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Transboundary water allocation under water scarce and uncertain conditions: a stochastic bankruptcy approach

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Abstract

Designing a feasible and stable water sharing mechanism for transboundary river basins is a big challenge. The stochastic and uncertain characteristics of water flow in these rivers is among the main reasons which make the formation of cooperative coalitions with feasible water allocations and self-enforceable allocation agreements difficult. When the water in these river basins is scarce the task becomes even more challenging. This article focuses on the application of stochastic game theoretic extension of the bankruptcy concept to transboundary water resource sharing under water scarce and uncertain conditions. Among the water allocation vectors obtained from stochastic bankruptcy rules only the ones from the stochastic constrained equal awards rule were self-enforcing under uncertainty. Furthermore, the authors also proposed an allocation rule that can be used under a stochastic setting. The proposed rule provides water allocations that are self-enforcing in the absence of uncertainty. Generally, the application of the stochastic bankruptcy approach could be a source of important strategic information which can serve for the sustainable sharing and management of these vital sources of fresh water, particularly during water scarcity.

Keywords: Bankruptcy game; Self-enforcing; Stochastic; Uncertainty; Water allocation; Water bankruptcy

1. Introduction

Currently, there are 276 transboundary rivers in the world (Ansink, 2009). These rivers could be sources of cooperative management which ensures the sustainability of these river basins or sources of water disputes which hinder the efficient utilization of these crucial water resources and endanger their sustainability for future generations (Gleick, 1993; Homer & Thomas, 1994; Swain, 2001, 2015; Wolf *et al.*, 2006; De Stefano *et al.*, 2012; Mianabadi *et al.*, 2014a, 2014b).

There are no internationally agreed upon mechanisms for allocating water in border-crossing rivers (Wolf, 1999). In most of these border-crossing river basins there are no basin-wide cooperative

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agreements either (Ansink, 2009). If agreements on the sharing of these rivers exist they are mostly bilateral or trilateral (Wolf, 1998). The majority of these river basins are without institutional framework as well (Ansink, 2009). If institutional frameworks exist in some of these river basins they usually lack an authority with the power needed to enforce the implementation of river sharing agreements (Ansink & Arjan, 2008). Therefore, these trilateral and bilateral cooperative water allocation agreements face high risk of being broken, mainly during the periods of water scarcity which could result from the impacts of climate change and increasing water demand (Bates *et al.*, 2008).

The main reasons for the lack of standardized mechanisms for allocating transboundary river basins' water are the socio-economic and environmental asymmetries among the riparian countries of these river basins. These asymmetries are also responsible for the absence of basin-wide cooperative coalitions and for the lack of institutional frameworks which oversee the implementation of basin-wide or sub-basin cooperative agreements if they exist (Ansink, 2009). In addition, the other main reasons for the absence of such cooperative agreements are the stochastic and uncertain characteristics of water flow in these river basins together with the uncertain future the riparian countries face in terms of water demand and availability. For these reasons the riparian countries do not want to cooperate and make commitments on fixed allocations of water which could hurt their claims in the future when uncertainties are realized (Wu & Whittington, 2006). Hence in the absence of a cooperative framework for basin-wide water sharing and management the riparian states might prefer to act unilaterally to avoid risk and maximize their utilities. These unilateral actions could lead to inefficient, unequitable and unreasonable utilization of these valuable resources and endanger their sustainability. This will further add more uncertainties concerning water availability through time and space in these river basins. As a result the possibility of conflicts happening in these river basins increases (Hensel *et al.*, 2006). Climate change on the other hand makes matters even more complicated since it affects the water availability, timing, quality, and demand (Bates *et al.*, 2008; Cooley *et al.*, 2011; Valipour, 2015a, 2016).

Avoiding trial and error policies and designing sustainable water management schemes are important for maintaining the integrity of water resources (Valipour, 2015b, 2015c). In most of the transboundary river basins with water sharing agreements, fixed water allocations are common (Wolf, 1998). On the contrary, as described above, the stochastic water flow is one of the characterizing features of these rivers. Therefore, fixed water allocation frameworks could lead to water allocations which are unacceptable in the face of uncertainty and stochastic flow of water. Such allocation schemes might not induce basin-wide cooperative river water sharing agreements. Even if these allocations can be a base for sharing agreements, the agreements which are based on them have increased probability of being broken in time. As a result, basin-wide cooperative water sharing agreements and mechanisms should be adaptable with the stochastic and uncertain nature of these shared river basins. Therefore, flexible water allocation mechanisms which result in feasible allocations, that can be a base for self-enforcing or stable agreements, are needed in order to reach basin-wide water sharing agreements. Self-enforcing agreements ensure that water allocations are in the best interest of each riparian state (Barrett, 1994). The self-enforcing property is among the main characteristics which allocation rules need to have, since there is no institutional set-up with enforcing power in most transboundary river basins.

Water scarcity is an issue of concern these days. The rise in demand and the impact of climate change are the two main factors which are responsible for water scarcity in most transboundary river basins (Cooley *et al.*, 2011; Ansink & Harold, 2015). When water scarcity prevails, the amount of water available in a river basin is less than the total water demand. The occurrence of such a scenario could threaten the integrity of these crucial ecological capitals and also could catalyze water conflicts. Such a scenario has already happened in some river basins, for instance Qezelozan-Sefidrood Basin (Madani *et al.*,

2014) and Tigris-Euphrates (Mianabadi *et al.*, 2015a, 2015b). It is also predicted that river basins which are under huge pressure from the increasing water demand and climate change will experience water scarcity in the near future. The Nile river basin is one of these river basins which are predicted to be water bankrupt in the near future (Brunnee & Stephen, 2002; Molden *et al.*, 2010).

Understanding water scarcity as much as understanding the physical features of a river basin is important. Understanding the strategic interaction between the sharing countries is essential too. Hence strategic research approaches are crucial for designing water sharing schemes in border-crossing river basins that can lead to equitable and reasonable water allocations. One of the most popular strategic approaches in recent years for designing water sharing mechanisms in transboundary river basins under scarcity is the bankruptcy game. The bankruptcy game is formulated by O'Neill (1982) from the bankruptcy problem. The bankruptcy problem is a resource sharing problem where the amount of divisible resource available for sharing is less than the total water demand (O'Neill, 1982; Curiel *et al.*, 1987). The condition where the amount of water available is less than the total water demand is known as the water bankruptcy scenario. The bankruptcy approach has been extended and used for allocation of water in transboundary river basins (Gallastegui *et al.*, 2002; Ansink & Marchiori, 2015; Ansink & Weikard, 2012; Zarezadeh *et al.*, 2012; Madani *et al.*, 2014; Mianabadi *et al.*, 2014a, 2014b, 2015a, 2015b).

Most of the applications of the bankruptcy theory to transboundary rivers have been done in deterministic settings. Hence, they failed to capture the stochastic behavior of river basins as well as the uncertainty riparian states are facing in terms of water availability and demand. In this research, stochastic cooperative bankruptcy game theoretic allocation approaches are surveyed for their application in water bankrupt transboundary river basins under stochastic and uncertain settings. The stochastic bankruptcy approach of Habis & Herings (2013) was proposed for water allocation under stochastic and uncertain settings in transboundary river basins with finite water availability expectations. This approach takes the stochastic and uncertain nature of transboundary river basins into account and could provide more realistic strategic information than the earlier approaches used in deterministic settings. In this article, the water sharing problem during water scarcity in a transboundary river basin was conceptualized as a stochastic bankruptcy scenario and stochastic bankruptcy scenario under uncertainty. Then strategic cooperative coalition formation was analyzed in these two settings. Furthermore, the authors also proposed an allocation rule and discussed the self-enforceability of water allocation vectors from it under stochastic and uncertain settings.

The rest of this paper is organized as follows. Section 2 introduces cooperative stochastic bankruptcy games and cooperative stochastic bankruptcy games under uncertainty. In section 3 the application of stochastic bankruptcy games and cooperative stochastic bankruptcy games under uncertainty for allocating water in transboundary river basins under scarcity are discussed. The approaches are further elaborated using the Nile river basin and a hypothetical water bankrupt river basin as an example. In this section the authors also introduce a water allocation method and examine its applicability in stochastic and uncertain settings. Section 4 summarizes and concludes the paper.

2. Method

2.1. *Stochastic cooperative games with transferable utility*

In deterministic transferable utility games, the payoffs of coalitions are known with a great deal of certainty, while in stochastic games this is not the case. Most of the shared resources in our world depict dynamic characteristics in terms of their availability. Transboundary river basins are one of these shared resources with changing features. One of their changing features is the amount of water available in them from time to time. Hence, in these basins the water allocation payoff of cooperative management is not known with a great deal of certainty. Therefore, stochastic cooperative games can be useful for assisting strategic decision making in such cases. These games usually are played in sequence of time periods with finite expectation. Let us consider time periods $t \in T = (0, 1)$. In period 1 and period 0, the state of nature s out of a finite set S and s' out of a finite set of states S' occurs with finite expectations, respectively.

In the ex-ante state players, or in our case riparian countries, discuss their strategies. In the ex-post stage, when the worth of the coalition is known with certainty, they engage in the transferable utility game and allocation of the worth of the coalition or the available water resource is made using allocation rules. If the allocations are made in the ex-ante stage the game will no longer be a transferable utility (Suijs, 2000; Borm & Suijs, 2002; Habis & Herings, 2013). Therefore, in stochastic cooperative games with transferable utility the division of the resource always occurs in the ex-post stage.

Definition: The stochastic cooperative game with transferable utility is given as a tuple (N, v_s, x_i, u_s) where $N = (1, 2, 3 \ldots n)$ is the set of players, $v_s{:}2^N \to \mathbb{R}$ is the characteristics function representing the stochastic worth of a coalition $v(C)$, $C \subseteq N$ with the assumption that $v_S(\emptyset) = 0$, and $u^i{:}\mathbb{R}^s \to \mathbb{R}$ represents the preference of a player i among the random payoffs with finite expectations (Suijs & Borm, 1999; Suijs, 2000). The utility function is assumed to be state separable, continuous and monotonically increasing (Koopmans, 1960).

2.2. Stochastic bankruptcy games

The bankruptcy problem is a division problem involving a perfectly divisible estate among agents whose cumulative demand or claim is higher than the available resources (O'Neill, 1982; Curiel *et al.*, 1987). Most studies on bankruptcy problems are carried out from two perspectives. The first one is from the game theoretic approach and the other is from an axiomatic perspective (Herrero & Villar, 2001). The bankruptcy scenario usually happens in various real life situations. Resource sharing problems in economic sectors and border-crossing rivers are notable examples. O'Neill (1982) extended the bankruptcy problem to the corresponding bankruptcy game by providing the game theoretical analysis of the problem under deterministic settings. Furthermore, O'Neill (1982) also proved that the core of the bankruptcy game is non-empty.

Definition: A bankruptcy game is defined as a pair (E, d) where $d = (d_1 \ldots\ldots d_n)$ represents the vector of claims satisfying the condition $\sum_{i \in N} d_i \geq E \geq 0$. When extended to the bankruptcy game with transferable utility the characteristics function describing the worth of the coalitions $v^{E,d}{:}2^N \to \mathbb{R}$ is written as:

$$v^{E,d}(C) = Max\left\{E - \sum_{i \in N/C} d_i, 0\right\}, C \subset N \tag{1}$$

Hence the worth of a coalition C in a bankruptcy game with transferable utility is the amount of resource which is not claimed by its complement (O'Neill, 1982). The worth of coalitions in the transferable utility game can be divided among the claimants using allocation rules. Allocation or division rules are rules that distribute the worth of the coalition to the claiming agents involved by satisfying efficiency $\left(v^{E,d}(C) = \sum_{i\in C} x_i\right)$ and individual rationality $(x_i \geq 0)$.

The concept of the bankruptcy problem was further extended to the stochastic bankruptcy problem by (Habis & Herings, 2013). They defined the stochastic bankruptcy problem as a tuple (S, E, d, u) where S is the finite states of nature, $E = (E_s)_{s\in S} \in \mathbb{R}^s$ is the resource available for sharing in each state, $d_s = \left(d_s^1 \ldots\ldots\ldots\ldots, d_s^n\right) \in \mathbb{R}^N$ is the state dependent vector of claims, $u = (u^i)_{i\in N}$, and $u{:}\mathbb{R}^S \to \mathbb{R}$ is the utility function describing the utility of each resource claimant. The stochastic allocation rule assigns $x_s \in \mathbb{R}^{S\times N}$ to every (S, E, d, u) given that the conditions $\sum_{i\in N} x_s^i = E_s$ and $0 \leq x_s \leq d_s$ are satisfied in each state $s \in S$.

The stochastic bankruptcy problem (S, E, d, u) can be transformed into a stochastic bankruptcy game (N, S, v, u) using O'Neill's (1982) approach. The worth of a coalition C at state s is the unclaimed state left after satisfying the total claim of its complement (O'Neill, 1982; Curiel *et al.*, 1987). It can be written as:

$$v_s(C) = Max\left\{E - \sum_{i\in N/C} d_s^i, 0\right\}, s \in S, C \subset N \tag{2}$$

The stochastic bankruptcy problem can be extended to the allocation of water in a water scarce border-crossing river basin. The coalition worth for the corresponding bankruptcy game for a water sharing problem can also be expressed using Equation (2). These games can assist the sustainable sharing and management of transboundary river basins by providing important strategic information since they can take into account the changing features of transboundary river basins.

2.3. Stochastic bankruptcy games under uncertainty

Stochastic bankruptcy games under uncertainty were introduced by Habis & Herings (2013) by associating stochastic bankruptcy games with transferable utility games and uncertainty (Habis & Herings, 2011). They provided a weak sequential core introduced by Kranich *et al.* (2005) as a solution concept.

Definition: A transferable utility game with uncertainty is a tuple (N, S, v, u) where $v = (v_1 \ldots\ldots v_s)$ are state dependent characteristic functions and $u = (u^1 \ldots\ldots u^n)$ are state separable, continuous and monotonically increasing utility functions indicating players' preferences (Habis & Herings, 2011).

Definition: A weak sequential core is the set of feasible allocations $\bar{x}$ for the grand coalition from which no coalitions ever have credible deviations (Kranich *et al.*, 2005; Predtetchinski *et al.*, 2006; Habis & Herings, 2011).

Allocation rules are important for distributing the worth of strategic cooperative coalition among the agents involved. Therefore, coalition worth distribution is an important issue when applying the bankruptcy approach to solve resource-sharing problems. Allocation rules which can be used to distribute the available resource or estate among the claimants must satisfy certain desirable properties. As mentioned earlier, efficiency and feasibility are the main ones. The most known bankruptcy allocation rules are the constrained equal awards rule, constrained equal loss, adjusted proportional and proportional allocation rules. Detailed axiomatic description and characterization of these rules can be found in Dagan (1996), Thomson (2003), Moreno-Ternero & Villar (2004), Branzei *et al*. (2008) and Thomson (2015). Most of these traditional bankruptcy allocation rules have been applied for allocating resources in deterministic settings. Habis & Herings (2013) tested these bankruptcy allocation rules in stochastic and uncertain settings with finite expectations. They found that only the stochastic constrained equal awards (*SCEA*) allocation rule yields allocations for the grand coalition that are not blocked in the ex-ante and ex-post period by allocations from deviations and credible deviations. Hence, allocation outcomes from stochastic proportional (*SP*), stochastic adjusted proportional (*SAP*) and stochastic constrained equal loss (*SCEL*) rules do not yield allocation outcomes that are self-enforcing under uncertainty.

The concept of self-enforceability or stability is deeply related to credibility. Credible allocations are allocations for the grand coalition which cannot be blocked by allocations with higher utility from sub-coalitions (Kranich *et al*., 2005). For bankruptcy games in deterministic settings, allocation rules that lead to allocations in the core are credible (Ray, 1989). These allocations cannot be blocked by allocations from a sub-coalition that results in higher payoffs with higher utility for their members. Stochastic bankruptcy games with uncertain grand coalition worth with finite expectation allocations in the weak sequential core are credible given that every bankruptcy game in each state $s \in S$ leads to allocations in the core while satisfying efficiency, marginality, individual rationality and most importantly self-enforceability (Kranich *et al*., 2005; Habis & Herings, 2013).

Habis & Herings (2011) analyzed these games in the absence of ex-ante commitment possibilities, considering the players are risk-averse utility maximizers. This is a perfect analogy with the behavior of riparian states in a contested transboundary river basin where the riparian countries compete and interact to maximize their benefits and also try to minimize their risk of exposure to water scarcity in the future. Therefore, since the riparian states do not want to enter into agreements in the ex-ante period which could compromise any future claim they can have on the river basin's water in the ex-post period. As a result, allocation rules used to allocate the available water in these games should be able to result in water allocation payoffs that cannot be blocked in the ex-ante stage in order to be self-enforcing or stable. The application of stochastic bankruptcy games and stochastic bankruptcy games with uncertainty for water sharing under water scarce scenarios could help choose or design allocation rules that can be a base for self-enforcing allocation agreements which cannot be blocked through time. Therefore, water sharing in the water bankrupt international river basins could be assisted hugely from applying these strategic cooperative games.

In a later section of this article the authors apply the stochastic bankruptcy allocation rules to allocate the predicted available water in the Nile river basin under stochastic and uncertain settings by making the following assumptions. First, the basin countries on the White Nile were grouped into an assumed coalition and the number of water claimants in the river sharing problem was made four. Such adjustment is justifiable for the reasons mentioned in Wu & Whittington (2006). In addition the authors assumed that the predictions for the available water in the medium term and long term will prevail in the same time period with equal probability. This is a reasonable assumption since it is highly uncertain whether the runoff volume of the Nile will increase or decrease in the future. Due to the lack of data the authors only considered the water demands of 8 out of the 11 riparian countries in the river basin.

The self-enforceability of the *SCEA* rule and the non-self-enforceability of the *SP*, *SAP* and *SCEL* allocation rules were also demonstrated additionally by applying them to a hypothetical river basin under scarcity with three riparian countries. Finally, the proposed method was applied to a hypothetical river basin as well, in order to show its applicability under stochastic and uncertain settings.

3. Results and discussion

3.1. Case study

Poor water governance, low institutional capacity, and unpredictable water availability are some of the main causes of conflict and uncertainty in border-crossing basins (Mianabadi *et al.*, 2015a, 2015b). Most of the river basins with these issues are located in northern and sub-Saharan Africa (De Stefano *et al.*, 2012). The Nile river basin, shown in Figure 1, is one of these most important river basins. Most of the basin area lies in arid and hyper-arid geographical zones with high climatic uncertainty (Nile Basin Initiative, 2012). It is the longest river in the world covering a distance of 6,695 km. It drains an area of 3.1 million square kilometers and its catchment covers approximately 10% of the African continent making it one of the largest and most important river basins in Africa (Food & Agriculture Organization (FAO), 2007; Nile Basin Initiative, 2012). The river basin is characterized by unevenly distributed water resources and uses. In addition, the basin's hydrology is highly variable through time and space (Jury, 2011; Nile Basin Initiative, 2012). The impact of climate change is expected to make matters even worse in the basin too (Eckstein, 2009).

The Nile river basin has always been a center of attention in the region due to the existing water sharing disputes. The United Nations Convention on the law of the non-navigational uses of international watercourses (United Nations, 1997) and The Helsinki Convention on the management of transboundary rivers and lakes put forward core principles for the management of transboundary water resources. But like in most other transboundary river basins there are no allocated water rights or allocations to the riparian countries which respect these principles in the Nile river basin.

Because of the fact that most of the Nile river basin lies across arid and semi-arid areas, and due to the increasing water demand in the region, water scarcity has been predicted to occur in the basin in the near future by various studies. A study carried out by Awulachew *et al.* (2012), taking into account the current unilateral water management and planning trends in the basin, predicted that the Nile river will be short of water in a few years. They found that the total water demand for the medium-term and long-term scenario would be 94.5 km^3 and 127 km^3, respectively. Hence, both the medium-term and long-term water demands are higher than the 84.1 km^3 short-term and the 88.2 km^3 long-term predicted average water that is expected to be available in the basin. The results concur with the predictions made by Molden *et al.* (2010), Brunnee & Stephen (2002) and Keith *et al.* (2013) who stated that the water availability in the basin for the medium-term and long-term future scenarios might not be enough to satisfy the total water demand in the basin.

It is still highly uncertain whether runoff volumes of the Nile will increase or decrease. But it is evident that the risks associated with climate change for now outweigh possible benefits (Nile Basin Initiative, 2012). Therefore, before this uncertainty is realized it is smart to prepare management schemes which can deal with all the possible scenarios. One of the management schemes is to design a stochastic self-enforcing allocation mechanism for allocating water under water scarce scenarios that could happen in the future. Therefore, with this motivation, assuming the medium-term and long-term predicted values of available runoff will be realized at the same time period with the same finite expectation, the authors applied the stochastic bankruptcy allocation rules for allocating the predicted available water. The results obtained concur with Habis & Herings' (2013) proposition. Hence the *SCEA* rule is the only classical bankruptcy rule which yields allocations for the grand coalition which are credible (Habis & Herings, 2013).

Table 1 shows the results obtained from *SCEA* rule for the medium-term water demands when the predicted available runoff for medium-term and long-term time periods were assumed to be realized

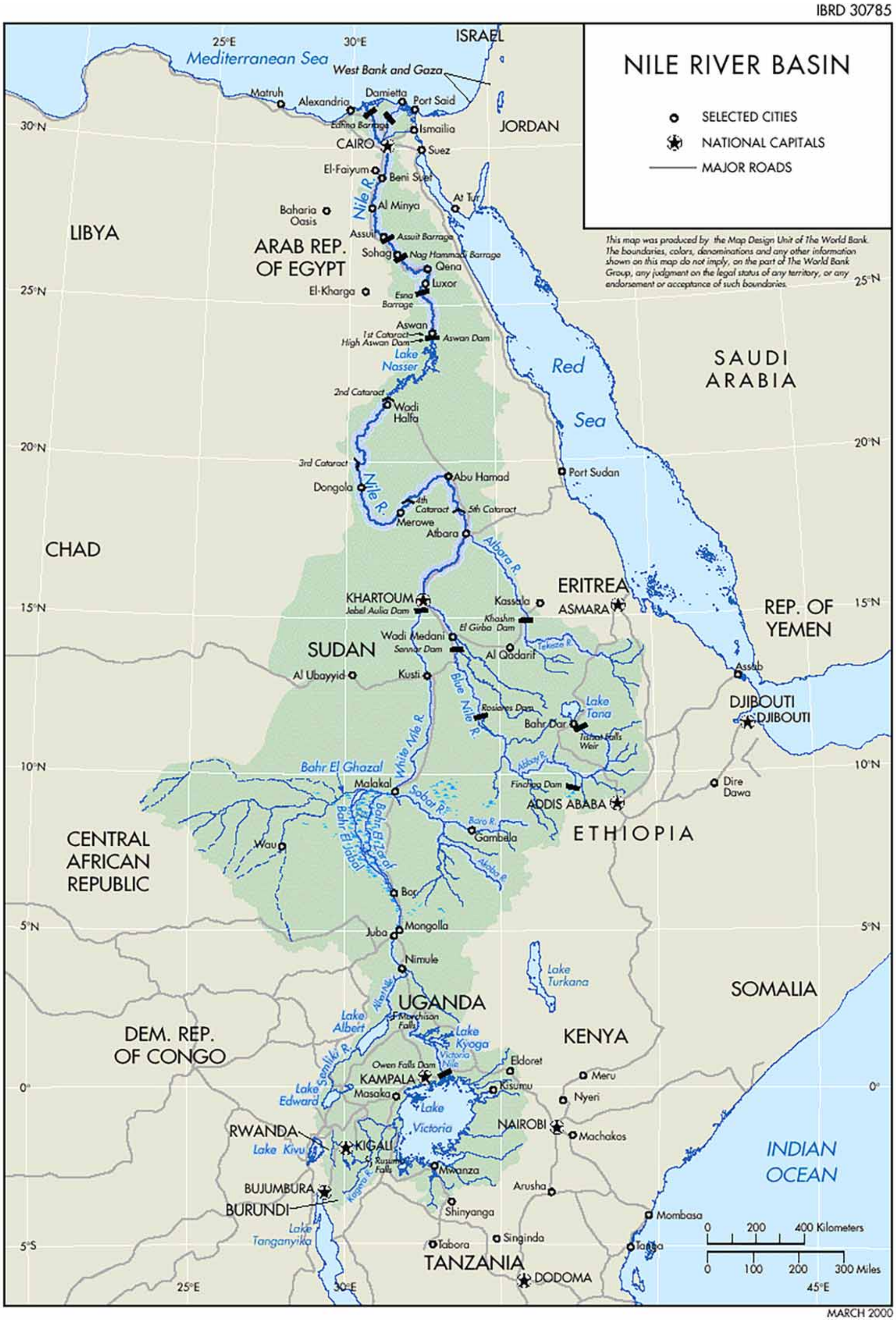

Fig. 1. The Nile river basin (World Bank, 2000).

Table 1. Water allocation vectors from *SCEA, SP, SAP* and *SCEL* for the medium-term predicted water demand in the Nile river basin.

Riparian state	Water demand (million.m^3) Awulachew *et al.* (2012)	Available water (million.m^3) Awulachew *et al.* (2012)	
Coalition of upstream states	2,170	84,100 88,200	
Ethiopia	4,190	88,200 84,100	
Sudan	39,239	84,100 88,200	
Egypt	48,942	84,100 88,200	
Core allocation vectors $x_i^E = \begin{bmatrix} x_1^1, x_2^1, x_3^1 \\ x_1^2, x_2^2, x_3^2 \end{bmatrix}$			
SCEA	*SP*	*SAP*	*SCEL*
$\begin{bmatrix} 2170, 4190, 38870, 38870 \\ 2170, 4190, 39239, 42601 \end{bmatrix}$	$\begin{bmatrix} 1930.35, 3727.26, 34905.49, 43536.90 \\ 2024.45, 3908.97, 36607.18, 45659.39 \end{bmatrix}$	$\begin{bmatrix} 1338.31, 2584.11, 35237.29, 44940.29 \\ 1447.39, 2794.73, 37127.44, 46830.44 \end{bmatrix}$	$\begin{bmatrix} 0, 1433, 36482, 46185 \\ 584.75, 2604.75, 37653.75, \\ 47356.75 \end{bmatrix}$

with the same probability. The *SCEA* awarded the hypothetical coalition of riparian states on the White Nile and Ethiopia which is the upstream country in the Blue Nile with 100% of their claims. Therefore, the rule fully recognizes the claims of countries with minimal water demands. On the other hand, *SCEA* rewarded Sudan and Egypt with 99.05% and 79.42% of their water demands when the medium-term water availability predications were realized. When the long-term available water was assumed to be materialized the *SCEA* rewarded the two countries with 99.64% and 79.89% of their water demands, respectively.

Table 2 also depicts the water allocation vectors obtained from *SCEA* for long-term water demands when medium-term and long-term predictions for the available water are expected to occur with the same probability. For the hypothetical coalition of countries containing the upstream riparian countries on the White Nile and for Ethiopia, *SCEA* allocated 100% of their demands when either the medium-term or long-term expected available water was materialized. While Sudan and Egypt were rewarded with 60.89% and 56.80% of their demands, respectively, if the available water predicted for the medium-term was realized. On the other hand, if the available water predicted for the long-term was realized *SCEA* allocated 64.91% and 60.54% of their water claims to Sudan and Egypt, respectively.

Allocations generated by marginal vectors where the same permutations of claimants are used in each state belong to the weak sequential core (Habis & Herings, 2011). When *SCEA* is applied to allocate water, the same permutation of players is used hence the allocations belong to the weak sequential core and cannot be blocked by any coalition in any state. These allocations are Nash equilibrium allocations and no riparian can do better by deviating in the ex-ante state. Therefore, these allocations for the riparian countries are self-enforcing under stochastic and uncertain conditions. For the other classical bankruptcy allocation rules, even though in the deterministic setting they provide allocations which are in the core, when extended to the stochastic setting under uncertainty it was proved by Habis & Herings (2011) that the allocations from these rules in the ex-post state have credible deviations in the ex-ante state. As a result, water allocations from the stochastic extension of the other bankruptcy allocation rules are not self-enforcing under uncertainty. This is because there is a possibility that sub-coalition of riparian countries could cooperate and block the allocations that can be achieved by these rules in the ex-post stage by minimizing the risk of the members of the sub-coalition while maximizing the total allocation of at least one riparian country. But for *SCEA* all allocations are credible in both ex-ante and ex-post time periods. Hence, they cannot be blocked in any state. Table 3 shows the allocation payoffs obtained for the hypothetical river basin with three water claiming countries, by applying the extension of classical bankruptcy allocation rules to stochastic and uncertain settings. The results obtained for this hypothetical river basin also show that only the allocations from *SCEA* are self-enforceable under stochastic and uncertain conditions.

If the uncertainty associated with the water availability is not considered, the allocation rules can be extended to their stochastic forms and can be applied state by state. In such a way, the stochastic extensions of all the bankruptcy allocation rules provide us with allocations which are in the core and self-enforcing. Tables 1 and 2 can be interpreted in this way too. But when uncertainty about the amount of resource for sharing in each state is considered, only the *SCEA* rule results in allocations which do not have credible deviations in the ex-ante stage. The application of the *SCEA* rule to the river basin provides certain desirable properties such as efficiency, individual rationality and marginality but the most important one is stability or self-enforceability. Self-enforceability of water allocation outcomes is crucial because the Nile river, like most transboundary river basins, lacks an institutional setup with

Table 2. Water allocation vectors from *SCEA, SP, SAP* and *SCEL* for the long-term predicted water demand in the Nile river basin.

Riparian state	Water demand (million.m^3) Awulachew *et al.* (2012)	Available water (million.m^3) Awulachew *et al.* (2012)
Coalition of upstream states	6,823	84,100 88,200
Ethiopia	15,178	84,100 88,200
Sudan	50,992	84,100 88,200
Egypt	54,668	84,100 88,200
Core allocation vectors $x_i^E = \begin{bmatrix} x_1^1, x_2^1, x_3^1 \\ x_1^2, x_2^2, x_3^2 \end{bmatrix}$		

SCEA	*SP*	*SAP*	*SCEL*
$\begin{bmatrix} 6823, 15178, 31049.5, 31049.5 \\ 6823, 15178, 33099.5, 33099.5 \end{bmatrix}$	$\begin{bmatrix} 4494.83, 9998.9, 33592.30, 36013.97 \\ 4713.96, 10486.36, 35229.98, 37769.70 \end{bmatrix}$	$\begin{bmatrix} 4068.96, 9536.98, 33409.03, 37085.03 \\ 4155.2, 9243.4, 35562.7, 39238.7 \end{bmatrix}$	$\begin{bmatrix} 0, 2932, 38746, 42422 \\ 0, 4298.67, 40112.67, \\ 43788.67 \end{bmatrix}$

Table 3. Allocation vectors from *SCEA, SP, SAP* and *SCEL* for hypothetical river basin.

Claimant n	Claim c_i	Estate E_S	
1	10	10	
		20	
		30	
2	20	10	
		20	
		30	
3	30	10	
		20	
		30	
Core allocation vectors $x_i^E =$	$\begin{bmatrix} x_1^1, x_2^1, x_3^1 \\ x_1^2, x_2^2, x_3^2 \\ x_1^3, x_2^3, x_3^3 \end{bmatrix}$		
SCEA	*SP*	*SAP*	*SCEL*
$\begin{bmatrix} 3.33, 3.33, 3.33 \\ 6.66, 6.66, 6.66 \\ 10, 10, 10 \end{bmatrix}$	$\begin{bmatrix} 1.66, 3.33, 5 \\ 3.33, 6.66, 10 \\ 5, 10, 15 \end{bmatrix}$	$\begin{bmatrix} 1.66, 3.33, 5 \\ 3.33, 6.66, 10 \\ 5, 10, 15 \end{bmatrix}$	$\begin{bmatrix} 0, 0, 10 \\ 0, 5, 15 \\ 0, 10, 20 \end{bmatrix}$

enforcing power for implementing basin-wide water allocation arrangements. Similar interpretation applies for the allocation outcomes obtained for the hypothetical river basin under scarcity mentioned above.

3.2. Proposed method

Stochastic allocation mechanisms can be very useful in managing border-crossing river basins under variable conditions in a more sustainable way through the course of time and in the face of uncertainty. In the above sections the importance of stochastic allocation rules and the application of the extended classical stochastic bankruptcy rules were discussed. Such approaches enable us to incorporate desirable properties such as self-enforceability and flexibility into water allocation rules. These desirable features of allocation frameworks are even more important for allocating water under water scarce and uncertain conditions (Drieschova *et al.*, 2008).

The allocation rule presented here was proposed for the deterministic setting by Degefu & He (2016). They extended the allocation rule proposed by Mianabadi *et al.* (2014a, 2014b, 2015a, 2015b). The innovative contribution of their allocation rule is that it considers that the available water is owned by all the riparian states and introduces a new way of taking the water contribution of the riparian countries to the grand cooperative coalition into account. The utility obtained from the same amount of water is different from country to country. Their risk from water scarcity varies as well. Hence, in addition, they proposed vulnerability to water scarcity and the adaptive capacity of the sharing countries to weigh their water claims. In this article authors investigate the applicability of the allocation rule in stochastic and stochastic with uncertainty settings when the values of predicted water available for sharing are expected with the same probability. Proofs for some of the properties of the allocation rule can be found in the Appendix (available with the online version of this paper).

$$
\begin{aligned}
&x_s^i = c_s^i - d_s^i \\
&d_s^i = \left[\frac{1 - \left(\dfrac{v_s^{ri} x_s^{si}}{\sum v_s^{ri} x_s^{si}} \right)}{n-1} \right] \times D_s \\
&d_s^i \leq c_s^i \\
&0 \leq x_s^i \leq c_s^i
\end{aligned}
\tag{3}
$$

c_s^i is the claim of the riparian country i at each state $s \in S$;
v_s^{ri} is the relative vulnerability of the riparian country i at state $s \in S$;
x_s^{si} is the marginal contribution of a riparian country to the coalition at state $s \in S$;
x_s^i is allocation to the riparian state i at state $s \in S$;
d_s^i is the deficit allocated to the riparian state i at state $s \in S$;
D_s is the total water deficit at state $s \in S$.

The worth of a non-cooperative coalition is defined using O'Neill's (1982) bankruptcy theory as shown in Equation (2). The allocation rule can be applied to allocate the available water in a stochastic setting, state by state, without considering the uncertainty and the results obtained were in the core. These allocation outcomes are feasible as well as self-enforceable. Table 4 shows the allocation vectors obtained by applying the allocation rule state by state to a hypothetical river basin under scarcity. It is assumed here that all the agents have equal weights.

Table 4. Water allocation vectors from the proposed allocation rule for the hypothetical river basin under bankruptcy.

Claimant n	Claim c_i	Estate E_S	Core allocation vectors $x_i^E = \begin{bmatrix} x_1^1, x_2^1, x_3^1 \\ x_1^2, x_2^2, x_3^2 \\ x_1^3, x_2^3, x_3^3 \end{bmatrix}$
A	100	100	$\begin{bmatrix} 33.33, 33.33, 33.33 \\ 0, 50, 150 \\ 0, 66.66, 233.33 \end{bmatrix}$
		200	
		300	
B	200	100	
		200	
		300	
C	300	100	
		200	
		300	

On the other hand, when the uncertainty associated with the occurrence of the estate is considered the allocation results obtained have credible deviation in the ex-ante state before the resolution of the uncertainty in the ex-post state. As a result, Table 4 can be interpreted in a different way. As can be seen from Table 4, there is a possibility that the members of a sub-coalition of the grand coalition might cooperate to decrease their risk while maximizing the total allocation of at least one riparian country. When uncertainty is considered in the stochastic setting, only the allocation outcomes which are in the weak sequential core are self-enforcing. Therefore, even though the allocations from the allocation rule are in the core they have credible deviation in the ex-ante state from sub-coalition of the grand coalition. As a result, the allocation payoffs are not in the weak sequential core. Therefore, the allocation rule, even though useful in allocating the water in the stochastic and deterministic settings, should be further developed to yield water allocations that are self-enforcing under uncertainty.

In this article the authors discussed the application of the bankruptcy allocation rules to allocate transboundary water under stochastic and uncertain settings. In order to increase its applicability, the approach should be further studied by taking into account the following. First, the temporal and spatial variation of the river water should be taken into account in order to enhance the applicability of the methodology in reality. Second, geographical, economic and political as well as military asymmetries among the countries should be taken into account in order to fully capture the barriers to design an allocation mechanism with basin-wide agreement. Third, the non-consumptive benefits of the river basin's water should be considered as well. Fourth, the role the approach discussed in this article can play, in terms of assisting decision makers in reaching basin-wide water allocation agreements in accordance with the principles put forward by the Helsinki Convention on the management of transboundary rivers and lakes (1966) and United Nations Watercourses Convention (1997), should be further investigated.

4. Conclusion

The issue of water scarcity could be a cause for conflicts among river-sharing parties. This issue becomes more complicated when sharing transboundary rivers since they are shared by sovereign countries. In addition to water scarcity the stochastic and uncertain nature of most of these river basins makes sharing these basins' water even more challenging. In this paper the application of stochastic bankruptcy games for the allocation of water under the bankruptcy scenario in border-crossing rivers was proposed. Furthermore, the application of stochastic bankruptcy allocation rules was demonstrated. In addition, an allocation rule for the allocation of transboundary water under the stochastic setting which uses a novel way of accounting the water contribution of riparian countries was proposed and discussed in detail using a hypothetical case example.

The results depicted that only applying the *SCEA* rule results in water allocation vectors which are self-enforcing. This is due to the fact that there is no possibility to increase the total reward of one of the water claiming countries while decreasing the risk faced by the other riparian states in any state. Hence, these allocations are in the weak sequential core. On the other hand, the water allocation vectors obtained from *SP*, *SAP* and *SCEL* allocations rules can be blocked in the ex-ante stage. The reason for this is that the riparian countries can cooperate in the ex-ante stage to increase the water allocation vector of at least one riparian state while decreasing the risks faced by the other riparian countries.

The proposed allocation rule was also applied to a hypothetical river basin under water scarcity to allocate the water in a stochastic setting and in a stochastic setting under uncertainty. The water allocation vectors obtained for the stochastic setting where the allocation rule was applied to allocate the available water were self-enforcing. This is because the allocation procedure is composed of a series of deterministic water allocation steps and these allocation vectors are in the core. On the other hand, when the proposed allocation rule is applied to allocate the water under a stochastic setting with uncertainty the allocation vectors obtained were not self-enforcing. This is because these allocation vectors can be adjusted by increasing the total allocation vector of one of the riparian states while decreasing the uncertainty faced by the other riparian countries in the ex-ante stage.

The Helsinki Convention on the management of transboundary rivers and lakes and the United Nations Watercourses Convention (1997) states the guiding principles for the management of transboundary river basins. The approach discussed in this article needs to be further developed according to these guidelines in order to capture the river sharing problem in reality fully. The authors hope that this research article contributes to the sustainable management and sharing of border-crossing river basins.

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Compliance with ethical standards

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References

Ansink, E. (2009). Game-Theoretic Models of Water Allocation in Transboundary River Basins. PhD Dissertation, Wageningen University.

Ansink, E. & Arjan, R. (2008). Climate change and the stability of water allocation agreements. *Environmental and Resource Economics 41*(2), 249–266.

Ansink, E. & Harold, H. (2015). The economics of transboundary river management. In: *Handbook of Water Economics.* Dinar, A. & Schwabe, K. (eds). Edward Elgar, Cheltenham, pp. 434–466.

Ansink, E. & Marchiori, C. (2015). Reallocating water: an application of sequential sharing rules to Cyprus. *Water Economics and Policy* 1(4), DOI: http://dx.doi.org/10.1142/S2382624X15500186.

Ansink, E. & Weikard, H. -P. (2012). Sequential sharing rules for river sharing problems. *Social Choice and Welfare 38*(2), 187–210.

Awulachew, S. B., Demissie, S. S., Hagos, F., Erkossa, T. & Peden, D. (2012). Water management intervention analysis in the Nile basin. In: *The Nile River Basin Water, Agriculture, Governance and Livelihoods.* Awulachew, S. B., Smakhtin, V., Molden, D. & Paden, D. (eds). Routledge-Earthscan, Abingdon, pp. 292–311.

Barrett, S. (1994). Self-enforcing international environmental agreements. *Oxford Economic Papers 46*(S), 878–894.

Bates, B., Kundzewicz, Z. W., Wu, S. & Palutikof, J. (2008). *Climate Change and Water.* Tec Paper Intergovernmental Panel on Climate Change *210.*

Borm, P. & Suijs, J. (2002). Stochastic cooperative games. In: *Theory and Decision Library C.* Vol. 31. Moulin, H. & Peters, H. (eds), Springer, US, pp. 1–26.

Branzei, R., Ferrari, G., Fragnelli, V. & Tijs, S. (2008). A flow approach to bankruptcy problems. *AUCO Czech Economic Review 2*(2), 146–153.

Brunnee, J. & Stephen, J. T. (2002). The changing Nile Basin regime: does law matter? *Harvard International Law Journal 43*(1), 105–159.

Cooley, H., Christian-Smith, J., Gleick, P. H., Allen, L. & Cohen, M. J. (2011). Climate change and transboundary waters. In: *The World's Water, Vol. 7. The Biennial Report on Freshwater Resources.* Gleick, P. H., Allen, L., Cohen, M. J., Cooley, H., Christian-Smith, J., Heberger, M., Morrison, J., Palaniappan, M. & Schulte, P. (eds). Island Press/Center for Resource Economics, Washington, DC, pp. 1–22.

Curiel, I. J., Maschler, M. & Tijs, S. H. (1987). Bankruptcy games. *Zeitschrift fur Operations Research 31*, A 143–A 159.

Dagan, N. (1996). New characterizations of old bankruptcy rules. *Social Choice and Welfare 13*(1), 51–59.

Degefu, D. M. & He, W. (2016). Allocating water under bankruptcy scenario. *Water Resources Management* doi:10.1007/s11269-016-1403-x.

De Stefano, L., Duncan, J., Dinar, S., Stahl, K., Strzepek, K. M. & Wolf, A. T. (2012). Climate change and the institutional resilience of international river basins. *Journal of Peace Research 49*(1), 193–209.

Drieschova, A., Giordano, M. & Fischhendler, I. (2008). Governance mechanisms to address flow variability in water treaties. *Global Environmental Change 18*(2), 285–295.

Eckstein, G. (2009). Water scarcity, conflict and security in a climate change world: challenges and opportunities for international law and policy. *Wisconsin International Law Journal 27*(3), 409–461.

FAO (2007). Nile Basin System. http://www.faonile.org/nbs/nilesystem.html (accessed 16 November 2015).

Gallastegui, M. C., Iñarra, E. & Prellezo, R. (2002). Bankruptcy of fishing resources: the northern European anglerfish fishery. *Marine Resource Economics 17*(4), 291–307.

Gleick, P. H. (1993). Water and conflict: fresh water resources and international security. *International Security 18*(1), 79–112.

Habis, H. & Herings, P. (2011). Transferable utility games with uncertainty. *Journal of Economic Theory 146*(5), 2126–2139.

Habis, H. & Herings, P. (2013). Stochastic bankruptcy games. *International Journal of Game Theory 42*(4), 973–988.

Hensel, P. R., McLaughlin, M. S. & Sowers, T. E. (2006). Conflict management of riparian disputes. *Political Geography 25*(4), 383–411.

Herrero, C. & Villar, A. (2001). The Three Musketeers: four classical solutions to bankruptcy problem. *Mathematical Social Sciences 42*(3), 307–328.

Homer, D. & Thomas, F. (1994). Environmental scarcities and violent conflict: evidence from cases. *International Security 19*(1), 5–40.

Jury, M. R. (2011). Climate variability and hydrologic response climatic factors modulating Nile River flow. In: *Nile River Basin Hydrology, Climate and Water Use.* Assefa, M. M. (ed.). Springer, Dordrecht, Heidelberg, London, New York, pp. 267–280.

Keith, B., Enos, J., Garlick, B., Simmons, G., Copeland, D. & Cortizo, M. (2013). Limits to population growth and water resource adequacy in the Nile River Basin, 1994–2100. In: *Proceedings of the 31st International Conference of the System Dynamics Society*. Held 21-25 July 2013, Cambridge, MA, USA.

Koopmans, T. C. (1960). Stationary ordinal utility and impatience. *Econometrica 28*, 287–309.

Kranich, L., Perea, A. & Peters, H. (2005). Core concepts for dynamic TU games. *International Game Theory Review 7*(1), 43–61.

Madani, K., Zarezadeh, M. & Morid, S. (2014). A new framework for resolving conflicts over transboundary rivers using bankruptcy methods. *Hydrology and Earth System Sciences 18*, 3055–3068.

Mianabadi, H., Mostert, E., Zarghami, M. & Van De Giesa, N. (2014a). A new bankruptcy method for conflict resolution in water resource allocation. *Journal of Environmental Management 144*(1), 152–159.

Mianabadi, H., Sheikhmohammady, M., Mostert, E. & Van De Giesen, N. (2014b). Application of the ordered weighted averaging (OWA) method to the Caspian Sea conflict. *Stochastic Environmental Research and Risk Assessment 28*(6), 1359–1372.

Mianabadi, H., Mostert, E., Pande, S. & Giesen, N. (2015a). Weighted bankruptcy rules and transboundary water resources allocation. *Water Resources Management 29*(7), 2303–2321.

Mianabadi, H., Mostert, E. & van de Giesen, N. (2015b). Chapter 7. Trans-boundary River Basin management: factors influencing the success or failure of international agreements. In: *Conflict Resolution in Water Resources and Environmental Management,* 1st edn. Hipel, K. W., Fang, L., Cullmann, J. & Bristow, M. (eds). Springer International Publishing, Switzerland, pp. 133–141.

Molden, D., Awulachew, S. B., Conniff, K., Rebelo, L. M., Mohamed, Y., Peden, D., Kinyangi, J., Van Breugel, P., Mukherji, A., Cascão, A., Notenbaert, A., Demissie, S. S., Neguid, M. A. & El Naggar, G. (2010). *Nile Basin Focal Project: Synthesis Report, CGIAR Challenge Program on Water and Food*, Colombo, Sri Lanka.

Moreno-Ternero, J. D. & Villar, A. (2004). New characterizations of a classical bankruptcy rule. *Review of Economic Design 10*(2), 73–84.

Nile Basin Initiative (2012). Summary: The State of the River Nile Basin Chapter 9 State of the River Nile Basin. http://sob.nilebasin.org (accessed 6 December 2015).

O'Neill, B. (1982). A problem of rights arbitration from the Talmud. *Mathematical Social Sciences 2*(4), 345–371.

Predtetchinski, A., Herings, P. & Perea, A. (2006). The weak sequential core for two-period economies. *International Journal of Game Theory 34*(1), 55–65.

Ray, D. (1989). Credible coalitions and the core. *Int. J. Game Theory 18*(2), 185–187.

Suijs, J. (2000). *Cooperative Decision-Making Under Risk.* Springer Science Business Media, New York.

Suijs, J. & Borm, P. (1999). Stochastic cooperative games: super additivity, convexity, and certainty equivalents. *Games and Economic Behavior 27*(2), 331–345.

Swain, A. (2001). Water wars: fact or fiction? *Futures 33*, 769–781.

Swain, A. (2015). Water wars. In: *International Encyclopedia of the Social & Behavioral Sciences,* 2nd edn, Vol. 25. Wright, J. D. (ed.). Elsevier, Oxford, pp. 443–447.

Thomson, W. (2003). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey. *Mathematical Social Sciences 45*(3), 249–297.

Thomson, W. (2015). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: an update. *Mathematical Social Sciences 74*, 41–59.

United Nations (1997). *Convention on the Law of the Non-Navigational Uses of International Watercourses, Adopted by the General Assembly of the United Nations*. Entered into force on 17 August 2014.

Valipour, M. (2015a). Optimization of neural networks for precipitation analysis in a humid region to detect drought and wet year alarms. *Meteorological Applications 2*(1), 91–100.

Valipour, M. (2015b). Future of agricultural water management in Africa. *Archives of Agronomy and Soil Science 61*(7), 907–927.

Valipour, M. (2015c). A comprehensive study on irrigation management in Asia and Oceania. *Archives of Agronomy and Soil Science 61*(9), 1247–1271.

Valipour, M. (2016). Long-term runoff study using SARIMA and ARIMA models in the United States. *Meteorological Applications 22*(3), 592–598.

Wolf, A. T. (1998). Conflict and cooperation along international waterways. *Water Policy 1*(2), 251–265.

Wolf, A. T. (1999). Criteria for equitable allocations: the heart of international water conflict. *Natural Resources Forum 23*(1), 3–30.

Wolf, A. T., Kramer, A., Carius, A. & Dabelko, G. D. (2006). Water Can Be a Pathway to Peace, Not War (No. 1). Woodrow Wilson International Center for Scholars. www.wilsoncenter.org/topics/pubs/NavigatingPeaceIssue1.pdf (accessed 25 December 2015).

World Bank (2000). The Nile River Basin. http://siteresources.worldbank.org/INTAFRNILEBASINI/About%20Us/21082459/Nile_River_Basin.htm (accessed 10 July 2015).

Wu, X. & Whittington, D. (2006). Incentive compatibility and conflict resolution in international river basins: a case study of the Nile Basin. *Water Resources Research 42*(2), 1–5.

Zarezadeh, M., Madani, K. & Morid, S. (2012). Resolving transboundary water conflicts: lessons learned from the Qezelozan-Sefidrood river bankruptcy problem. In: *Proceedings of World Environmental and Water Resources Congress 2012*. American Society of Civil Engineers, Reston, pp. 2406–2412.

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