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Complex systems and inter/transdisciplinary research: A review FREE

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I. INTRODUCTION AND BACKGROUND

Complex systems are made up of many components that interact with each other, and non-linearity in the dynamics plays an essential role. The interesting properties of these systems arise as a result of the interaction of these components and cannot be studied by observing only a small part of the system. The large number of these makes fluctuations an inherent part of their dynamics. One of the notable characteristics of these systems is that order can arise spontaneously, which is called self-organization, and that "in practice," the present does not always determine the future (as Newton teaches), regardless of the number of computers we have available. This is what we call chaos.[1] There are diverse examples of complex systems, such as the Earth's global climate, disordered materials, lasers and ultrashort pulses, social organizations, the human brain, transportation and communication systems, and air turbulence, essential for the airline industry.

In 2021, three laureates shared the Nobel Prize in Physics[2,3] for their studies of complex phenomena. Syukuro Manabe and Klaus Hasselmann laid the foundation of our knowledge of the Earth's climate and how the humanity influences it, and Giorgio Parisi was rewarded for his revolutionary contributions to the theory of disordered and random phenomena.

Complex systems invite inter/transdiciplinary research. Interdisciplinary research is understood as that whose subject of study and research question integrates different disciplines and methodological approaches, while by transdisciplinary research, it is understood that its subject of study and research question cross different disciplines transversally, generating knowledge that cannot be produced with the disciplines separately.

During December 2021, the second version of the International Workshop on Statistical Physics (IWoSP) was held in Antofagasta, Chile, in online and face-to-face modes. The main topics covered were Statistical Mechanics and Fundamentals, Systems out of Equilibrium, Nonlinear Phenomena, Dynamical Systems, Complex Systems, and Inter/Transdisciplinary research. The purposes of IWoSP meetings are to keep updating the scientific community on the new developments and tendencies in statistical mechanics and interdisciplinary research, to incentive collaborative international science programs, and to identify and discuss the most relevant advances in the area.

The current issue gathers the contribution of leading scientists that delineate the state of the art in areas strongly influenced by Complex Systems and Inter/Transdisciplinary Research. The authors have made special effort in writing 26 high-level articles with an introduction general enough so as to reach a wide audience and introduce people outside their fields to their research.

II. SUMMARY OF AREAS COVERED

A. Non-interdisciplinary articles

In recent decades, extensive research has explored non-linear diffusion equations, seeking their application in physical and complex systems displaying deviations from standard behavior. The article by Curilef[1] has a dual purpose: first, to enhance our understanding of non-linearity in diffusive systems by deriving non-linear diffusion equations for two specific examples and second, to introduce a method for analytically solving a type of non-linear diffusion equations. This method utilizes a power-law ansatz, maximizing the Tsallis entropy and producing solutions for coupled equations describing spatial and temporal distributions. The study holds promise for uncovering additional non-linear phenomena in nature through analytical or semi-analytical approaches.

The authors Cisternas *et al.*[5] consider the nontrivial behavior of a set of macroscopic magnetic rotors, positioned at the vertices of a regular polygon. By adjusting the rotation planes of the magnets, it is possible to induce qualitative transitions and find new configurations. A complete understanding of the system can be obtained using group and bifurcation theories. Symmetry considerations are also relevant in the splitting of the phase space into basins of attractions associated with the stable equilibria. In this article, the analyses are carried out exhaustively for a small number of magnets, but the authors are confident that there are patterns that persist for larger symmetric systems.

Self-propelled entities may exhibit surprising collective behaviors under the presence of the appropriate interaction; for example, in the flocking transition, the collective moves in unison. Escaff[6] considers an apolar interaction that leads to the formation of two counter-propagating flocks. For short-range interactions, the system forms two counter-propagating clusters, which collide, exhibiting typical behaviors of dissipative solitons. They interpenetrate, continuing their movement or forming a bound state where the clusters remain together. All these self-organized behaviors were analyzed by using mean-field strategies. Consequently, there remain open questions about the role of fluctuations, especially in the meta-stability of bound states.

Michea and Velazquez[7] explore the thermodynamic equilibrium of a binary astrophysical system in the presence of negative heat capacity to extend Thirring conditions of stability. They addressed the orbital motion within the quadrupole–orbit approximation to propose a Langevin equation to describe the evolution of collective degrees of freedom under the incidence of thermal fluctuations associated with internal degrees of freedom. According to their preliminary calculations, thermal and dynamical stability is possible within a quasi-stationary sense: the binary system eventually develops instabilities (escape or collapse) that depend on the internal temperatures, the angular momentum, and the quadrupole–orbit parameter.

Atenas *et al.*[8] investigate complexity and disequilibrium in a system with long-range interactions, known as the d-HMF (dipole-type Hamiltonian mean field) model. It is shown that the behavior of the López–Mancini–Calbet complexity aligns with expectations, which peaks at a particular value and diminishes at higher temperatures. On the other hand, the quasi-stationary states are an example of intriguing phenomena observed in nonequilibrium dynamics of systems with long-range interactions. Could the complexity or the disequilibrium glimpse something about the nature of these states?

The article written by González Díaz[9] explores the derivation of the diffusion equation using the maximum caliber principle and the continuity equation. It reveals an inverse relationship between a particle's mass and diffusion coefficient, with higher mass resulting in lower diffusion and vice versa. Monte Carlo simulations support these findings, demonstrating diffusion behavior for different masses by simulations. The study underscores the utility of the maximum caliber principle in understanding non-equilibrium physics and offers insights into the diffusion coefficient's dependence and particle mass. Open questions for further exploration include finding an exact relationship for kinetic theory, comprehensive identification of its components, and investigating the implications of the inverse relationship between diffusion and mass in various physical systems.

Various models that have been advanced to study processes in complex systems, ranging from the dynamics of vortices in type-II superconductors to the development of social crisis, involve power-law nonlinear density-dependent diffusion. In their contribution, Curilef *et al.*[10] investigate a thermodynamic-like treatment for this type of complex systems, based on non-additive entropies, that incorporates the effects of time-dependent, external driving forces. In particular, the authors derive a lower bound on the work performed by the driving forces during a transformation. The bound is formulated using a free-energy-like functional expressed in terms of an appropriate non-additive entropy.

Curado *et al.*[11] reconsidered Jüttner's generalization of the Maxwell distribution of velocities for a relativistic gas. Jüttner's probability distribution function (PDF), written in the variable rapidity, is not Lorentz invariant. This proposal presents a change in curvature at the origin at high energy. However, in one dimension, using the additivity law of rapidity and the central limit theorem, these authors were able to obtain a new PDF whose curvature at the origin does not change for any energy value, which agrees with computer dynamics simulations. According to this proposal, the temperature is a Lorentz invariant quantity. Further studies are required to obtain the distributions of a relativistic gas in two and three dimensions.

Qi *et al.*[12] address the issue of infrastructure network vulnerability. They consider a model where a network is attacked by removing nodes with an optimal strategy. Since damage is related not only to the node itself, but also to the complex interactions between nodes given by the network topology, the authors propose a defense strategy based on hiding links, where the probability to hide a link is given by some topological property of the node, such as its degree or betweenness centrality, thus decreasing the attacker's information about the network. The model is applied to a scale-free network, and a realistic IEEE-30 bus system, showing that link hiding provides an effective strategy to reduce network damage, which can be useful to design defense for actual infrastructure systems.

Guo *et al.*[13] address the issue of synchronization in hypernetworks, dealing, in particular, with the effect of the coupling between nodes within each network layer, and time delay for the interaction within a layer. Taking a system of Rössler chaotic oscillators as a model, the authors study both single and two layer networks, showing how the synchronized regions (SRs) of an individual layer are affected by the existence of an additional one. They find that the inner coupling has an important impact on the synchronization of the whole hypernetwork, such as the possibility of having stable synchronization of the hypernetwork by coupling an unstable and a stable network. As to time delay, for unbounded SRs, the time delay can change the size and type of the SR.

B. Interdisciplinary articles

Descalzi *et al.*[14] in their interdisciplinary article study localized chaos by modeling of transmission of a light pulse through an optical fiber. Localized chaos means that the power spectrum appears irregular, while the moduli are smooth, asymmetric, and well-defined curves in time. This article is interdisciplinary as it integrates the methodology of the study of nonlinear optics with the study of complex systems. The equation that models the stable transmission of pulses has the following essential ingredients: Energy dissipation, which is a result of the absorption of light in the optical fiber, energy injection to compensate for the loss, nonlinearity coming from the instantaneous response (Kerr effect) and the non-instantaneous response (Raman effect) of matter, and dispersion.

Cartes[15] in his article aims to understand the riots that affected Chile's capital, Santiago, in October 2019. To achieve this, an epidemiological non-local model, previously used to explain the French riots from 2005 successfully, is extended and implemented. The extension takes into account the effects of the subway network on the riot's distribution, which seems essential in the disorder activity. While the model adequately described the riots' aggregated temporal evolution, it could not deliver a result close to the actual disorder spatial distribution. The main reason for this failure is attributed to the lack of a population transport mechanism, which seems vital to explain Santiago's riots. The work is founded on different areas of knowledge, including non-linear science, crime science, and transport engineering. All of them are essential to studying the subject of crowd dynamics, making the article an interdisciplinary one.

Individual behaviors and social relations influence each other. However, understanding the underlying mechanism remains challenging. Through simulating the weak prisoner's dilemma in finite populations, Zhang *et al.*[16] found that neither a simple reward measure nor a pure punishment mechanism can extensively promote cooperation. Instead, a combination of appropriate punishment and reward mechanisms can promote cooperation's prosperity regardless of how large or small the temptation to defect is. These results demonstrate that dispensing rewards and punishments impartially in society is essential to social harmony.

Velazquez *et al.*[17] in their interdisciplinary article address the question of the student workload, namely, the development of quantitative methods to determine the training time required by students to fulfill the learning outcomes of a course. They exploited the activity logs of digital platforms to collect empirical evidence about the complex character of educational systems. Their analysis revealed statistical patterns analogous to the ones observed in financial markets with respect to the usage of available time of individuals to perform different academic activities. Atenas *et al.*[18] exploited the precedent findings to develop two agent-based models to describe the learning phenomenon and introduce a new estimation method for the academic credits based on the observed academic performance of students after a follow-up period.

Tsallis and Oliveira[19] in their interdisciplinary work present strong numerical evidence of an isomorphism connecting the energy q-exponential distribution with a specific geographic growth random model based on preferential attachment through exponentially distributed weighted links. This connection is fully analogous to the well known examples of isomorphism between models within Boltzmann–Gibbs thermostatistics and nontrivial random geometrical models. Such examples include the Kasteleyn–Fortuin theorem related to bond percolation, the zero-state limit of the Potts ferromagnet related to random resistor networks, and the de Gennes isomorphism of the zero-component limit of the n-vector model with self-avoiding random walk.

In his interdisciplinary research, Muñoz[20] studies how connectivity affects the wealth distribution in an economic system. An agent-based money exchange model is considered, with spending propensity, which determines the amount of money agents can risk in each transaction, and over a complex network, so that interactions are only possible between previously connected nodes. This allows one to consider the fact that, in general, not all interactions are possible, due to, e.g., location, transportation, or friendship issues. Results show that network topology weakly affects the wealth distribution, which may explain why the Pareto distribution of wealth is so robust, for a large variety of economic systems. However, it is also observed that the behavior deviates from the Pareto case for large values of the characteristic exponent of scale-free networks, which may have implications for the topological properties of interaction networks in actual societies.

Davis *et al.*[21] illustrate an interdisciplinary application of statistical inference and the use of information theory, which aims to correct evaluation biases found in quite general contexts of science and education. Grade assignments in tests, as well as scores from recommendation letters and similar numerical assessments, are inherently uncertain due to human bias. In this work, we have presented a statistical method, based on Jaynes' maximum entropy principle, providing an unbiased correction to an initial grade. This correction takes into account the uncertainty in the trust we put on the evaluator. Our results naturally agree with our intuition that extreme scores should most of the time be moderated, meaning shifted toward the center of the grading scale. These results open the way toward a systematic study of biases in human evaluation.

In his interdisciplinary research, Granado *et al.*[22] use intracranial electroencephalography (iEEG) recordings to analyze deep brain electrical activity, bringing together insights from neurology, mathematics, and physics that allow describing the dynamic activity of the deep brain. In this paper, they investigate the dynamics of preictal and basal signals in patients with refractory epilepsy using entropy and complexity quantifiers. The results show that minutes before an epileptic seizure, the system evolves from a highly dissipative chaotic state of the basal period to a state where entropy reaches a maximum and complexity is significantly reduced, corresponding to the preictal period.

The article of Camargo *et al.*[23] embodies inter/transdisciplinary research by integrating concepts from nonlinear dynamics, network theory, and macroeconomics to model and analyze economic synchronization phenomena. Their research explores the intricate dynamics between coupled economies in an idealized macroeconomics. They investigate how these systems transit between chaotic and cyclical behaviors and the conditions that lead to their synchronization. The study provides insights into the complex nature of economic cycles, the factors that can disrupt harmony between interconnected economies, and the potential for predicting and mitigating financial crises. This work contributes to a deeper understanding of economic resilience and the propagation of economic phenomena on a global scale.

The connection between information geometry and thermodynamics has long posited that information geometry curvature diverges at phase transitions. Recent research on Bose–Einstein gases challenged this curvature to converge to zero rather than diverge. Examining information geometry curvature at a finite number of particles, which increases, curvature values decrease proportionally to the power of the number of particles. The temperature at which maximum curvature occurs approaches the defined critical temperature. In the thermodynamic limit, curvature maintains a finite value, contradicting the idea that it diverges at phase transitions. This study provides insights into the interplay of information geometry and thermodynamics in finite systems. The interdisciplinary aspect of this research of Pessoa[24] lies in the connection between information geometry and thermodynamics, challenging conventional beliefs by examining curvature behavior at phase transitions in finite systems.

In their interdisciplinary article, Kao *et al.*[25] study the sliding mode control method for coupled delayed fractional reaction–diffusion Cohen–Grossberg neural networks on a directed non-strongly connected topology. A novel fractional integral sliding mode surface and the corresponding control law are designed to realize global Mittag–Leffler synchronization. The sufficient conditions for synchronization and reachability of the sliding mode surface are derived via the hierarchical and Lyapunov methods. Finally, simulations are provided to verify the theoretical findings. The interdisciplinary nature of this research is evident from the fusion of mathematics, fractional calculus, and neural networks. It addresses complex problems in diverse fields, from fluid dynamics to engineering control.

Feng *et al.*[26] propose a model to optimize investment portfolios, based on the Markowitz mean-variance portfolio model, but considering that asset returns follow an asymmetric power-law distribution, which in turn suggests that fractal methods are more appropriate to deal with this system. Thus, the authors consider fractal expectation and variance measures, allowing them to determine asset returns and risks. The authors find analytical expressions for the investment weights for assets under this model and apply their results to the industry indexes of Shanghai Stock Exchange. The analysis shows that the fractal analysis for the portfolio improves on the traditional approach, based on nonfractal measures, and, thus, provides a better technique to optimize investments.

Chen *et al.*[27] study the dynamics of epidemics over a network, where traffic is allowed along vertices, as a model for the spread of biological or computational virus, where people travel or information transport is inherent to the network and, thus, to the propagation of the disease. In this work, the authors consider the classical SIS model over a scale-free network, adding a parameter for the allocation of protection resources in each node, which, if enough, may avoid infection of a node, but which may be inhomogeneously distributed along the network. Thus, the authors study how heterogeneity in protection resources modifies the threshold needed for widespread contagion and how the effectivity of a given resource distribution depends on the traffic flow.

Dong *et al.*[28] proposed reverse transition entropy, which is combined with refined composite multi-scale analysis and generalized fractional-order entropy to construct the refined composite multi-scale reverse transition generalized fractional-order complexity-entropy curve. Their measure aims to characterize and identify different complex time series, such as the static and dynamic transition probabilities of the temporal structure. They applied their methodology to track the dynamical changes of rolling bearing and turbine gearbox time series, observing better performance than that achieved with other methods.

Chen and Yong[29] studied the controllability of complex networks for understanding how to control a complex social network. The extended precedent work, under the assumption that all nodes are compliant, passing on information neutrally without any preferences, by another model where there is the presence of stubborn agents, or zealots, who hold steadfast to their beliefs and seek to influence others. They reported that the presence of zealots alters the energy cost at a quadratic rate with respect to their own fixed beliefs, among other results.

III. CONCLUSIONS

Complexity can be studied from different view angles. The first 12 articles mentioned in this review on Complex Systems and Inter/Transdisciplinary Research study complex systems from a non-interdisciplinary point of view. Topics covered include Condensed Matter Physics, Magnetic Ordering, Soliton-like Behaviors, Stochastic Processes, Information Theory Entropy, Game Theory, and Network Theory. The following 14 articles mentioned in this review can be considered Inter/Transdisciplinary. As previously mentioned, interdisciplinary research means that the subject of study and research questions integrate different disciplines and methodological approaches, while transdisciplinary research cross different disciplines transversally. Examples of Interdisciplinary research included in the Focus Issue are Nonlinear Optics—Localized Chaos, Social Issues—Epidemiological Models, Social Networks—Game Theory, Economics—Teaching–Learning Processes, Wealth distribution in an economic system—Network Theory, Statistics—Information Theory, Brain Activity—Nonlinear Dynamics, Operations Research—Fractals Theory, SIS Model—Phase Transitions, Wind Turbines–Complex Time Series—Social Networks, Social Networks—Network Theory. We hope that this collection will be enjoyed by our readers. Although the covered topics are vast, they are far from complete. Future contributions will be expanded to cover new hot topics, such as *machine learning* and *artificial intelligence*, which are now receiving a progressive and increasing interest for the wide audience, impacting many areas beyond academic research.

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