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Estimation of the Multidimensional Transient Functions Oculo-Motor System of Human

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Abstract. Proposed a new method of constructing nonparametric dynamic models of the oculomotor system system (OMS) in the form of human multidimensional transition functions on the basis of experimental data "input-output". As the test signals used bright points on the long duration of the computer screen. OMS response is measured using information technology Eye-tracking and recorded on video. As a result data processing of the experiment we receive function based "pupil coordinate – time". Using the method of least squares (Ordinary Least Squares, OLS) defined transition functions of the first, second and third order - integral transformations of Volterra kernels , representing a model of OMS. Completed experimental studies using computer simulations confirm the adequacy of the constructed approximation model as a real system

STATEMENT OF THE PROBLEM

The *i*nnovative technology of Eye tracking which is rapidly developing nowadays - is the process of determination of the point where look being sent to or the determination of eye movements relatively to the head [1-3]. This high-tech innovation has been further developed and effectively used in the construction of a mathematical model of process of tracking eye movement to detect anomalies in data tracking to quantify the motor symptoms of Parkinson's disease [4,5]. Using nonlinear dynamic model of Wiener and Volterra-Laguerre [6] and their identification is based on a random effects test [7], which requires the application of methods of correlation analysis and generate a large amount of experimental data (long-term experimental studies).

In order to build a model of Volterra [8] oculo-motor system (OMS) a person is encouraged to use the test deterministic effects, for example, step signals (the most appropriate for the study of the dynamics of OMS) [9], which simplifies the computational algorithm to identify and significantly reduce the time of processing of experimental data. There is a method and computer algorithms identifying deterministic nonlinear dynamical systems in the form of Volterra models using multi-test signals [10].

The purpose and research problems

The purpose of work is development method for constructing nonparametric dynamic model of oculo-motor system, taking into account its inertial and nonlinear properties, based on experimental studies of "input-output" and also computational tools and software for the information technology processing experimental data.

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To achieve this goal were set this following tasks:

- development methods for constructing nonlinear dynamic model of OMS as a Volterra kernels which characterizing both nonlinear and inertial properties of the nature objects;
- development information technology of obtaining experimental data for identification OMS based on pupil's movement tracking using video registration;
- development computational methods of identification multidimensional dynamic (transient) characteristics OMS using test inputs as a Heaviside functions of different amplitudes;
- verification constructed model OMS.

THE VOLTERRA MODEL AND IDENTIFICATION OMS

Basis for creation of mathematical (informational) model of investigated object are the results of measurements of its input and output variables, and the solution of the problem associated with the identification of the experimental data and process them with the noise measurements.

To describe the objects of unknown structure appropriate to use the most universal nonlinear nonparametric dynamic models - Volterra model [8]. The nonlinear and dynamic properties investigated object is uniquely described by a sequence of invariant with respect to the type of input signal is of multidimensional weight functions - Volterra kernels.

For continuous nonlinear dynamical system connection between the input $x(t)$ and output $y(t)$ signals with zero initial conditions can be represented by a series of Volterra

$$\begin{aligned} y(t) &= \sum_{n=1}^{\infty} y_n(t) = \int_0^t w_1(\tau)x(t-\tau)d\tau + \int_0^t\int_0^t w_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1\, d\tau_2 + \\ &+ \int_0^t\int_0^t\int_0^t w_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1\, d\tau_2\, d\tau_3 + \ldots, \end{aligned} \qquad (1)$$

where $w_n(\tau_1,\ldots,\tau_n)$ – Volterra kernel n-th order, function is symmetric with respect to real variables $\tau_1,\ldots,\tau_n$; $y_n(t)$ – the n-th partial component of response system (n-dimensional convolution integral); t – current time.

For nonlinear dynamical system multiple-input and multiple-output used multivariate Volterra series, which has the form:

$$\begin{aligned} y_j(t) &= \sum_{i_1=1}^{\nu}\int_0^t w_{i_1}^j(\tau)x_{i_1}(t-\tau)d\tau + \sum_{i_1=1}^{\nu}\sum_{i_2=1}^{\nu}\int_0^t\int_0^t w_{i_1 i_2}^j(\tau_1,\tau_2)x_{i_1}(t-\tau_1)x_{i_2}(t-\tau_2)d\tau_1\, d\tau_2 + \\ &+ \sum_{i_1=1}^{\nu}\sum_{i_2=1}^{\nu}\sum_{i_3=1}^{\nu}\int_0^t\int_0^t\int_0^t w_{i_1 i_2 i_3}^j(\tau_1,\tau_2,\tau_3)x_{i_1}(t-\tau_1)x_{i_2}(t-\tau_2)x_{i_3}(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \ldots, \end{aligned} \qquad (2)$$

where $w_{i_1 i_2 \ldots i_n}^j(\tau_1,\ldots,\tau_n)$ – Volterra kernel n-th order in $i_1, i_2, \ldots, i_n$ inputs and j-th output ($j=1,2,\ldots,\mu$), the functions symmetric with respect to real variables $\tau_1,\ldots,\tau_n$; $y_j(t)$ – system response for the j-th output at the current time t for zero initial conditions; $x_1(t),\ldots,x_\nu(t)$ – input signals; ν, μ – quantity of inputs and outputs, respectively.

In the context the problem stated above - identification OMS - need to use the model (2) for the mathematical description of the object [8]: two pair rectus muscles (input object) provide eye movement up and down, left and right, and various combinations "**FIGURE 1**"; measured responses - the coordinates $u(t)$ and $v(t)$ current position the pupil relative to the initial position u_0 and v_0 (the outputs of the object). In this case in model (2) adopting $\nu=2$ and $\mu=2$.

In this paper to simplify the experiment and data identification, problem solved for the case horizontal pupil's movement ($\nu=1$ and $\mu=1$), i.e. based on the model (1).

Problem identification (model constructing) as (1) or (2) consist to determine the Volterra kernels based on experimental data "input-output" OMS. Construction of the model is the selection of test actions $x(t)$ and development of algorithm, which enables for the measured response $y(t)$ allocate partial components $y_n(t)$ and determined on the basis of their Volterra kernels $w_n(\tau_1,\ldots,\tau_n)$, $n=1,2,\ldots$ [9].

Here we provide some basic advice for formatting your mathematics, but we do not attempt to define detailed styles or specifications for mathematical typesetting. You should use the standard styles, symbols, and conventions for the field/discipline you are writing about.

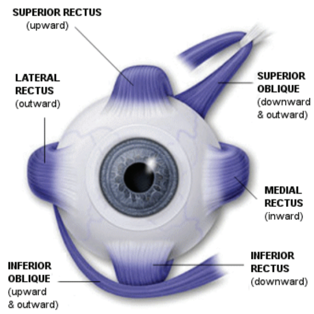

FIGURE 1. Direct eye muscles

Computing Method of Multidimensional Transient Functions for Identification OMS

Taking into account specificity investigated object to identification used test multistage signals. If test signal $x(t)$ represents an identity function (Heaviside function) – $\theta(t)$, the result of identification the transition function of the first order $\widehat{h}_1(t)$ and the diagonal section n-th order $\widehat{h}_n(t,...,t)$.

To determine the sections subdiagonal transition functions n–th order ($n\geq2$) OMS tested using the n step test signal with given amplitude and different intervals between signals. With appropriate processing responses get subdiagonal section n-dimensional transition functions $h_n(t-\tau_1,\ldots, t-\tau_n)$, which represent n-dimensional integral of Volterra kernel n-order $w_n(\tau_1,\ldots, \tau_n)$:

$$h_n(t-\tau_1,...,t-\tau_n)=\int_0^\infty\ldots\int_0^\infty w_n(t-\tau_1-\lambda_1,...,t-\tau_n-\lambda_n)d\lambda_1\ldots d\lambda_n. \tag{3}$$

Method for determination sections of n-dimensional transition functions based on the statement, proof of which is similar to that given in [10].

The Method of Constructing Approximate Model of Volterra Nonlinear Dynamical System

Is developing a method of constructing approximate Volterra model [11] identification method based on the Volterra series time-domain response is based on the approximation $y(t)$ at an arbitrary deterministic signal $x(t)$ in the form of integral power of the polynomial N-th order (N - order approximation model)

$$\widetilde{y}_N(t)=\sum_{n=1}^{N}\hat{y}_n(t)=\sum_{n=1}^{N}\underbrace{\int_0^t\ldots\int_0^t}_{n\ times} w_n(\tau_1,...,\tau_n)\prod_{i=1}^{n}x(t-\tau_i)d\tau_i. \tag{4}$$

Let the input test signals OMS are fed alternately $a_1x(t)$, $a_2x(t)$, ,…, $a_Lx(t)$; a_1, a_2,…,a_L – distinct real numbers satisfying the condition $|a_j|\leq1$ for $\forall j=1,2,...,L$, then

$$\widetilde{y}_N[a_jx(t)]=\sum_{n=1}^{N}\hat{y}_n[a_jx(t)]=\sum_{n=1}^{N}a_j^n\underbrace{\int_0^t\ldots\int_0^t}_{n\ times} w_n(\tau_1,...,\tau_n)\prod_{i=1}^{n}x(t-\tau_i)d\tau_i=\sum_{n=1}^{N}a_j^n\hat{y}_n(t) \tag{5}$$

The partial components in the approximation model $\hat{y}_n(t)$ are found using the least square method (LSM) This makes it possible to obtain such evaluation in which the sum of squared deviations of responses identified the NDS $y[a_jx(t)]$ on the model $\hat{y}_N[a_jx(t)]$ response is minimal, i.e., OMS provides a minimum criterion

$$J_N=\sum_{j=1}^{L}\left(y[a_jx(t)]-\widetilde{y}_N[a_jx(t)]\right)^2=\sum_{j=1}^{L}\left(y_j(t)-\sum_{n=1}^{N}a_j^n\hat{y}_n(t)\right)^2\to\min, \tag{6}$$

where $y_j(t)=y[a_jx(t)]$. Minimization of the criterion (6) is reduced to solving the system of normal equations of Gauss, which in vector-matrix form can be written as

$$\mathbf{A}'\mathbf{A}\hat{\mathbf{y}}=\mathbf{A}'\,\mathbf{y}, \tag{7}$$

where

$$\mathbf{A}=\begin{bmatrix} a_1 & a_1^2 & \cdots & a_1^N \\ a_2 & a_2^2 & \cdots & a_2^N \\ \cdots & \cdots & \cdots & \cdots \\ a_L & a_L^2 & \cdots & a_L^N \end{bmatrix}, \quad \mathbf{y}=\begin{bmatrix} y_1(t) \\ y_2(t) \\ \cdots \\ y_L(t) \end{bmatrix}, \quad \hat{\mathbf{y}}=\begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \cdots \\ \hat{y}_N(t) \end{bmatrix}.$$

From (7) we obtain

$$\hat{\mathbf{y}}=(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\mathbf{y}. \qquad (8)$$

In (8), matrix operations, we obtain

$$\begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \cdots \\ \hat{y}_N(t) \end{bmatrix}=\begin{bmatrix} \sum_{j=1}^{L} a_j^2 & \sum_{j=1}^{L} a_j^3 & \cdots & \sum_{j=1}^{L} a_j^{N+1} \\ \sum_{j=1}^{L} a_j^3 & \sum_{j=1}^{L} a_j^4 & \cdots & \sum_{j=1}^{L} a_j^{N+2} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{j=1}^{L} a_j^{N+1} & \sum_{j=1}^{L} a_j^{N+2} & \cdots & \sum_{j=1}^{L} a_j^{2N} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sum_{j=1}^{L} a_j y_j(t) \\ \sum_{j=1}^{L} a_j^2 y_j(t) \\ \cdots \\ \sum_{j=1}^{L} a_j^N y_j(t) \end{bmatrix}. \qquad (9)$$

THE RESULTS OF THE RESEARCH

For successful implementation of the technology of experimental determination of the dynamic characteristics of OMS and human diagnostics in technical and medical applications, it's necessary to have data sets - the coordinates of the position of the pupil on plane, namely, the values of the horizontal and vertical eye rotation angles with respect to the initial position.

Tracking the angles of pupil rotation with the help of video recording involves the use of video cameras to produce images of the pupil over time at regular intervals, which would be clearly fixed the position of the pupil when it moves. We need a high-speed camera that would record not only visible to the human eye movement OMS (saccades), and more informative studies for micro-movements - tremors (small oscillations of the eye), drift (slow flowing micro-movements) and mikrosccades (rapid eye movement lasting 10-20 ms).

For OMS studies the selected camera shooting speed is 120 (frames/sec). For an array of pupil coordinate values when moving eyes based on the footage, it is necessary to divide the video into individual frames. One study time is 408,17 ms (50 frames). Processing video recording data associated with the release of each frame of video on the center of the pupil of the eye "**FIGURE 2.**"

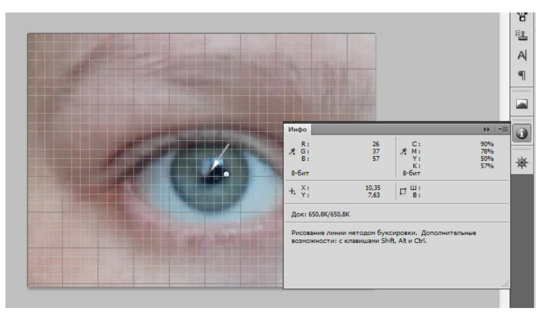

FIGURE 2. Determination of the pupil center

Result image analysis - the pixel coordinates of the pupil center on the frame of video x_i, y_i, i = 1,2, ..., m (m - the number of video frames, m=50). According to the coordinates of the pupil center and the results of the calibration is calculated direction of gaze, tied to the subject observer of the image. The accuracy assessment of coordinates in pixels ±2 pel.

Using tools developed by data processing means based nonparametric dynamic model of the human form of OMS in transition and transition of two-dimensional functions (3).

Results of Identification OMS Transient Function

Approbation tracking technology of the pupil's behavior based on video registration is performed on the task of analysis of work the oculomotor apparatus along the horizontal axis.

Where in the input (test) signal - distance from the base of the perpendicular, dropped from the center of pupil eye to the plane, in which is formed the perturbation – the light source to the point source (light spots) in the horizontal plane. Measure the response (the output) is a function of the current deviation of pupil in the frame image of the OMS from the starting point, depending on the time.

In mm: x=31,9, y=27,4
In pixels: x=90, y=78

In mm: x=33,9, y=27,4
In pixels: x=94, y=77

In mm: x=36,0, y=27,5
In pixels: x=99, y=77

In mm: x=37,9, y=26,9
In pixels: x=37,9, y=26,9

FIGURE 3. Result eye image analysis

To determine the diagonal section of the transient response second order object is tested at first step signal with an amplitude of the a (horizontal distance to light spot from the starting point, represents the original position the pupil)

Measured response of the eye $y_1(t)$, $y_2(t)$, $y_3(t)$ to the input test signals $a_1\theta(t)$, $a_2\theta(t)$ and $a_3\theta(t)$ (L=3) for values of the test signal amplitudes a_1=0,33 a_2=0,66 and a_3=1, shown in "**FIGURE 4.**"

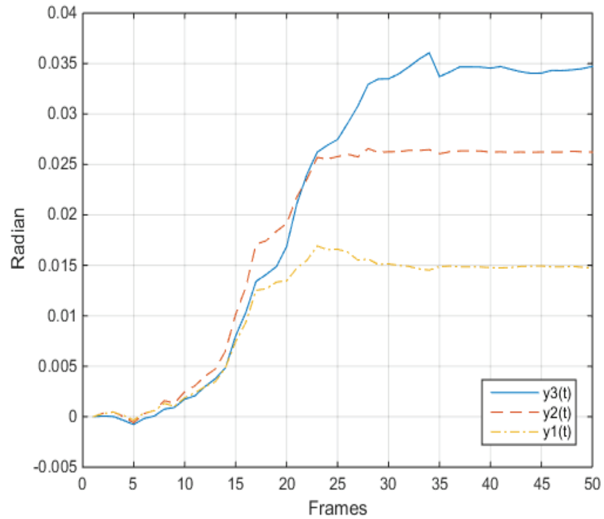

FIGURE 4. Response of the eye $y_1(t)$, $y_2(t)$ and $y_3(t)$.

Obtained graphs of OMS transient functions first $\hat{h}_1(t)$, second $\hat{h}_2(t,t)$ and third order $\hat{h}_3(t,t,t)$ shown in "**FIGURE 5,**" respectively.

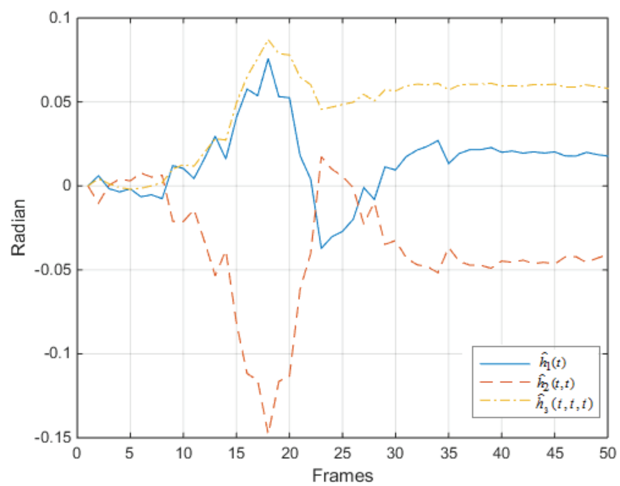

FIGURE 5. Transient functions $\hat{h}_1(t)$, $\hat{h}_2(t,t)$ and $\hat{h}_3(t,t,t)$

The model response is calculated on the basis of estimates of the transient functions $\hat{h}_1(t)$, $\hat{h}_2(t,t)$ and $\hat{h}_3(t,t,t)$

$$\tilde{y}(t,a) = a\hat{h}_1(t) + a^2\hat{h}_2(t,t) + +a^3\hat{h}_3(t,t,t). \qquad (10)$$

Comparison of the response of the constructed model $\tilde{y}(t,a)$ with the response of the identified OMS (with experimental data) $y(t,a)$ shown in "**FIGURE 6.**"

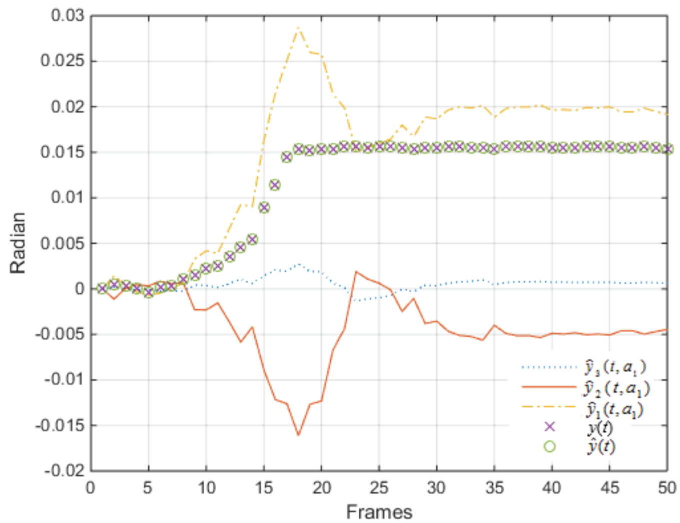

FIGURE 6. Comparison of responses the OMS of identify $y(t)$, model $\hat{y}(t)$ and partial component of response OMS first $\hat{y}_1(t,a_1)$, second $\hat{y}_2(t,a_1)$ and third order $\hat{y}_3(t,a_1)$ at an amplitude a_1=0.33

Provided graphs are practically the same (standard deviation σ=0.0009) which confirms effectiveness computational algorithm of identification and adequacy of the constructed model based on experimental data "input-output".

CONCLUSIONS

Proposed a new method and information technology of construction nonparametric dynamic models of human oculomotor system (OMS) given its nonlinear and inertial properties on the basis of experimental data "input-output". This uses a mathematical model in the form of integral-power polynomial Volterra (multidimensional transition r functions). Has been the further development of information technology «Eye tracking» and developed software tools identify OMS. Basis on these experimental studies OMS for different amplitudes of input signals (distance eye point perturbations on the initial position on the screen). Using the method of least squares consruct nonparametric dynamic model of the human OMS in the form of transition and diagonal sections of the two-dimensional and three-dimensional transition functions. This mean square error (MSE) of identification is at σ=0.0009.

These results identify OMS provide opportunities early diagnosis of neurodegeneration in Parkinson's disease and may be used in diagnostic studies in determining disease stages; as well as for hardware vision correction, in man-machine systems at professional selection of operators for visual control feature fast process, for training in the sport.

REFERENCES

1. J. Kepler, and U. Linz, Biomechanical Modelling of the Human Eye. Netz Werk für Forschung, Lehre und Praxis, Linz, 231 (2004).
2. E. D. Guestrin and M. Eizenman, General Theory of Remote Gaze Estimation Using the Pupil Center and Corneal Reflections, IEEE Trans. Biomed. Eng., 53 (6), 1124-1133. DOI:10.1109/tbme.2005.863952 (2006).
3. O. V.,Komogortsev and A. Karpov, Automated Classification and Scoring of Smooth Pursuit Eye Movements in Presence of Fixations and Saccades. Journal of Behavioral Research Methods, 45 (1), 1–13 (2013).
4. D. Jansson and A. Medvedev, Volterra Modeling of the Smooth Pursuit System with Application to Motor Symptoms Characterization in Parkinson's Disease. European Control Conference (ECC), 1856-1861. DOI: 10.1109/ecc.2014.6862207 (2014).

5. D. Jansson, Stochastic Anomaly Detection in Eye-Tracking Data for Quantification of Motor Symptoms in Parkinson's Disease. Advances in Experimental Medicine and Biology,.823, 63-82. DOI:10.1007/978-3-319-10984-8_4 (2015).
6. D. Jansson and A. Medvedev, System identification of Wiener systems via Volterra-Laguerre models: Application to human smooth pursuit analysis. European Control Conference (ECC), 2700-2705. DOI: 10.1109/ECC.2015.7330946 (2015).
7. P. Z. Marmarelis and V. Z. Marmarelis, Analysis of Physiological Systems, The White Noise Approach, Plenum Press, New York, 487 (1978).
8. F. J. Doyle, R. K. Pearson and B. A Ogunnaike, Identification and Control using Volterra Models. Published Springer Technology & Industrial Arts, 314. (2001).
9. A. Fomin, M. Masri, V. Pavlenko, A. Fedorova, Method and information technology for constructing a nonparametric dynamic model of the oculomotor system. EasternEuropean Journal of Enterprise Technologies, 2/9 (74), 64-69. DOI: 10.15587/1729-4061.2015.41448 (2015)
10. V. D. Pavlenko, O. O. Fomin, A. N. Fedorova, and M. M. Dombrovskyi, Identification of Human Eye-Motor System Base on Volterra Model. Herald of the National Technical University «KhPI». Subject issue: Information Science and Modelling, Kharkov, NTU «KhPI», 21 (1193), 74-85 (2016).
11. D. Graupe, Identification of Systems, 2nd ed., R. E. Krieger Publ. Co., New York (1976).