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The Single-mode and Two-mode Squeezed Light Generated in Ring Microresonators: Theoretical Limitations and Experimental Possibilities

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Abstract. The correlation properties of photon pairs generated in high-Q whispering-gallery mode microresonators in the four wave mixing process are analyzed theoretically. The degree of two-mode squeezing and photon entanglement are investigated depending on different parameters of the pump or microresonator, and the optimal regime for maximum squeezing is found. Various experimental configurations are analyzed, and the most optimal options for compact generation of two-mode squeezed light are found.

INTRODUCTION

A new class of high-Q whispering-gallery mode (WGM) microresonators has been developed recently, that are extremely promising sources of non-classical light. Currently, the most well known devices used for this purpose are nonlinear bulk crystals which exploit the parametric down-conversion (SPDC) or the spontaneous four wave mixing (SFWM) processes. Such sources can create radiation with unique properties for different purposes. However, in order to create radiation with a very high degree of two-mode squeezing or high correlation between photons, long crystals or setups consisting of several crystals [1] are required, which can not be used in small-scale (*e.g.* on-chip) applications. For these applications, the use of the WGM microresonators is a very good solution [2]. They already demonstrated good prospects for quantum optics tasks [3]. They allows to achieve a high efficiency of non-classical light generation and, what is more, can be used to store and process quantum information [4].

The goal of this paper is the theoretical analysis of correlation properties in generated photon pairs. We extend the previous theoretical works in this area [5, 6] taking into account the recent experimental achievements in fabrication of the very high-Q WGM microresonators [7, 8] and focus on the optimal regimes and parameters for greatest possible squeezing. We calculate analytically the degree of squeezing and the purity of the squeezed state generated by means of the four-wave mixing process in the WGM resonators with third-order nonlinearity $\chi^{(3)}$ and analyze its dependence on the most important parameters, namely the losses due to coupling with the transmission line, internal losses in the cavity and the optical pump power.

Analysis of the squeezing

We consider the model of the WGM resonator as the ring cavity shown at Fig. 1.

The pump wave with frequency ω_p, power P in watts and the amplitude $A_p = \sqrt{P/\hbar\omega_p}$ along with the vacuum noise $\hat{V}_1$ is incident on the first beamsplitter (T_1, R_1) through which interaction with the internal central mode of the resonator $\hat{a}$ and the external field $\hat{d} = \sqrt{T_1}\hat{a} - \sqrt{R_1}(A_p + \hat{V}_1)$ is carried out. Similar expressions can be written for the "+" and "−" satellite modes of the cavity: $\hat{d}_- = \sqrt{T_1}\hat{b}_- - \sqrt{R_1}\hat{V}_{1-}$ and $\hat{d}_+ = \sqrt{T_1}\hat{b}_+ - \sqrt{R_1}\hat{V}_{1+}$. It is in this external non-classical field at the output where single-mode and two-mode squeezing are observed and analyzed using a homodyne detector. The internal modes of the resonator, in turn, experience four-wave mixing, as a result of which the energy from the central mode $\hat{a}$ flows into satellites $\hat{b}_+$, $\hat{b}_-$, which are equally spaced in frequency from the central mode to the region of free spectral range. Losses in such a system can be successfully modeled with the help of another beamsplitter (T_2,R_2), which also introduces an admixture from vacuum noise $\hat{V}_2$. The vacuum noise introduced in this way satisfies the following commutation relations:

$$[\hat{V}_i(\omega), \hat{V}_j^\dagger(\omega')] = \delta_{ij}\delta(\omega - \omega'). \quad (1)$$

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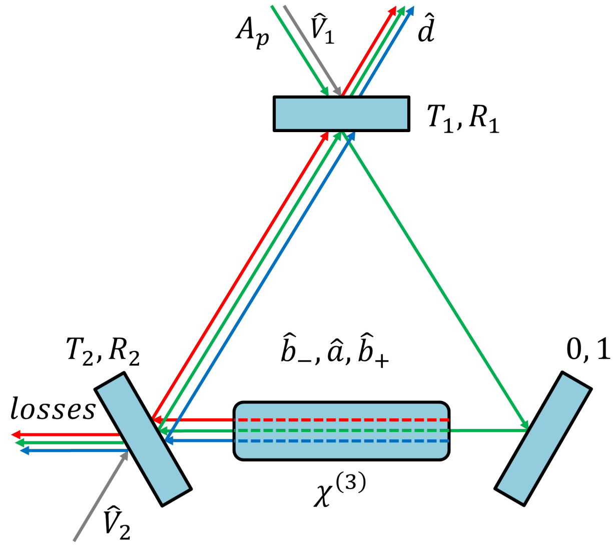

FIGURE 1. A model of the ring microresonator with three mirrors: one ideal and two translucent ones with transmittance and reflection coefficients T_1, R_1 and T_2, R_2. Classical pumping with the corresponding quantum noise falls on the upper mirror, and there are also vacuum noise on the lower left mirrors. Inside, the cavity is filled with a Kerr medium with third-order nonlinearity $\chi^{(3)}$.

Let us write down the equations for the evolution of the annihilation operator of the main cavity mode and add to the right-hand terms corresponding to the interaction of this mode with the pump field $A_p e^{-i\omega_p t}$ through the first beamsplitter, two vacuum modes $\hat{V}_1$ and $\hat{V}_2$ and losses corresponding to them. Following the notation of the article [6], the Langevin equation for the internal pumped cavity mode can be written as follows:

$$\dot{\hat{a}} + (\kappa + i(\omega - \omega_p - g\hat{a}^\dagger \hat{a}))\hat{a} = \sqrt{2\kappa_1}(A_p + \hat{V}_1) + \sqrt{2\kappa_2}\hat{V}_2, \tag{2}$$

where $\kappa = \kappa_1 + \kappa_2$ and $\kappa_{1,2}$ is related to the transmission and reflection coefficient $T_{1,2} \approx 2\kappa_{1,2}$, $R_{1,2} \approx 1$. To solve the equation (2) we use frequency domain and a standard procedure of linearisation after which the equation on $\hat{a}$ is linked to the equation on $\hat{a}^\dagger$. As a result, we have single-mode squeezing, and for the analysis of squeezing in the output field we use the language of spectral density, which together with the solution methods are considered in detail in the review [9]. When the internal losses are negligible in comparison with the coupling $\kappa_2 \ll \kappa_1$, the spectral density of mode d at the frequency near the central mode can be represented as the sum of the contributions from the cosine and sine quadratures of squeezed light:

$$S_0[\hat{d}^\varphi] = \frac{e^{2r}}{2} cos^2(\varphi - \varphi_0) + \frac{e^{-2r}}{2} sin^2(\varphi - \varphi_0), \tag{3}$$

where r determines the degree of squeezing and φ denotes the angle of homodyning deferred from a certain reference value φ_0. At $\varphi = \varphi_0 + \pi/2$ we observe maximum squeezing $Sq = 1/(2\min_\varphi S_0[\hat{d}^\varphi])$, and at $\varphi = \varphi_0$ — the maximum anti-squeezing $ASq = 2\max_\varphi S_0[\hat{d}^\varphi]$ in comparison with vacuum spectral density. At the same time the ratio ASq/Sq remains equal to one, and this corresponds to a pure state. In the presence of significant internal losses, the situation changes. The ratio ASq/Sq is always more than one, and the purity of this state $P = ASq/Sq$ will not reach one for any other parameters.

Similar equations for the two side modes $\hat{b}_-$ and $\hat{b}_+$ of the resonator are as follows:

$$\dot{\hat{b}}_+ + (\kappa + i(\omega_+ - 2g\hat{a}^\dagger \hat{a}))\hat{b}_+ - ig\hat{a}^2 \hat{b}_-^\dagger = \sqrt{2\kappa_1}\hat{V}_{1+} + \sqrt{2\kappa_2}\hat{V}_{2+}, \tag{4}$$

$$\dot{\hat{b}}_-^\dagger + (\kappa - i(\omega_- - 2ig\hat{a}^\dagger \hat{a}))\hat{b}_-^\dagger + ig(\hat{a}^\dagger)^2 \hat{b}_+ = \sqrt{2\kappa_1}\hat{V}_{1-}^\dagger + \sqrt{2\kappa_2}\hat{V}_{2-}^\dagger. \tag{5}$$

In these two equations, ω_- and ω_+ are frequencies corresponding to the satellite modes. Let us pay attention to the connection between these equations through mode $\hat{a}$. The mode in the linearization method splits into classical

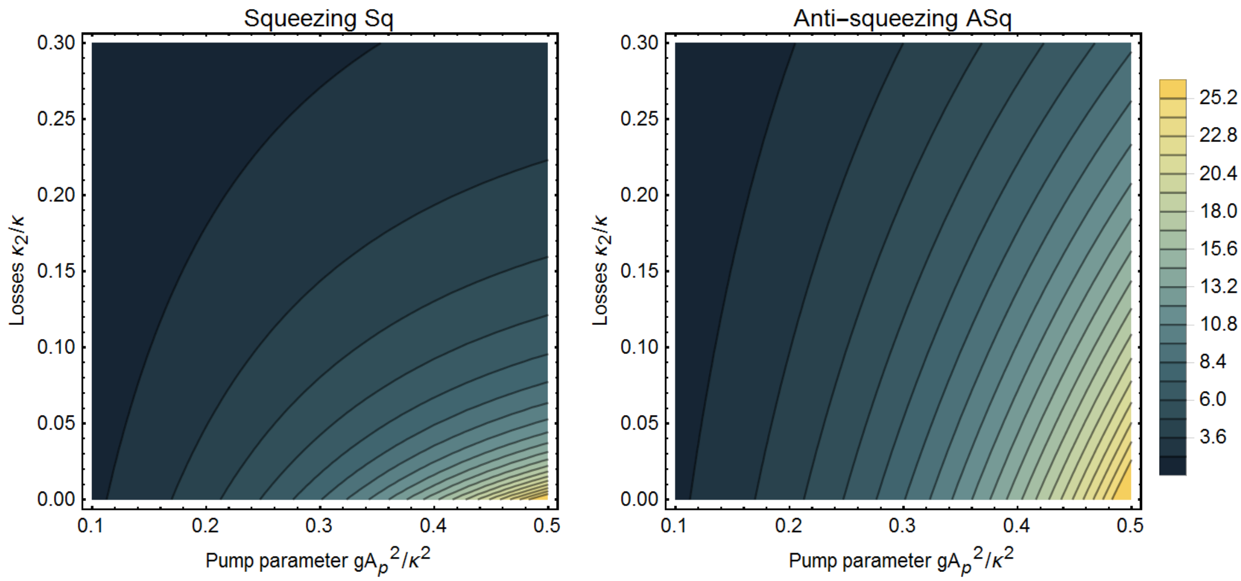

FIGURE 2. Squeezing Sq and anti-squeezing ASq of the resulting single-mode or two-mode light as a function of both losses and pump parameter.

and quantum parts $\hat{a} \to A + \hat{a}$, and in the zeroth approximation only the classical part is left in the equation. In the frequency domain the classical part of central mode is determined by a nonlinear equation $A(\kappa + ig|A|^2) = \sqrt{2\kappa_1}A_p$, the solution of which is well known and determines nonlinear resonance. The solution seems similar to the case of the central mode. However, instead of the spectral density of the modes separately, it is necessary to analyze the correlation between them. For this we will consider $S^\varphi(\hat{d}_+ + \hat{d}_-)$.

The spectral density for the central output mode $\hat{d}$, depending on the phase φ of the local oscillator during homodyne detection, turns out to be exactly equal to the spectral density of $\hat{d}_+ + \hat{d}_-$ i.e. $S[\hat{d}^\varphi] = S[\hat{d}_+^\varphi + \hat{d}_-^\varphi]$. This reflects the fact that the linearized equation (2) for the quantum part of $\hat{a}$ and the conjugate equation for $\hat{a}^\dagger$ have exactly the same form and coefficients as linearized equations (4,5). In the presence of internal losses, the expression for the spectral density will be a linear combination of two expressions similar to the case without losses (3). For simplicity, we write the formula for the detuning $\xi = \omega - \omega_p - g|A|^2$ equals to zero:

$$S[\hat{d}^\varphi] = \frac{(c_1+c_2)^2}{2} cos^2(\varphi - \varphi_1) + \frac{(c_1-c_2)^2}{2} sin^2(\varphi - \varphi_1) + \\ + \frac{(1+\sqrt{\kappa_2/\kappa_1}(c_1+c_2))^2}{2} cos^2(\varphi - \varphi_2) + \frac{(1+\sqrt{\kappa_2/\kappa_1}(c_1-c_2))^2}{2} sin^2(\varphi - \varphi_2). \tag{6}$$

Here c_1, c_2, φ_1, φ_2 are expressed in terms of the resonator and pump parameters, and in fact, there remains only a dependence on two essential parameters: the relative losses κ_2/κ (or coupling κ_1/κ) and the pump parameter gA_p^2/κ^2. The dependence of the state squeezing Sq and anti-squeezing ASq on these parameters is presented in the Fig. 2. The figure shows that, at zero losses, the degree of squeezing and anti-squeezing completely coincide, but with an increase in losses, a significant discrepancy appears. At the same time, a monotonic dependence of both quantities on the pump parameter is visible: with an increase in the pump power, the state Sq also increases. However, this does not mean that by increasing power we improve the situation, since the purity of the state also decreases.

Thus, both the squeezing and the purity of the state depend on the pump power. With fixed relative internal losses, or with a fixed cavity loading, pumping can be changed in the experiment. The dependence of purity P on squeezing Sq is shown in the Fig. 3 for different losses κ_1/κ. It is remarkable that this dependence turns out to be almost linear, and if we have a desire to increase squeezing, we will inevitably have to proportionally sacrifice the purity of the state. Let us pay attention to the fact that for any non-zero losses there is an upper limit of permissible light squeezing: for a $\kappa_2/\kappa = 0.05$ this boundary is 20, for $\kappa_2/\kappa = 0.1$ it is 10, for $\kappa_2/\kappa = 0.2$ it is 5. Thus, we can conclude that the maximum possible squeezing that is available in this scheme is defined as inverse losses $Sq_0 = \kappa/\kappa_2$. With linear

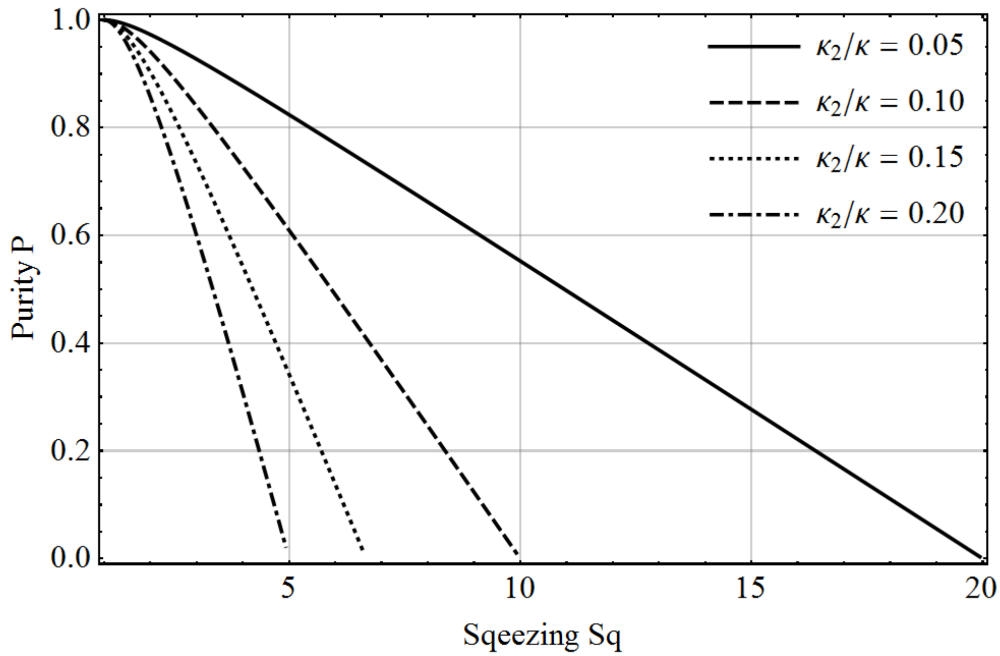

FIGURE 3. Parametric dependence of purity P of the resulting single-mode or two-mode light on its squeezing Sq.

accuracy, the approximate formula for the dependence of P on Sq therefore becomes as follows:

$$P \approx 1 - \frac{\kappa_2}{\kappa} Sq. \tag{7}$$

Thus, an obvious fact arises: to obtain simultaneously a high Sq and high P, it is necessary to work with very low relative internal losses or greatly overload the resonator. In this case, it is necessary to use in the experiment ring microresonators with maximum intrinsic Q factor, or Q_2, and to be able to create strong loading. To better understand what κ_2/κ experimentally is let us look at an absorption spectrum of the resonator. The spectrum contains absorption lines that are characterized by a depth K and a full width κ. Using these parameters, one can easily find the degree of losses using a system of equations with loaded Q factor :

$$K = \frac{4Q\Gamma^2}{Q_1 + Q_2}, \tag{8}$$

$$1/Q = \frac{1}{Q_1} + \frac{1}{Q_2} = \frac{2\kappa_1}{\omega} + \frac{2\kappa_2}{\omega}, \tag{9}$$

where $\Gamma = 1$ for a single-mode resonator. Solving this system with respect to γ_2/γ, we find that the degree of loss does not depend on the loaded Q factor of the resonator or on the line width, but depends only on the line depth K, and in the overloaded mode it is determined by $\kappa_2/\kappa \approx K/4$.

CONCLUSION

Summing up, we have performed a theoretical analysis to describe the correlation properties in generated photon pairs. Analytical expressions have been found for the spectral densities of noises in pumped mode and two satellite modes. These expressions have allowed us to analyze the correlation between these noises in terms of quadrature squeezing and anti-squeezing. It has been discovered that the single-mode squeezing in the central mode equals to the two-mode squeezing of satellites. A significant difference between squeezing and anti-squeezing has been demonstrated by the example of the dependence on the losses in the system and the pump parameter. According to calculations, the purity of the state depends on squeezing in an approximately linear way, and the losses act as the slope of the line. In order to achieve the best correlation properties, it is necessary to work in a regime when the ratio of the loaded Q-factor of the resonator to the intrinsic Q-factor is close to zero, which can only be achieved with very large values of the resonator intrinsic Q-factor and the ability to greatly overload the resonator. For an experimental understanding of the restrictions on the maximum available squeezing, an expression has been obtained that relates the losses coefficient to the depth of the absorption line of the WGM microresonator.

It is worth mentioning that the analysis described in the article can be performed taking into account the geometric dispersion, which can be controlled using the design of new resonator shapes. In this case, improvements over the described results are possible.

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