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Continuous Variable Teleportation Protocol for Split-Squeezed Bose Einstein Condensates

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Abstract. In the present paper, a quantum teleportation protocol for split-squeezed Bose-Einstein Condensates is proposed that transfers a spin coherent state between two locations using entanglement introduced by squeezing interactions. One axis squeezing is used for this purpose, two axis anti squeezing further reduces the noise. The protocol uses continuous variable quadrature measurements that works for short interaction times in the Holstein-Primakoff approximation. The performance of the protocol is also analyzed by calculating the fidelity to show that it exceeds the classical bound.

INTRODUCTION

There are different physical systems for which entanglement has been realized experimentally such as superconductors [1] and photons [2] for example. Bose Einstein Condensates are currently being investigated to achieve entanglement between two BEC clouds. In Refs. [3, 4], theoretical analysis have been performed on the entanglement created by $S_1^z S_2^z$ type interaction between two distinct BECs, and their implementation in teleportation have been investigated [5, 6]. In Ref. [7, 8], another approach is proposed for creating entanglement between physically separated BECs, using squeezing interactions. In the scheme, squeezing in spin systems, as original proposed by Kitagawa and Ueda [9], is performed. Here first a maximally S^x-polarized spin coherent state is squeezed by one-axis twisting using a $(S^z)^2$ Hamiltonian. After spin sqeezing is performed, it is spatially split into two ensembles. In the present work, we study this split squeezed BEC for the purpose of achieving quantum teleportation between two BECs.

SPLIT SPIN-SQUEEZED BOSE-EINSTEIN CONDENSATES

We consider split-spin squeezed BEC as discussed in detail in Ref. [7], that considers the split squeezing operation to be equivalent to a combination of squeezing each of the split BECs individually then applying the $S_1^z S_2^z$ operation that creates the entanglement between the two BECs [3]. The effective Hamiltonian is

$$H = (S_1^z + S_2^z)^2 = (S_1^z)^2 + (S_2^z)^2 + 2S_1^z S_2^z \tag{1}$$

We use one-axis squeezing, as opposed to two-axis counter twisting that achieves maximum noise reduction, as this has been achieved experimentally.

Generally a two-component BEC is described by a spin-coherent state of the form,

$$|\alpha,\beta\rangle\rangle = \frac{(\alpha b^\dagger + \beta a^\dagger)^N}{\sqrt{N!}}|0\rangle \tag{2}$$

where $b^\dagger$ and $a^\dagger$ are bosonic creation operators of the two hyperfine spin states respectively, and α,β are arbitrary complex coefficients satisfying $|\alpha|^2+|\beta|^2 = 1$. The spin degree of freedom in Hilbert space is described by the Schwinger boson (total spin) operators defined as,

$$S^x = b^\dagger a + a^\dagger b \qquad S^y = -ib^\dagger a + ia^\dagger b \qquad S^z = b^\dagger b - a^\dagger a \tag{3}$$

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The commutation relation for spin operators is $[S^i, S^j] = 2i\varepsilon_{ijk}S^k$ where ε_{ijk} is the Levi-Civita symbol.

In our approach, the discrete collective spin operators are mapped to the continuous canonical position and momentum operators, in the same way as the quadratures of the light. This is the Holstein-Primakoff approximation that maps a part of the Bloch sphere around the polarized spin vector from which deviations approximated by the flat phase space of continuous variables. This approximation is only valid for states lying close to the initial spin polarized state, for large deviations from the initial polarized S^x ground state, the mapping becomes increasingly inaccurate. The approximation fails and the full spin degree of freedom for the system need to be taken into account.

In the Holstein-Primakoff approximation, one sets the macroscopically occupied mode to be a constant and it is treated as a classical variable while the remaining mode can be treated as quantum mechanically. The ground mode described by the bosonic operator b is populated, and it is taken as $b \approx \sqrt{N}$ where N is the particle number so that $b^\dagger b \approx N$. With this the spin operators transform as

$$S^x \approx N - a^\dagger a \approx N$$

$$S^y \approx x\sqrt{2N} \qquad x = \frac{a + a^\dagger}{\sqrt{2}} \qquad S^z \approx p\sqrt{2N} \qquad p = \frac{a - a^\dagger}{\sqrt{2}i} \tag{4}$$

Within the HP approximation, we can describe the system in terms of continuous variables that considers the degree of freedom analogous to the quadratures of the light.

In paper [10], the importance of implementing quantum information tasks using continuous variable(CV) operators and measures is considered. CV has been extensively used to model entanglement and teleportation for light field quadratures efficiently than the discrete variable approach used for qubits. CV teleportation protocol for light system was introduced in [10], the nonlocal resource shared by Alice and Bob is the EPR state with perfect correlations in both position and momentum as proposed by [11]. If (x_1, p_1), (x_2, p_2) are two entangled light modes, so ideal EPR state is the one for which $\mathrm{var}(x_1 - x_2) \to 0$ and $\mathrm{var}(p_1 + p_2) \to 0$.

EPR PAIR CORRELATIONS

The interaction Hamiltonian for one axis squeezing in (1) leads to a decrease in noise in one of the quadratures and this quadrature can be used to create EPR-like correlations that we expect to have suppressed noise fluctuations. Within the HP approximation, one of the spin operators can be treated as classical variable. Initially, the BEC is prepared in S^x polarised state, so $S_i^x \approx N_i$ where i labels the two BECs. The other two spin operators (S_i^y, S_i^z) form two quadratures (x_i, p_i) analogous to light as shown in (4). Working in the Heisenberg picture, the time evolution of operators under the HP approximation is

$$S_i^z = Constant = S_i^z(0) \qquad S_i^y(t) \approx S_i^y(0) + 4N_i t(S_i^z(0) + S_j^z(0)) \tag{5}$$

We transform the spin operators to minimize the variance of S_i^z and S_i^y for $i \in 1,2$ as,

$$S_i^{z'} = S_i^y \sin\theta + S_i^z \cos\theta \qquad S_i^{y'} = S_i^y \cos\theta - S_i^z \sin\theta \tag{6}$$

with the optimum squeezing angle is given by,

$$\tan 2\theta = \frac{4\sin 4t \cos^{N-2} 4t}{1 - \cos^{N-2} 8t}. \tag{7}$$

We calculate the variances using (6) and (7) as,

$$Var(S_1^{y'} - S_2^{y'}) = \langle (S_1^{y'} - S_2^{y'})^2 \rangle - (\langle S_1^{y'} - S_2^{y'} \rangle)^2 \qquad Var(S_1^{z'} + S_2^{z'}) = \langle (S_1^{z'} + S_2^{z'})^2 \rangle - (\langle S_1^{z'} + S_2^{z'} \rangle)^2 \tag{8}$$

With this, we plot variances of EPR-Pair like quantities in Fig. 1. For the S^z quadrature, we see that the fluctuation in noise is observed to decrease for a time of the order of $t = 1/2N$, however the other quadrature remains noisy. Since the entanglement states are detected in the region $0 < t < 1/2N$,we get one of the EPR pair$(S_1^{z'} + S_2^{z'}) \to 0$ to be used in CV teleportation protocol.

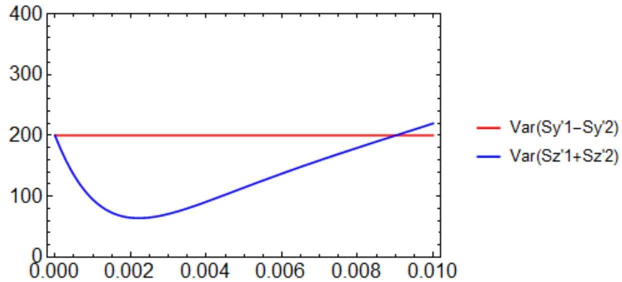

FIGURE 1. Variation of EPR-like variances $(S_1^{y'} - S_2^{y'}, S_1^{z'} + S_2^{z'})$ as a function of interaction time t for $N_L = N_R = 100$ from one-axis squeezing Hamiltonian interaction.

TELEPORTATION PROTOCOL

We now describe the basic idea of CV teleportation protocol for BEC system. Through the one-axis squeezing and splitting interactions, we get the correlations in BEC cloud, and the prepared entangled state has analogous correlations to an EPR state [7]. Initially, both Alice and Bob shares the entangled state. In order to perform the teleportation, Alice mixes the input state with her EPR state at a beam splitter. After that, there is a quadrature measurement performed by Alice and results are sent to Bob through the classical channel. Bob uses these classical results from Alice's measurement in order to perform the correct phase-space displacement on his mode so as to retrieve the original unknown input state. That completes the teleportation process.

In the Heisenberg picture, we summarise the whole protocol as: - Input mode: $S_{in}^{y'}, S_{in}^{z'}$

- EPR Pair: $(S_1^{y'}, S_1^{z'}), (S_2^{y'}, S_2^{z'})$
- Beam Splitter Output:

$$S_u^{y'} \rightarrow \frac{S_{in}^{y'} - S_1^{y'}}{\sqrt{2}} \qquad S_u^{z'} \rightarrow \frac{S_{in}^{z'} - S_1^{z'}}{\sqrt{2}}$$

$$S_v^{y'} \rightarrow \frac{S_{in}^{y'} + S_1^{y'}}{\sqrt{2}} \qquad S_v^{z'} \rightarrow \frac{S_{in}^{z'} + S_1^{z'}}{\sqrt{2}} \tag{9}$$

- Bob's Mode 2:

$$S_2^{y'}(t) = S_{in}^{y'}(t) - (S_1^{y'} - S_2^{y'}) - \sqrt{2}S_u^{z'}(t) \qquad S_2^{z'}(t) = S_{in}^{z'}(t) + (S_1^{z'} + S_2^{z'}) - \sqrt{2}S_v^{z'}(t) \tag{10}$$

As shown earlier, for split-squeezed BECs using one-axis squeezing, $S_1^{z'}(t) + S_2^{z'}(t) \rightarrow 0$, so Bob's mode 2 in S^z quadrature can be classically corrected and this approach transmits information for one of the input variable on Bloch sphere (θ, ϕ) achieving half-teleportation.

Teleported State using one-axis squeezing interaction (after classical correction):

$$S_{tel.}^{z'}(t) = S_{in}^{z'}(t) + (S_1^{z'}(t) + S_2^{z'}(t)) \rightarrow S_{in}^{z'}(t) \qquad S_{tel.}^{y'}(t) = S_{in}^{y'}(t) - (S_1^{y'}(t) - S_2^{y'}(t)) \tag{11}$$

Teleported State using two-axis anti-squeezing interaction (after classical correction):

$$S_{tel.}^{z'}(t) = S_{in}^{z'}(t) + (S_1^{z'}(t) + S_2^{z'}(t)) \rightarrow S_{in}^{z'}(t) \qquad S_{tel.}^{y'}(t) = S_{in}^{y'}(t) - (S_1^{y'}(t) - S_2^{y'}(t)) \rightarrow S_{in}^{y'}(t) \tag{12}$$

It is clear from (11) and (12) that full teleportation is possible with two axis squeezing interactions. Generalization towards this direction will be investigated in the future.

CONCLUSION

We have introduced a teleportation protocol for quantum transfer the information from one BEC to another via shared entanglement within the HP approximation. Here we have incorporated entanglement through squeezing interactions, and we have discussed two squeezing mechanisms: one axis squeezing and two axis anti-squeezing. We observed that the one axis squeezing interaction leads to noise suppression in one of the quadratures for time $t = 1/2N$, we get one ideal EPR pair correlation$(S_1^{z'} + S_2^{z'}) \to 0$. Hence the CV approach is used for transmitting information for one angle on the Bloch sphere, and the process can be described as "half-teleportation". Two axis squeezing, on the other hand, suppresses noise in both quadratures obtaining two EPR pairs and with this we can achieve full teleportation for angles θ and ϕ of Alice's original state as proposed from CV approach. However, practical implementation for two axis squeezing is more difficult as compared to one axis squeezing. The CV approach for BEC systems shows the potential for implementing future quantum information tasks with BEC as the storage medium.

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