 3.5.

The lower panel shows that the larger move in nominal rates in the 4% inflation case results in a sharper initial drop in the real long rate, by about 9 basis points for two periods. Long rates subsequently move up more strongly in the high-inflation situation. The larger drop in real rates in this case causes the recession (defined as the period during which the output gap is growing) to end after one quarter, whereas the recession in the low-inflation case lasts two quarters. The recovery similarly ends sooner in the 4% inflation scenario. Output overshoots and cycles a little in the high-inflation case. Although output also overshoots slightly in the low-inflation case, the approach to equilibrium is more gradual.

As in the previous simulation pair, we compute a simulation for an interest-rate level operating target and a nominal income growth ultimate target. The high- and low-inflation cases use $\lambda$ equal to 14 and 6.5, respectively. In both simulations, monetary policy drops nominal short rates sharply as an output gap opens and inflation falls below target, leading to a shortfall in nominal income growth. The easing is reversed quickly, however, as a drop in the real long rate prompts a rebound in real output that pushes nominal income growth roughly back to target. In both the high- and low-inflation cases, after a few quarters the nominal short rate actually gets pushed a bit above its new long-run equilibrium, and this overage is transferred to the real long rate. Very slowly, the real long rate drifts toward the new natural rate, bringing the output gap eventually to zero. Although output is a little higher in the second through sixth quarters in the high-inflation case, the difference is small.

B. Temporary Unanticipated Shocks

This section generally considers a temporary 0.4 percent shock to aggregate demand; in most simulations, the shock occurs in the first period of the solution and lasts for one
quarter. Section 3.2.2 also considers a longer-lasting temporary shock and a reaction function that includes forward-looking elements.

**Operating Instrument—Log-Difference Nominal Short Rates:** With a log-difference reaction function and \( \lambda \) set equal to 30, short rates in the high-inflation case decline about 4 percentage points over the span of a few quarters, while short rates fall \( 1\frac{1}{2} \) percentage points (to 65 basis points) in the low-inflation case. (This simulation is shown in figure 3.) Real long rates drop considerably further initially with 4% inflation—35 basis points, as opposed to 20 basis points in the zero-inflation case. The lower real rates permit a somewhat steeper recovery of output, but the pattern is not markedly different. The relatively modest variation in output across the two cases reflects the small difference in long rates measured in percentage points.

**Operating Instrument—Nominal Short-Rate Levels:** Figure 4 shows results for temporary unanticipated shocks and an interest-rate-levels reaction function. Again, the high-inflation case permits a substantially more aggressive response measured in terms of percentage point movement in nominal short rates. (The policy responsiveness parameter \( \lambda \) is equal to 10 and 1.65, respectively, in the two cases.) Consequently the decline in real long rates is more than twice as steep. But because the percentage point difference in long rates is relatively small, the trajectory of output during the recovery is only a little steeper, ending the recovery about a quarter earlier.

Figure 5 considers a more severe shock—an unanticipated 1% drop in demand that persists for four quarters. For these simulations, we also modify the assumption concerning the recognition of the shock. We assume that although the shock is not anticipated in advance, once it begins, the
magnitude and duration of the shock are recognized accurately. We assume further that the policy reaction function incorporates a contemporaneous and forward-looking response to this shock, once it is recognized. Specifically, the reaction function is

$$i_t = \rho^\pi_i + \pi^T + \lambda(z_{t-1} - z_t) + G_t + G_{t+1} + G_{t+2} + G_{t+3}$$

(12)

where $G$ denotes the shock, which may represent reduced government spending or some other factor depressing aggregate demand.

In this simulation, nominal short and real long rates drop immediately, but the nominal short rate can drop much more sharply—and stay low for longer—under the high-inflation case. Over the first year, the real long rate averages about 25 basis points lower in the high-inflation case. With real rates unable to fall as sharply in the low-inflation case as in the high-inflation case, the decline in output is significantly sharper in the first case. The recoveries in the two cases proceed in parallel, with the level of output higher in the 4% inflation case, and the output gap is closed about one quarter sooner in this case.

This shock pattern seems particularly informative because the depth of the ensuing recession, measured as the percentage point shortfall of output below potential, matches the depth of a "typical" recession in postwar U.S. data. In the 4% inflation scenario, the output gap reaches a trough of 2.8%, while in the zero inflation case, the gap troughs at 3.6%. The 0.8 percentage point absolute difference in output gaps in the two scenarios probably would be regarded by many as economically meaningful.\(^{11}\)

\(^{11}\) Recession depths may be estimated either from a log detrended output series (with the trend broken in 1973) or from an output gap series implied by the unemployment rate and an inverted Okun's law.
Discontinuous Interest-Rate Response: Many of the policies discussed above are more vigorous than those pursued historically by the monetary authority. However, the central bank could respond even more vigorously to an adverse demand shock than in the continuous responses depicted in figures 2 and 4. In those simulations, we use the largest policy coefficient that yields a continuous path of interest rates that lies above zero. An alternative strategy would employ more vigorous policy coefficients that send the nominal rate quickly to the zero bound. Such a “bang-bang” policy might be vigorous enough to offset much of the constraint imposed by the lower bound under the continuous policies explored above.\(^{12}\)

\(^{12}\) The model used for this simulation substitutes the linear version of the term structure equation. At each time period, if the solution value of the short rate falls below zero, the short rate is set to zero and exogenized (for that period only). We then re-solve for the other variables in the model conditional on that exogenized value for the short rate. The solution then proceeds on a period-by-period basis, imposing the zero lower bound in this way.

Figure 6 depicts such a simulation. The simulation uses the aggressive nominal income targeting parameter (\(\lambda = 1.5\)) of figure 5 for both the zero inflation and the 4% inflation case. The simulation is conducted so that if the nominal rate were to transgress its lower bound for a given period, it is constrained for that period to remain at its lower bound, and the other variables in the model are solved for conditional on that constraint. Comparing figure 6 with figure 5, the ability to drop the short rate immediately to its zero bound has a relatively small effect on the real rate and the output gap. Thus while the other interest-rate level simulations depicted in figures 2, 4, and 5 do not allow policy to discontinuously drop the short rate, it is likely that this restriction has relatively little effect on the simulated paths of real rates and output.
IV. Additional Considerations

The differences identified in the previous section between output under high- and low-inflation cases may depend in part on certain aspects of the model’s specification and parameters. In this section we consider the following modifications:

- The existence of a term premium in long-term interest rates, which implies a lower steady-state level of real short rates, may limit the ability of monetary policy to stimulate economic activity.
- If short real rates, in addition to long real rates, affect spending, the ability of long rates to jump down, cushioning the effects of an adverse spending shock, would be less relevant and the behavior of short rates would be more relevant.
- If bond markets are partly backward looking, bond rates may be less apt to jump down when news becomes available about adverse spending shocks.

A. Term Premium in Interest Rates

As noted, the previous simulations assume that real long-term interest rates contain no term premium. If there is a term premium in long rates, however, the ability of monetary policy to ease in response to an adverse spending shock would be more constrained, because the equilibrium short real rate would be lower than the equilibrium long rate by the amount of the term premium. Whitesell (1990) estimated that the equilibrium real rate on three-month Treasury bills between 1978 and 1992 was between $\frac{1}{2}$ and $1\frac{5}{8}$ percentage points below that on ten-year Treasury notes. In this section we assume a constant term premium in long-
term real rates of 1 percentage point. In contrast to the estimation results underlying the previous simulations, we assume that the equilibrium real short-term rate and the equilibrium real long rate excluding the term premium have already been lowered by some exogenous event and are 1.1% rather than 2.1%.

We compute a simulation that is comparable to that shown in figure 1, except for the term premium. Because nominal short rates start out at 1.1%, rather than 2.1%, and because the reaction function is specified in log-difference form, the initial percentage point drop in short rates in the low-inflation scenario is only about one-half that in the 4% inflation case. Consequently, real long rates cannot fall quite as far initially with a term premium. Although the relative difference in the behavior of long rates across the two cases seems small, it lasts for half a year and is large enough to generate a relatively more robust recovery in the high-inflation case with a term premium; the recovery is completed noticeably more quickly with higher inflation. Across the low-inflation scenarios, output runs about 0.1% lower under the term-premium case.

B. Shorter Term Real Rates in IS Curve

Our simple characterization of aggregate spending makes the output gap a function only of long-term real rates. And...
yet it is likely that many important expenditure categories—auto purchases, business equipment expenditures, and even new home purchases—depend at least in part on shorter term real rates. If expenditures do depend on shorter term rates, then the dynamics of the monetary transmission mechanism might be significantly altered, and our simulations might not be representative of the response of the economy to output shocks at low inflation. In particular, the ability of the real long rate to jump down in both high- and low-inflation scenarios would be less relevant, and the behavior of the short rate would be more relevant, to the potential disadvantage of a low-inflation policy if a constraint on the inability of short nominal rates to decline also prevented short real rates from falling appreciably. On the other hand, monetary policy likely has more control over a shorter duration real rate than a longer duration one, despite the zero bound, unless short real rates were quite close to zero before the shock. Consequently, it is not clear that the presence of short rates in the IS curve necessarily disadvantages a low-inflation policy.

To test the importance of this modification to our specification, we simulate the model with an IS curve with both a short-duration (eight quarters) and a long-duration (40 quarters) real rate. For comparison with the simulation depicted in figure 1, the operating instrument is a log-difference reaction function.\(^{15}\) In this simulation, we simulate the model with an IS curve with both a forward-looking real rate, and the rate at which the backward-looking weights decay into the past, which expands to

\[
\rho_t^b = (1 - \delta) \sum_{j=0}^{\infty} \delta^j (i_{t-j} - \pi_t).
\]

By varying the degree of “backward lookingness” \(w\) in the real rate, and the rate at which the backward-looking weights \(\delta^j\) decay into the past, we can get an idea of the sensitivity of our results to the real-rate specification.

Setting \(\delta = 0.98\) (which implies weights decaying into the past at the same rate as the weights decay into the future for the forward-looking real rate), \(w = 0.95, \lambda = 50,\) and the interest elasticity to its benchmark value, the model is stable with a log-difference reaction function.\(^{15}\) In this simulation, the real long rate still jumps down considerably, despite the very large parameters on the backward-looking component of real rates. But it takes three to four quarters for the rate to reach its trough, unlike the case in figure 1, where long rates reach their low point essentially immediately. As in figure 1, real rates are lower in the high-inflation scenario than in the low-inflation case for the first four quarters or so, but the relative differences between the purely forward-looking case and the mixed forward/backward case are quite small. Consequently, the relative paths of output are similar. Overall, incorporating backward-looking behavior in the bond markets does not alter the qualitative properties of the model simulations.\(^{16}\)

\(^{14}\) The simulation uses an eight-quarter bond as an approximation to an eight-quarter duration bond. At the steady-state interest rate of 1.6%, an eight-quarter bond has a duration of 7.55 quarters. The jointly estimated coefficient on the short-duration real rate is zero. If we exclude the long-duration real rate from the IS equation, the estimated coefficient on the short-duration real rate is 0.079, ten times smaller than the estimated coefficient on the long real rate. (One would expect the coefficient to be smaller, since shortening the duration makes the long rate move more closely with the short rate, increasing its volatility.) We simulate the model with both long- and short-duration real rates in the IS curve by imposing a weight of \(\delta^t\) on the long rate and \(\delta^s\) on the shorter duration rate, using the coefficients estimated for each (0.79 for the long rate, 0.079 for the shorter rate).

\(^{15}\) Interestingly, the stability of the model is sensitive to the exact combination of \(w, \delta,\) and IS interest elasticity. For example, with \(w = 0.5, \delta = 0.9,\) and the interest elasticity in equation (4), the model does not have a stable, unique solution. The backward-looking long rate places too much weight on the recent past, and thus exerts a destabilizing force on output and inflation.

\(^{16}\) For more details on the simulations described in this section, see Fuhrer and Madigan (1994).
behavior both in financial markets and in product markets and whose broad properties correspond with those of large-scale macroeconomic models. Our results indicate that the argument is correct, qualitatively speaking. Although long-term real rates in forward-looking bond markets do decline in response to news about adverse spending shocks, thus cushioning the reduction in output, the decline in real rates can be constrained by the inability of nominal rates to fall below zero.

We find that for relatively small and short-lived spending shocks, as well as for permanent and large shocks, the path of output in the zero-inflation case is only modestly below that in the higher inflation case—on the order of 0.1% or 0.2%; the recession and the recovery tend to be completed one quarter later with higher inflation rates. But for large shocks persisting a few quarters, differences in output paths across high- and low-inflation scenarios can be larger—on the order of 1% of output. These results appear to hold for several types of monetary policy reaction functions when monetary policy responds quite vigorously to shocks. More measured responses of monetary policy could reduce the differences across high- and low-inflation cases, depending on the form of the monetary policy reaction function, and could even eliminate them for small shocks. For example, a reaction function that adjusted rates in terms of arithmetic differences rather than log differences and that responded cautiously to deviations of target variables from their targets might not be constrained by the zero bound on interest rates in cases of small shocks.

Consideration of several complications not included in the basic simulations produced mixed results. Allowance for a term premium in long-term interest rates tends to augment differences across inflation scenarios. Including shorter term rates in the IS curve also tends to be unfavorable to lower inflation, although our simulations suggest that this effect is not large. Assuming that bond markets are not fully forward-looking seems to have little effect on the conclusions.

The quantitative results of this paper clearly are no doubt model-specific. They depend on the specification and parameterization of the model. In addition, their practical implications depend on how quickly a central bank can recognize shocks and how vigorously it can respond to them. The assessment of these issues no doubt varies considerably with the nature of the shock. Moreover, other economic policies—for example, fiscal policy—may be useful in fighting recession even when conventional interest-rate monetary policy is hamstrung by the zero bound on nominal interest rates. Conceivably, monetary policy actions other than open-market operations, such as lending through the discount window, could also provide stimulus by lowering spreads of private interest rates over yields on government securities. In any case, though, this paper has provided an initial quantification and a starting point for future research on the issue of the relevance of the zero bound.

REFERENCES


