

# Discussion: “A Novel Explicit Equation for Friction Factor in Smooth and Rough Pipes” (Avci, A., and Karagoz, I., 2009, ASME J. Fluids Eng., 131, p. 061203)

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*This paper shows a comparison of the most classic and historical formulations to obtain the friction factor for cylindrical pipes with novel approaches. Although there has been some evolution of the expressions, basically, the ideas and results remain very close to the historical formulations. [DOI: 10.1115/1.4003157]*

## 1 Brief Historical Review and Discussion

It was back in the late 1930s when Colebrook and White developed their well-known and widespread relation to obtain the Darcy–Weisbach friction factor, very often named “*f*.” They derived their expression from the combination of previous research by Prandtl, back in 1935, and von Karman, in 1938. Classical Refs. [1–5] explain all these concepts in detail.

In a mathematical arrangement, the results by Colebrook and White are equivalent to say that the proposed value for “*f*” can be derived from

$$\frac{1}{\sqrt{f}} = -2 \log \left( \frac{\varepsilon}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right) \quad (1)$$

to be used in the Darcy–Weisbach equation and obtain the pressure loss in a cylindrical pipe. The referred pressure loss can be obtained from

$$\frac{\Delta p}{\rho g} = f \frac{L}{D} \frac{v^2}{2g} = \frac{8fL}{g\pi^2 D^5} Q^2 \quad (2)$$

More recently, in 1983, Haaland proposed an explicit formula to obtain “*f*” as follows:

$$\frac{1}{\sqrt{f}} = -1.8 \log \left( \left( \frac{\varepsilon}{3.7} \right)^{1.11} + \frac{6.9}{\text{Re}} \right) \quad (3)$$

Finally, in the June 2009 issue of the ASME Journal of Fluids Engineering, Avci and Karagoz published a paper [6] that introduces a “novel formulation.” They proposed a new expression for the friction factor, following the formula

$$f = \frac{6.4}{\{\ln(\text{Re}) - \ln[1 + 0.01 \text{Re} \varepsilon(1 + 10\sqrt{\varepsilon})]\}^2} \quad (4)$$

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The whole set of previously mentioned formulations, with their different expressions, has been considered and comparison between them is accomplished in the present paper.

When considering a low pipe relative roughness, for example,  $\varepsilon=0.0003$  (that is,  $k_s/D=0.0003$ ), a plot as shown in Fig. 1 is obtained. In this plot, for high Reynolds numbers (higher than  $10^6$ ), a 1.2% difference between the classic formulations and the new equation is found.

A second and higher pipe relative roughness, for example,  $\varepsilon=0.03$  (that is,  $k_s/D=0.03$ ), has been considered. The results of the comparison for a wide range of Reynolds numbers are presented in Fig. 2. For high Reynolds numbers (higher than  $1.5 \times 10^5$ ), again, a 1.1% difference between the classic formulations and the new equation is found. The two classical formulations give almost the same value, while the proposed new approach differs from them, giving a slightly higher friction factor.

## 2 Conclusions

From the observed trends in the comparison between the classical approaches and the new proposed Eq. (4), one can conclude that

- For a wide range of the pipe relative roughness, the three considered equations give almost the same friction factor, “*f*,” with only a 1% difference for the new approach.
- The relevance of the proposed equation is limited if the classical formulations are considered.
- The new approach presented in Ref. [6] only solves the explicitness of some classical equations, as for instance, Eq. (1). However, the explicitness was already overcome with formulations as the one by Haaland [4].
- Therefore, the only relevance of Ref. [6] could be its Fig. 3. In that figure, the “Princeton superpipe” results are shown. Nevertheless, there is a lack of explanation about the way these results were obtained, and they are not explicitly referred to. A detailed comment on the results of the Princeton superpipe would be of great interest.

Although a deeper analysis on the discrepancies of the new formulation (Eq. (4)) when compared with the classical approaches would be interesting, no improvement in using this new formulation is found.

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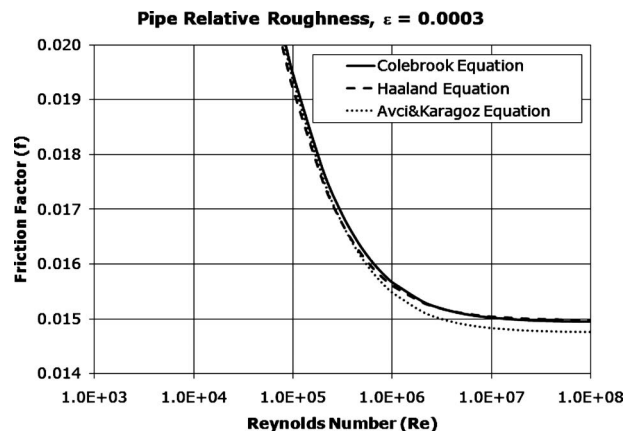


Fig. 1 Comparison of Eqs. (1), (3), and (4) for low pipe relative roughness

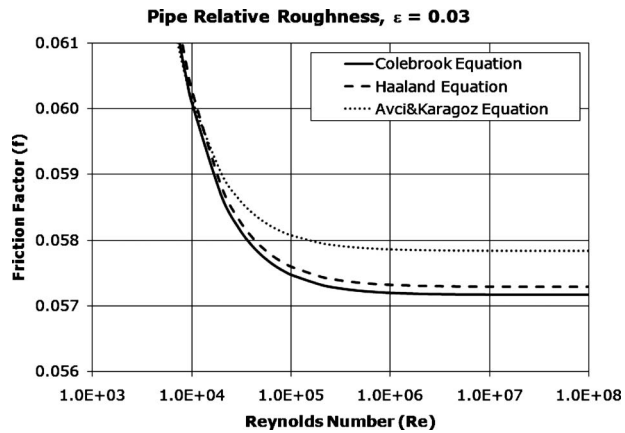


Fig. 2 Comparison of Eqs. (1), (3), and (4) for high pipe relative roughness

**Nomenclature**

D = pipe diameter, m  
 f = friction factor

g = gravitational constant, m/s<sup>2</sup>  
 k<sub>S</sub> = pipe roughness, m  
 L = pipe length, m  
 Δp = pressure drop in a pipe  
 Q = flow rate, m<sup>3</sup>/s  
 Re = vD/ν = Reynolds number  
 v = mean velocity, m/s  
 ε = pipe relative roughness  
 ν = fluid kinematic viscosity m<sup>2</sup>/s  
 ρ = fluid density, kg/m<sup>3</sup>

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