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Entanglement-Based Quantum Clock Synchronization

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Abstract. Synchronizing clocks using quantum entanglement works on the principle that two clocks Alice and Bob share between them a singlet state which is a stationary state that is immune to evolution under bare atomic Hamiltonian. A major obstacle to its realization is the hidden assumption of a common phase reference between the clocks. Without hidden assumption, a clock state of Alice or Bob is not a uniquely defined quantum state because the phase of the state is arbitrary. This results in an unknown relative phase in a two-particle entangled state defined by the clocks. We show that using entanglement purification, an entanglement-based clock synchronization is achieved despite earlier results showing the contrary. This closes the loophole for entanglement based quantum clock synchronization protocols, which is a non-local approach to synchronize two clocks independent of the properties of the intervening medium.

INTRODUCTION

An outstanding issue with many entanglement-based clock synchronization protocols is that they implicitly assume a common phase reference [1, 2, 3]. The origin of this problem is that definitions of superposition states of qubits such as $(|0\rangle + |1\rangle)/\sqrt{2}$ are defined only up to a phase convention that is defined locally. Establishing a common phase reference is equivalent to already having synchronized clocks, defeating the purpose of the quantum synchronization protocol. Worse still, any quantum algorithm that Alice and Bob execute may require careful synchronization in order to not introduce additional phases due to precession of the qubits. This problem affects schemes quantum superposition and entanglement based schemes [1, 2, 3]. Proposals to overcome this issue for schemes using quantum superposition have been introduced, which require a two-way exchange of clock qubits [1]. This however involves sending clock qubit atoms (e.g., Cs, Rb, Sr) between the two parties, which is highly challenging for long-distance intercontinental or space-based communications. In view of photonic long-distance space-based entanglement distribution now being demonstrated [4] and the rapid development of quantum memories [5] a protocol compatible with this technology is an attractive prospect. For example, long-distance entanglement can be established using photons, then stored on qubits where the clocks are present, after which the protocol of Ref. [6] can be executed.

We show contrary to previous arguments [3], that it is possible to produce an entangled state with controlled phase without Alice and Bob having any knowledge of each other's phase definitions. The main observation is that the full distillation protocol as originally given by Bennett and coworkers [7, 8] ensures that a controlled entangled state can be produced. The important ingredient in the distillation protocol is the random bilateral rotations, which leaves the singlet state invariant. The ability of the random bilateral rotations to isolate the singlet state—and only the singlet state—allows for a controlled entangled state to be produced in the local basis. This overcomes the necessity of a common phase reference, which is the major criticism made in Ref. [3]. Once this is prepared, it is possible to execute the original quantum clock synchronization protocol of Ref. [6] despite the presence of additional phases, differing basis conventions, and noise. We assume that Alice and Bob do have clocks ticking at the correct frequency, such that they can keep track of the precession for the duration of the algorithm, but the clocks have in general a relative time offset (the clocks are syntonized but not synchronized) [9].

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RELATIVE PHASE IN CLOCKS

Suppose Charlie prepares the singlet state

$$|\psi^-\rangle^{(C)} = \frac{|1\rangle_A^{(C)}|0\rangle_B^{(C)} - |0\rangle_A^{(C)}|1\rangle_B^{(C)}}{\sqrt{2}} \tag{1}$$

and sends it to Alice and Bob. Here the definitions of the states are with respect to Charlie's basis convention, which may be different to Alice's and Bob's. Thus the state $|0\rangle_A^{(C)}$ means a qubit state in Alice's possession, in the basis convention of Charlie, and so on. We assume that Alice, Bob, and Charlie all have different basis conventions, which we can relate according to $|\sigma\rangle^{(A)} = e^{-i\theta_\sigma^{(A)}}|\sigma\rangle^{(C)}$, $|\sigma\rangle^{(B)} = e^{-i\theta_\sigma^{(B)}}|\sigma\rangle^{(C)}$, where $\sigma \in \{0,1\}$. If the bases are transformed consistently using the same convention globally, then the state (1) is invariant, for example

$$|\psi^-\rangle^{(B)} = \frac{|1\rangle_A^{(B)}|0\rangle_B^{(B)} - |0\rangle_A^{(B)}|1\rangle_B^{(B)}}{\sqrt{2}}, \tag{2}$$

where we chose the irrelevant global phase $\theta_0^{(B)} + \theta_1^{(B)} = 0$ for simplicity. However, as pointed out by Ref. [3], without the availability of synchronized clocks, it is not possible for Alice and Bob to know about their mutual basis conventions. Thus the appropriate basis to view the state is in Alice and Bob's respective local bases

$$|\psi^-\rangle^{(\mathrm{loc})} = \frac{|1\rangle_A^{(A)}|0\rangle_B^{(B)} - e^{i(\theta_0^{(A)}+\theta_1^{(B)}-\theta_1^{(A)}-\theta_0^{(B)})}|0\rangle_A^{(A)}|1\rangle_B^{(B)}}{\sqrt{2}}. \tag{3}$$

We emphasize that $|\psi^-\rangle^{(\mathrm{loc})} = |\psi^-\rangle^{(B)} = |\psi^-\rangle^{(C)}$ are all in fact the same state, but they appear different due to different phase conventions. The effect of Alice and Bob choosing different phase conventions is equivalent to having an unknown relative phase in the singlet [1, 3]. We may define the relative difference between the basis choices of Alice and Bob by defining a rotation operator $U^{(AB)}|\sigma\rangle^{(B)} = |\sigma\rangle^{(A)}$, $U^{(BA)}|\sigma\rangle^{(A)} = |\sigma\rangle^{(B)}$ which in this case is $U^{(AB)} = U^{(BA)\dagger} = e^{i\sum_\sigma(\theta_\sigma^{(B)}-\theta_\sigma^{(A)})|\sigma\rangle\langle\sigma|}$. Operators then transform as $O^{(A)} = U^{(AB)}O^{(B)}U^{(AB)\dagger}$ and similarly for Bob's operators.

In addition to the different phase conventions, when Alice and Bob perform their entanglement purification circuit, they will not know precisely when the other starts their first quantum operation. Due to the precession of the qubits, there will be an additional phase offset in the singlet state, which without loss of generality we can attribute to Alice's side. Hence the arriving singlet will have a form in the local basis (up to a global phase)

$$|\psi_\varphi^-\rangle^{(\mathrm{loc})} = T|\psi^-\rangle^{(\mathrm{loc})} = \frac{1}{\sqrt{2}}(|1\rangle_A^{(A)}|0\rangle_B^{(B)} - e^{i\varphi}|0\rangle_A^{(A)}|1\rangle_B^{(B)}), \tag{4}$$

where the time delay operator is $T = e^{-i\omega\delta t|1\rangle\langle 1|}$, $\varphi = \theta_0^{(A)} + \theta_1^{(B)} - \theta_1^{(A)} - \theta_0^{(B)} - \omega\delta t$, and δt is the time difference between Alice and Bob's first quantum operation.

Furthermore, in addition to the systematic error introduced by the different phase conventions and time offset, there may be a stochastic error which reduces the purity of the state. We model this process using the noisy channel with both bit and phase flips, which for our state will appear as

$$|\psi_\varphi^-\rangle^{(\mathrm{loc})}\langle\psi_\varphi^-|^{(\mathrm{loc})} \to \rho_\varphi^{(\mathrm{loc})} = \frac{p}{4}I + (1-p)|\psi_\varphi^-\rangle^{(\mathrm{loc})}\langle\psi_\varphi^-|^{(\mathrm{loc})}, \tag{5}$$

where p is the probability that error will be introduced on sending the qubit through a noisy channel to Alice and Bob, and I is the 4×4 identity matrix. We assume that N imperfect Bell pairs (5) are shared between Alice and Bob, and φ is unknown to both of them. The task is then to achieve clock synchronization by first purifying the above state to a sufficiently high fidelity, then executing the entanglement-based synchronization protocol without knowledge of any shared timing information.

STATE PURIFICATION

The effect of random bilateral rotations [7] is to put any state in the form of a Werner state as defined by the local Bell basis. As discussed in Ref. [7], instead of applying an infinite set of random bilateral unitaries, it is equivalent to

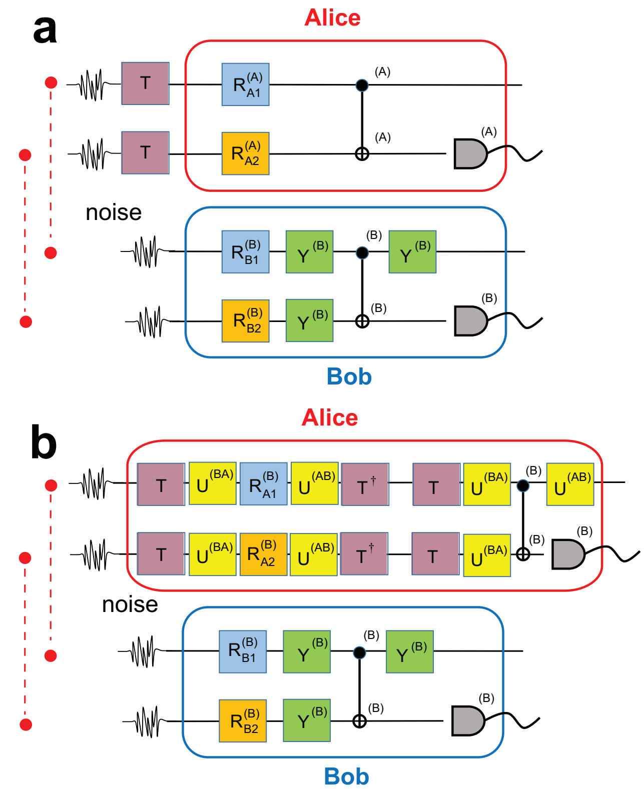

FIGURE 1. The quantum circuit for entanglement purification. (a) The circuit as performed by Alice and Bob; (b) an equivalent circuit where all circuit elements have been transformed to the same basis choice. $\mathscr{B} = R_A \otimes R_B$ are random bilateral rotations which are predecided by Alice and Bob, U^{AB} transforms from Bob's basis convention to Alice's, T includes the effect of a time delay between the start of Alice and Bob's operations.

consider a finite set generated by $\mathscr{G}_M = \sqrt{M_A^{(A)}} \otimes \sqrt{M_B^{(B)}}$ with $M \in \{X,Y,Z,I\}$. The first term in (5) in proportional to the identity, which is invariant under bilateral operations. Since the identity is diagonal under any basis choice, we can choose equally the local basis

$$I = |\psi^-_{\varphi=0}\rangle^{(\text{loc})}\langle\psi^-_{\varphi=0}|^{(\text{loc})} + |\psi^+_{\varphi=0}\rangle^{(\text{loc})}\langle\psi^+_{\varphi=0}|^{(\text{loc})} + |\phi^-_{\varphi=0}\rangle^{(\text{loc})}\langle\phi^-_{\varphi=0}\rangle^{(\text{loc})}|\phi^+_{\varphi=0}\rangle^{(\text{loc})}\langle\phi^+_{\varphi=0}|^{(\text{loc})}. \tag{6}$$

For the second term in (7), we obtain by explicit computation

$$\sum_n \mathscr{B}_n|\psi^-_\varphi\rangle^{(\text{loc})}\langle\psi^-_\varphi|^{(\text{loc})}\mathscr{B}_n^\dagger = \frac{I - |\psi^-_{\varphi=0}\rangle^{(\text{loc})}\langle\psi^-_{\varphi=0}|^{(\text{loc})}}{3}\sin^2(\frac{\varphi}{2}) + |\psi^-_{\varphi=0}\rangle^{(\text{loc})}\langle\psi^-_{\varphi=0}|^{(\text{loc})}\cos^2(\frac{\varphi}{2}), \tag{7}$$

where the sum is over the full group

$$\mathscr{B}_n \in \{\mathscr{G}_I, \mathscr{G}_X\mathscr{G}_Y, \mathscr{G}_Y\mathscr{G}_Z, \mathscr{G}_Z\mathscr{G}_X, \mathscr{G}_X\mathscr{G}_Y\mathscr{G}_X\mathscr{G}_Y, \mathscr{G}_Y\mathscr{G}_Z\mathscr{G}_Y\mathscr{G}_Z, \mathscr{G}_Z\mathscr{G}_X\mathscr{G}_Z\mathscr{G}_X, \mathscr{G}_X\mathscr{G}_Z, \mathscr{G}_X\mathscr{G}_Z\mathscr{G}_X\mathscr{G}_Z, \mathscr{G}_X\mathscr{G}_X, \mathscr{G}_Y\mathscr{G}_Y, \mathscr{G}_Z\mathscr{G}_Z\}, \tag{8}$$

and is averaged over the number of group elements used in the rotation. We compute the Werner state to be

$$\rho'_W = I\frac{p}{4} + (1-p)\left|\psi^-_{\varphi=0}\right\rangle^{(\text{loc})}\left\langle\psi^-_{\varphi=0}\right|^{(\text{loc})}\cos^2\left(\frac{\varphi}{2}\right) + \frac{1-p}{3}\left[I - \left|\psi^-_{\varphi=0}\right\rangle^{(\text{loc})}\left\langle\psi^-_{\varphi=0}\right|^{(\text{loc})}\right]\sin^2\left(\frac{\varphi}{2}\right). \tag{9}$$

The fidelity calculated using (9) agrees with that directly calculated from (7) which gives $F = \langle\psi^-|\rho_\varphi|\psi^-\rangle = \frac{p}{4} + (1-p)\cos^2\left(\frac{\varphi}{2}\right)$, since singlet states are invariant under bilateral rotations. A circuit that accomplishes this task is shown in Fig. 1 and is described in Ref. [10]. Eq. (9) shows that after the bilateral rotations, the state is correctly prepared in the local basis. Starting from a state (7) which had off-diagonal terms written in the local basis, the bilateral rotations produced a state (9) that is diagonal. Also, we see that the fidelity, F, contains an extra phase factor originating from the combination of the time delay and the different basis conventions. This is natural since the phase φ will produce a state that is different from a singlet state, which will result in a loss of fidelity. Since the purification protocol only works unless $F > 0.5$, in practice this will mean that the phase will need to be controlled to some extent. This can be achieved by having reasonably (but not exactly) synchronized clocks, so that the protocol can be executed accurately such that $|\varphi| < \pi/2$.

SUMMARY AND CONCLUSIONS

In summary, with entanglement distillation it is possible for Alice and Bob to share a singlet state in their local basis, despite not having any information about their mutual basis conventions, and including any time offset between execution of their quantum gates. The key ingredient is the incorporation of bilateral random unitaries in the entanglement purification protocol, which was not included in Ref. [3, 6]. This deterministically produces a Werner state for a singlet state in the local basis, thereby overcoming the ambiguity due to different phase definitions. This solves a major existing issue in the entanglement-based clock synchronization protocol, where it was previously thought that a common phase reference (which implies synchronized clocks) are required to perform the purification. We envision that the entanglement-based clock synchronization would be particularly useful in the context of the space-based quantum network [4, 11], where satellites are each in possession of an high-precision clock. Such entanglement based schemes are a powerful way to synchronize clocks without the use of a classical channel containing the timing information, which is susceptible to the properties of the intervening medium.

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