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On Resonance Regimes of Drill String Nonlinear Dynamics

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Abstract. The paper focuses on investigation of resonance regimes of a drill string nonlinear dynamics under the effect of a variable axial compressive force. The drill string is modelled in the form of a rotating elastic isotropic rod with hinged ends. Deformations of the drill string are assumed to be finite. Using Galerkin's approach a mathematical model of the drill string lateral vibrations reduces to a nonlinear ordinary differential equation for the generalized time function. Applying the harmonic balance method, the amplitude-frequency characteristics of the resonances on basic and higher frequencies are determined. As a result of numerical analysis of the impact of the dynamic system parameters on the resonance curves, considerable nonlinear effects of the amplitude-frequency characteristics of the drill string vibrations are revealed. Recommendations to choose optimal constructive and dynamic characteristics of drill strings are provided.

INTRODUCTION

The problem of ensuring dynamic stability of different mechanical structures is one of the main problems in machine dynamics. It has the special importance for rod elements, including rotating drill strings applied in oil and gas industry. Stability of their nominal motion depends on efficiency of the developed models and methods of their calculation. Flexibility of a drill string in view of its large length and effects of variable external loadings, in particular, a compressing axial force, can result in finite deformations of the string at drilling of oil and gas wells. Therefore, when investigating the drill string dynamics, it is necessary to consider its deformability to determine amplitudes of displacements with detecting dangerous resonant oscillating regimes of the drill string.

Solutions to several problems concerning the theory and basic principles of modelling of drill string dynamics can be found in [1], where the influence of longitudinal and transverse damping on zones of a parametrical resonance is examined. In [2] the authors analyze a drill string in vertical and deviated holes using Galerkin's method under the assumption that the string is hinged at both ends. It is shown that approximation by Galerkin's method can be successfully applied to research the drill string dynamics. In the work [3] the authors obtain that high speeds of a string rotation are in stability zones for any loadings affecting a bit. However, application of these speeds in practice does not seem possible. In [4] the authors indicate that even small initial curvature of a drill string, approximated by a finite series of smooth functions, considerably influences the frequency characteristics of the system. However, no destructive oscillations are observed.

Nonlinear systems with generalized (modal) parameters [5] are widely applied to model motion of separate or coupled elements of different constructions and machines, describing nonlinear oscillations of systems with one DOF or with discrete masses. Besides, such equations can be used to simulate nonlinear oscillations of systems with distributed parameters. They can be reduced to ODEs, applying the methods of variables separation, e.g. the Galerkin approach [2, 6], the Rayleigh-Ritz method [7, 8], and the finite element method [9, 10]. Based on the solutions of these equations one can conduct an exhaustive analysis of dynamics of the mechanical systems, and choose their optimal constructive parameters and dynamic characteristics.

In this paper the resonance phenomena of drill strings taking into account nonlinear complicating factors are examined. Derivation of their mathematical models is described in [11, 12]. Numerical analysis of these models

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shows that lateral vibrations of a drill string, modelled as a rotating elastic isotropic rod of symmetric cross-section, make the main contribution to the general oscillatory process, whereas the contribution of longitudinal and torsional vibrations is negligible in comparison with lateral ones. Therefore, a mathematical model of lateral vibrations of the elastic rod with initial curvatures to analyze the resonance phenomena of drill strings under the influence of an external compressive load is used here. The results of this research will provide deeper insight into the nature of the drill string behaviour under the lateral vibrations with determination of their amplitude-frequency characteristics.

STATEMENT OF THE PROBLEM

Let us consider a global Cartesian coordinate system *Oxyz*. The axis of the drill string is assumed to be bent only in the *Oyz-plane* (*z*-axis is directed along the drill string axis), i.e. flat bending of the elastic isotropic rod of length l with symmetric cross-section is examined. A nonlinear mathematical model of the drill string lateral vibrations, based on Novozhilov's theory of finite deformations [13], can be obtained from either the model [11] without taking into account the effects of a supersonic air flow, or the model [12] neglecting the terms of longitudinal displacement $w(z,t)$ along the z-axis, and adding an initial curvature v_0 to the model:

$$\rho A\frac{\partial^2 v}{\partial t^2}+EI_x\frac{\partial^4 v}{\partial z^4}-\rho I_x\frac{\partial^4 v}{\partial z^2\partial t^2}+\frac{\partial}{\partial z}\left(N(z,t)\frac{\partial(v+v_0)}{\partial z}\right)-\frac{EA}{1-\nu}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right)^3-\rho A\omega^2 v=0, \tag{1}$$

where ρ is the mass density, A is the cross-section area of the drill string, $v(z,t)$ is the displacement of the flexural center of the cross-section along the y-axis owing to bending, E is Young's modulus, I_x is the axial inertia moment, ν is Poisson's ratio, ω is the angular speed of the rod.

Boundary conditions for the rod with hinged ends are written as

$$v(z,t)=0,\quad EI_x\frac{\partial^2 v(z,t)}{\partial z^2}=0\qquad (z=0,\ z=l) \tag{2}$$

The longitudinal compressive loading $N(z,t)$ is supposed to be periodically varying and is presented in the form:

$$N=N_0+N_t\cos\overline{\Omega}t, \tag{3}$$

where N_0 and N_t denote constant and variable in time components, respectively; $\overline{\Omega}$ is the frequency of external effects.

MODELLING OF RESONANCE REGIMES

Let us define the dimensionless time $\tau=t\cdot\Omega_0$ [14], where Ω_0 is the frequency of the drill string natural vibrations. Then applying the Galerkin method, the lateral displacement $v(z,t)$ in the *Oyz*-plane is given by

$$v(z,t)=\sum_{i=1}^{n}f_i(t)\sin\left(\frac{i\pi z}{l}\right). \tag{4}$$

The initial curvature of the drill string has a smooth form. Hence, it can be approximated by a periodic trigonometric function:

$$v_0(z)=f_0\sin\left(\frac{\pi z}{l}\right) \tag{5}$$

Considering the lateral vibrations of the drill string on the general form of bending of its axis, i.e. at $n=1$ in (4), using the dimensionless time, and taking into account (5) we obtain an ordinary differential equation for the generalized time function $f(\tau)$ (hereinafter the index "1" of the function $f(\tau)$ is omitted):

$$\frac{d^2 f}{d\tau^2}+(1+\beta\cos\Omega\tau)f+\alpha f^3=F_0+F_1\cos\Omega\tau, \tag{6}$$

where

$$\beta=\frac{a_3}{a_2},\quad \alpha=\frac{a_4}{a_2},\quad F_0=\frac{d_1}{a_2},\quad F_1=\frac{d_2}{a_2},\quad \Omega=\frac{\overline{\Omega}}{\Omega_0} \tag{7}$$

and

$$\begin{gathered}\Omega_0=\sqrt{\frac{a_2}{a_1}},\quad a_1=\frac{\rho}{2l}\left(Al^2+I_x\pi^2\right),\quad a_2=\frac{1}{2l^3}\left[EI_x\pi^4-N_0\pi^2l^2-\rho A\omega^2l^4\right],\\ a_3=-\frac{N_t\pi^2}{2l},\quad a_4=\frac{3EA\pi^4}{8l^3(1-\nu)},\quad d_1=f_0\frac{N_0\pi^2}{2l},\quad d_2=f_0\frac{N_t\pi^2}{2l}.\end{gathered} \tag{8}$$

Basic Resonance

Investigation of the resonance regimes of the drill string motion can be reduced to analysis of amplitude-frequency characteristics of their lateral vibrations.

In nonlinear system (6) along with vibrations, which frequency coincides with frequency of the external force, higher and subharmonic oscillations can arise [15]. The general method to solve such a system is expansion of the function $f(\tau)$ into the Fourier series with undefined coefficients. In the resonance case difference of phases between natural vibrations and external effects may have a great impact on the magnitude of amplitudes and the frequency of vibrations.

Considering the resonance on the basic frequency a solution of (6) can be approximated by a simple harmonic with frequency Ω:

$$f(\tau)=r_0+r_1\cos(\Omega\tau-\varphi_1) \tag{9}$$

On substituting (9) into (6) and applying the method of harmonic balance, we obtain the following system of equations defining the dependence between the amplitudes r_0, r_1 and the frequency Ω:

$$\begin{gathered}r_1^2\left(\lambda(r_0,r_1)-\Omega^2\right)^2=(F_1-\beta r_0)^2,\\ \frac{\beta r_1^2}{2(F_1-\beta r_0)}\left(\lambda(r_0,r_1)-\Omega^2\right)+r_0\left(\lambda(r_0,r_1)-\alpha\left(2r_0^2-\frac{3r_1^2}{4}\right)\right)=F_0,\end{gathered} \tag{10}$$

where $\lambda(r_0,r_1)=1+3\alpha\left(r_0^2+\frac{r_1^2}{4}\right)$.

Resonance on Higher Frequencies

In nonlinear dynamic systems in view of existence of nonlinear quadratic or cubic terms the resonance on higher frequencies can occur [16]. Therefore, to analyze the resonance phenomena in details an approximate solution of (6) is written as follows:

$$f(\tau)=r_0+r_1\cos(\Omega\tau-\varphi_1)+r_3\cos(3\Omega\tau-\varphi_3). \tag{11}$$

Substituting (11) into (6), using the harmonic balance method and eliminating the unknown phase angles φ_1 and φ_3 through some trigonometric transformations, we get a system of equations for the unknown amplitudes of vibrations r_0, r_1, r_3 and the frequency Ω:

$$r_1^2\left[A_1\left(A_1 - 3A_3\frac{r_3^2}{r_1^2}\right) + \frac{3\alpha^2}{16}r_1^2\left(3r_3^2 - \frac{A_1}{A_3}r_1^2\right)\right] = F_1^2,$$
$$A_3^2 r_3^2 = \frac{\alpha^2}{16}r_1^6, \tag{12}$$
$$\frac{\beta}{2}r_1^2\left[A_1 - \frac{3\alpha}{32}\left(\frac{\alpha^2}{A_3}r_1^6 + 16A_3r_3^2\right)\right] = F_1\left(F_0 - A_0r_0\right),$$

where

$$A_0 = 1+\beta+\alpha r_0^2 + \frac{3\alpha}{2}\left(r_1^2 + r_3^2\right),\ A_1 = -\Omega^2 + 1 + 3\alpha r_0^2 + \frac{3\alpha}{4}\left(r_1^2 + 2r_3^2\right),\ A_3 = -9\Omega^2 + 1 + 3\alpha r_0^2 + \frac{3\alpha}{4}\left(2r_1^2 + r_3^2\right).$$

The amplitude-frequency characteristics (11), (12) depend on geometrical and physical parameters of the dynamic system. It allows to examine the effects of these parameters on the resonance regimes of the drill string lateral vibrations to separate the resonant frequencies from drilling operating frequencies or to control them.

NUMERICAL ANALYSIS AND DISCUSSIONS

Numerical analysis of the basic and higher resonances of the nonlinear dynamic system (6), based on the amplitude-frequency relations obtained above, is conducted in the Wolfram Mathematica computational package. The influences of the drill string length, angular speed of rotation, axial compressive load and the magnitude of its initial curvature on the branches of resonance curves are investigated.

The dimensions and material properties of a hinged supported steel drill string are: $D = 0.2\,\text{m}$ (outer diameter of the drill string), $d = 0.12\,\text{m}$ (inner diameter), $E = 2.1\times10^5\,\text{MPa}$, $\rho = 7800\,kg/m^3$, $\nu = 0.28$.

In Fig. 1-4 resonance curves for various values of the drill string length, namely $l = 100\,\text{m}$ (tiny points), $l = 250\,\text{m}$ and $l = 500\,\text{m}$ (bold points) with the angular speed of rotation $\omega = 1.05\,\text{rad/s}$ are shown. The longitudinal compressive load is defined as: $N(z,t) = 1.7 + 0.5\cos(\omega\tau)\,\text{kN}$.

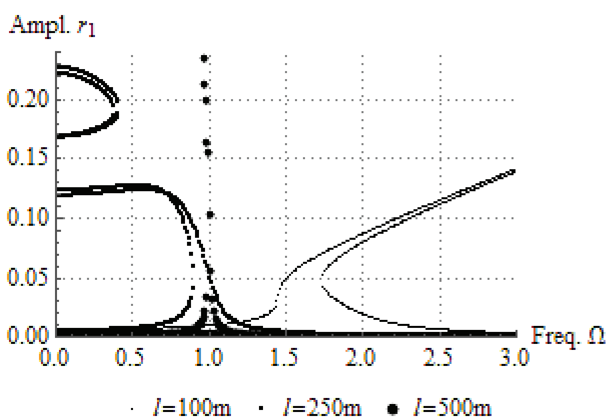

FIGURE 1. Influence of the drill string length on the resonance curves of its nonlinear lateral vibrations on the first harmonic, $f_0 = 0.3\,\text{m}$.

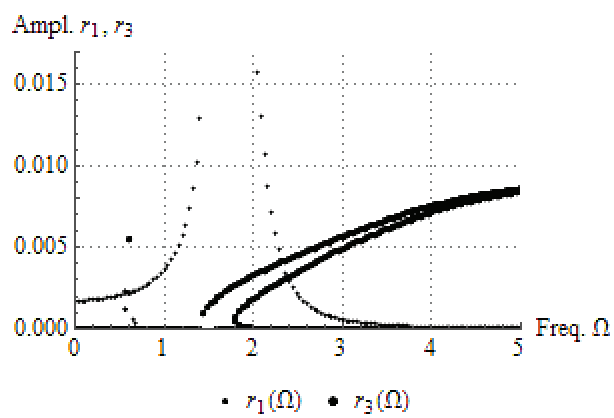

FIGURE 2. Resonance curves of 1st and 3rd harmonic vibrations of the drill string at the following values of parameters: $l = 100\,\text{m}$, $f_0 = 0.3\,\text{m}$.

As shown in Fig. 1, resonance curves ($l = 100\,\text{m}$) stretch out to the right because of existence of geometrical nonlinearity in the system. Meanwhile, shifting of the resonances curves towards the growth of external vibration frequency Ω takes place due to the initial curvature of the drill string axis. It is worth noting that the increase in the drill string length results in stretching the resonance curves out to the left, which is typical for mechanical systems with softening characteristics, and leads to instability of the system in the lower frequencies region. Moreover, the anomaly in the loop form for the simple harmonic appears at the range of amplitudes from 0.17 to 0.23 m.

Allowing for the third harmonic in the approximate solution (13), the increase in the external frequency Ω causes the sharp bias of the resonance curves $r_3(\Omega)$ in the higher frequencies direction, which corresponds to much smaller amplitudes compared to the resonance curves $r_1(\Omega)$ (Fig. 2). In addition, one more resonance curve $r_3(\Omega)$ appears to the left of the basic resonance (Ω changes from 0.5 to 0.7) due to the influence of the third harmonic on the oscillatory process.

However, such a high value of the initial curvature of the drill string can be considered only in theoretical research, and if neither friction nor rigid contacts with borehole walls is taken into account. When the value of the initial curvature $f_0 \leq 0.01\,\text{m}$, no shifting of the resonance curves to a zone of higher frequencies of the external effect is observed, as illustrated in Fig. 3.

As can be seen from Fig. 4, when the amplitude-frequency characteristics of the basic resonance drop down, oscillations on the third harmonic take place, that is the rise in amplitude-frequency characteristics of the resonance on higher frequencies is observed in the bifurcation zones of the basic resonance. Changes of resonance curves due to the increase in the angular speed of rotation and the axial compressive load are given in Fig. 5, 6, respectively.

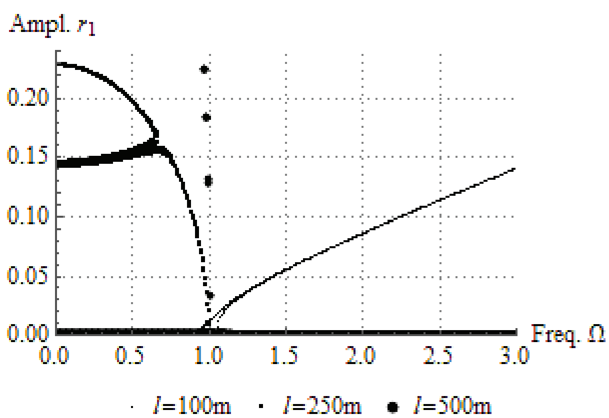

FIGURE 3. Influence of the drill string length on the resonance curves of its nonlinear lateral vibrations on the first harmonic, $f_0 = 0.01\,\text{m}$.

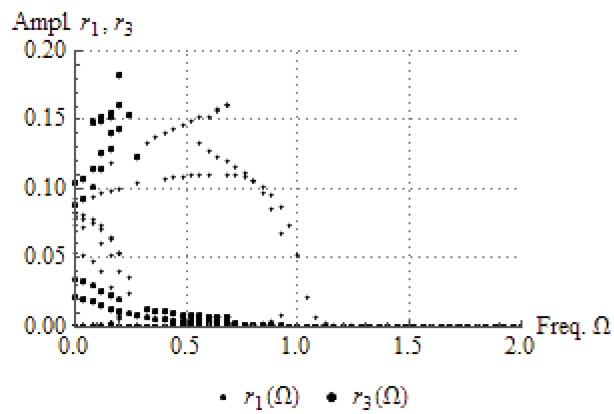

FIGURE 4. Resonance curves of 1st and 3rd harmonic vibrations of the drill string at the following values of parameters: $l = 250\,\text{m}$, $f_0 = 0.01\,\text{m}$.

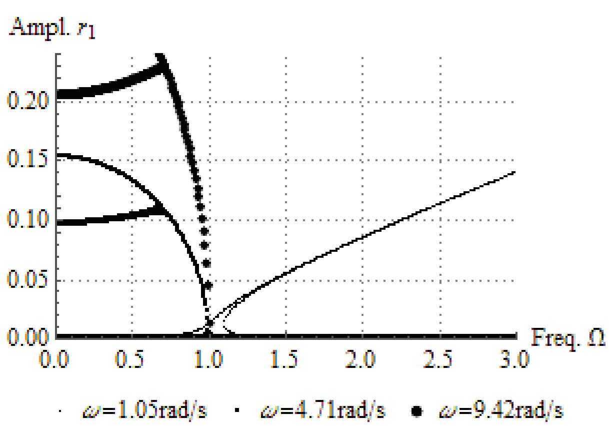

FIGURE 5. Influence of the drill string angular speed of rotation on the resonance curves of its vibrations on the first harmonic at $l = 100\,\text{m}$, $f_0 = 0.02\,\text{m}$.

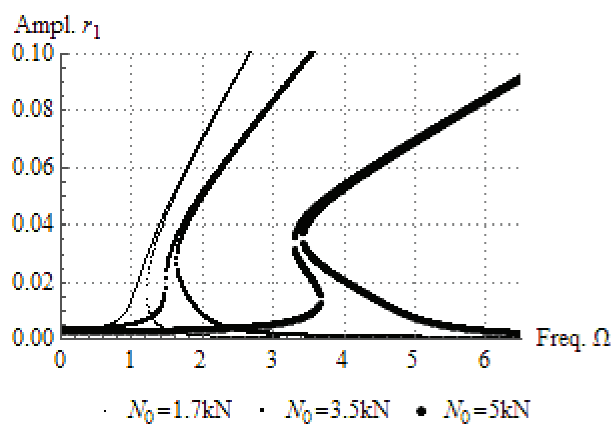

FIGURE 6. Impact of the axial compressive load on the resonance curves of the drill string nonlinear vibrations on the first harmonic at $l = 100\,\text{m}$, $D = 0.18\,\text{m}$, $\omega = 1.05$ rad/s.

Fig. 5 demonstrates that the resonance appears in the system on lower frequencies of the external effect when the angular speed of the drill string ($l = 100\,\text{m}$) increases significantly. At the same time the increase in the axial compressive load up to $5.5\,\text{kN}$ results in stretching the branches of the resonance curves out to the right with simultaneous considerable shift of the curves to the region of higher frequencies (Fig. 6). Similar results were obtained at high value of the initial curvature f_0 (Fig. 1, 2).

Consequently, in the bifurcation points of the amplitude-frequency characteristics of the drill string vibrations, presented on the constructed figures, one can determine instability zones of the resonance on basic and higher frequencies of the external effect.

CONCLUSION

As a result of the qualitative and quantitative analysis of the nonlinear model of the drill string motion it was established that emergence of the resonance on higher harmonics in the system has a considerable impact on stability of the oscillatory process. The rise in the amplitude-frequency characteristics of the resonance on the third harmonic in bifurcation zones of the amplitude-frequency characteristics of the basic resonance was observed. It is worth indicating that the significant increase in the drill string length and the angular speed of its rotation causes occurrence of considerable nonlinear effects of the drill string amplitude-frequency characteristics, which is typical generally for the dynamic systems with softening characteristics.

The results of this research show that geometrical nonlinearity of the models, describing the drill string dynamics, make a great contribution to the results of dynamic analysis of the drill string stability. By these reasons, modelling of resonance regimes of the drill string dynamics along with the analysis of its stability has a great importance for development of drilling equipment and improving its dynamic characteristics. In doing so, it is essential to take into account the geometrical nonlinearity of the system and the initial curvature of the drill string.

In future works the authors are going to investigate stability and resonance regimes of the drill string nonlinear dynamics under the nonlinear effects of a supersonic air flow, and to consider the nonlinear damping of the drill string vibrations.

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