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Dust Ion-acoustic Shock Waves in Magnetized Pair-ion Plasma with Kappa Distributed Electrons

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Abstract. We have performed a theoretical and numerical analysis of the three dimensional dynamics of nonlinear dust ion-acoustic shock waves (DIASWs) in a magnetized plasma, consisting of positive and negative ion fluids, kappa distributed electrons, immobile dust particulates along with positive and negative ion kinematic viscosity. By employing the reductive perturbation technique, we have derived the nonlinear Zakharov–Kuznetsov–Burgers (ZKB) equation, in which the nonlinear forces are balanced by dissipative forces (associated with kinematic viscosity). It is observed that the characteristics of DIASWs are significantly affected by superthermality of electrons, magnetic field strength, direction cosines, dust concentration, positive to negative ions mass ratio and viscosity of positive and negative ions.

INTRODUCTION

Ion acoustic waves in unmagnetized dusty plasma were first studied theoretically by Shukla and Silin [1] and observed experimentally by Barken et al. [2]. Dust is an omnipresent component of space and astrophysical plasmas [3]. Presence of dust in electron–ion plasma generates different kinds of modes. Two of these modes, which are frequently studied are, high frequency dust-ion-acoustic waves (DIAWs) having mobile ions and static dust particulates and other is low frequency dust acoustic waves (DAWs) having mobile dust grains. These two modes were studied theoretically and experimentally by many researchers [1, 4, 5].

Presence of negative ions along with electrons and positive ions modify the nonlinear wave structures in plasmas [6]. In such plasmas, negative ions are produced due to the attachment of electrons with neutral atoms. This type of electronegative plasmas are found in astrophysical environments and can also be produced in laboratory. The nonlinear structures in electronegative plasma in the presence or absence of magnetic field are studied by many researchers [7–9]. Usually, nonlinear structures in plasmas are studied by considering Boltzmann distributed electrons due to its lighter mass as compared to heavy ions. Many observations confirmed that kappa dustribution is more appropriate for modelling electrons in astrophysical plasma environments as compared to Maxwell–Boltzmann distribution. Due to high thermal speed of electrons as compared to ions, the probability of electrons to become superthermal is more. Moreover, kappa distribution is more suitable model for observed velocity distribution in space plasmas [10].

Shocks are generated in the plasma when dissipation is more dominant than dispersion in the presence of nonlinearity [11]. The investigation of the effect of negatively charged dust on ion-acoustic shock waves in Q-machine was reported by Luo et al. [12] and Anderson et al. [13]. During last two decades, many researchers showed their interest in studying the solitons and shocks [10, 14]. Three dimensional ion acoustic solitary and shock waves in e-p-i plasma with high energy superthermal electrons and positrons were investigated by El-Bedwehy and Moslem [15]. They derived ZK-Burgers equation and examined the dependence of ion acoustic solitary and shock waves on different plasma parameters. Recently, Tantawy [16] examined the features of dust ion-acoustic shock waves in magnetized plasma, containing nonextensive electrons, positive ions and dust particles. They derived the ZKB equation and determined its solution to see the effect of different plasma parameters on dust-ion-acoustic solitary waves. Both positive and negative polarity shocks were formed. Wang and Zhang [17] observed the characteristics of DIASWs in a dusty plasma consisting of Maxwellian electrons, negative and positive ions along with charged dust grains having dust

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charge fluctuations. It was observed that the dust charge fluctuation leads to the dissipation, which is responsible for production of shock waves in plasma. In the present study, our aim is to study the effect of superthermality of electrons, dust concentration and ratio of positive to negative ion concentration on the propagation characteristics of dust ion-acoustic shock waves in dusty plasma.

FLUID MODEL EQUATIONS

We consider a three dimensional magnetized two fluid plasma system consisting of positive and negative ions, kappa distributed electrons and negatively charged dust grains. This plasma system is assumed to be in external magnetic field $\mathbf{B} = B_0\hat{z}$. For simplification, it is assumed that DIASW is propagating in x–z plane. The normalized fluid equations are expressed as

$$\frac{\partial n_{p,n}}{\partial t} + \nabla.\left(n_{p,n}\mathbf{U_{p,n}}\right) = 0, \tag{1}$$

$$\frac{\partial \mathbf{U_p}}{\partial t} + (\mathbf{U_p}.\nabla)\mathbf{U_p} = -\nabla\phi + \Omega_c(\mathbf{U_p}\times\hat{\mathbf{z}}) + \eta_\mathbf{p}\nabla^2\mathbf{U_p}, \tag{2}$$

$$\frac{\partial \mathbf{U_n}}{\partial t} + (\mathbf{U_n}.\nabla)\mathbf{U_n} = \nabla\phi + \Omega_c(\mathbf{U_n}\times\hat{z}) + \eta_n\nabla^2\mathbf{U_n}, \tag{3}$$

where $\mathbf{U_p} = u_p\hat{i} + v_p\hat{j} + w_p\hat{k}$, $\mathbf{U_n} = u_n\hat{i} + v_n\hat{j} + w_n\hat{k}$ and Ω_c is the ion cyclotron frequency. In Eqs. (1), (2) and (3), n_p and n_n are the number densities of positive and negative ions normalized by its corresponding equilibrium value, $\mathbf{U_p}$ and $\mathbf{U_n}$ is the positive and negative ion fluid velocity normalized by ion-acoustic speed $c_p = \sqrt{\frac{T_e}{m_p}}$ and $c_n = \sqrt{\frac{T_e}{m_n}}$, $\phi\left(=\frac{e\phi}{T_e}\right)$ is the normalized electrostatic wave potential. Normalized number density of kappa distributed electrons is given by, $n_e = \mu\left(1 - \frac{\phi}{\kappa-\frac{3}{2}}\right)^{-\kappa+\frac{1}{2}}$ and normalized Poisson's equation is written as

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = \mu(1 + A_1\phi + A_2\phi^2) - n_p + \nu n_n + \delta_d, \tag{4}$$

where $A_1 = \frac{(2\kappa-2)}{(2\kappa-3)}, A_2 = \frac{(2\kappa-1)(2\kappa+1)}{2(2\kappa-3)^2}, \mu = \frac{n_{e0}}{n_{p0}} = 1 - \delta_d - \nu, \delta_d = \frac{z_{d0}n_{d0}}{n_{p0}}$ and $\nu = \frac{n_{n0}}{n_{p0}}$.

DERIVATION OF ZKB EQUATION

Shock waves are formed as a result of balance between nonlinearity and dissipation, and dissipation can be caused by the inter-particle collisions, dust charge fluctuation or Landau damping. The ion kinematic fluid viscosity (e.g. due to shear stress of the inertial fluid motion) modifies the wave structures. So in dissipative system, reductive perturbation technique leads to the formation of ZK-Burgers equation, featuring an extra dissipation term. According to this technique, the stretching coordinates can be chosen as [15, 16]

$$\xi = \epsilon^{1/2}x, \qquad \eta = \epsilon^{1/2}y, \qquad \zeta = \epsilon^{1/2}(z-\lambda t) \qquad \text{and} \qquad \tau = \epsilon^{3/2}t, \tag{5}$$

where ϵ is a small parameter characterizing the strength of the nonlinearity and λ is the phase velocity. The expansion of perturbed quantities can be written as

$$n_{(p,n)} = 1 + \epsilon n_{(p,n)1} + \epsilon^2 n_{(p,n)2} + \ldots\ldots\ldots, \qquad u_{(p,n)} = \epsilon^{\frac{3}{2}}u_{(p,n)1} + \epsilon^2 u_{(p,n)2}\ldots\ldots\ldots, \qquad v_{(p,n)} = \epsilon^{\frac{3}{2}}v_{(p,n)1} + \epsilon^2 v_{(p,n)2} + \ldots\ldots. \tag{6}$$

$$w_{(p,n)} = \epsilon w_{(p,n)1} + \epsilon^2 w_{(p,n)2} + \ldots\ldots. \qquad \text{and} \qquad \phi = \epsilon\phi_1 + \epsilon^2\phi_2 + \ldots\ldots\ldots \tag{7}$$

Using Eqs.(5)-(7) in Eqs. (1)-(4) and collecting the lowest order terms in ϵ, we obtain the following equations

$$n_{p1} = \frac{\phi_1}{\lambda^2}, \qquad u_{p1} = -\frac{1}{\Omega_c}\frac{\partial\phi_1}{\partial\eta}, \quad n_{n1} = -\frac{\phi_1}{\lambda^2}, \qquad u_{n1} = \frac{1}{\Omega_c}\frac{\partial\phi_1}{\partial\eta}, \quad v_{p1} = \frac{1}{\Omega}\frac{\partial\phi_1}{\partial\xi}, \qquad w_{p1} = \frac{\phi_1}{\lambda}, \tag{8}$$

$$v_{n1} = -\frac{1}{\Omega}\frac{\partial\phi_1}{\partial\xi}, \qquad w_{n1} = -\frac{\phi_1}{\lambda}. \tag{9}$$

From first order evolution Eq. (9), the phase velocity of dust-ion-acoustic waves comes out to be $\lambda^2 = \frac{1+\nu}{\mu A_1}$. Now collecting next higher order terms in ϵ, we get

$$\frac{\partial n_{p1}}{\partial \tau} - \lambda \frac{\partial n_{p2}}{\partial \zeta} + \frac{\partial u_{p2}}{\partial \xi} + \frac{\partial v_{p2}}{\partial \eta} + \frac{\partial w_{p2}}{\partial \zeta} + \frac{\partial (n_{p1} w_{p1})}{\partial \zeta} = 0, \quad \frac{\partial n_{n1}}{\partial \tau} - \lambda \frac{\partial n_{n2}}{\partial \zeta} + \frac{\partial u_{n2}}{\partial \xi} + \frac{\partial v_{n2}}{\partial \eta} + \frac{\partial w_{n2}}{\partial \zeta} + \frac{\partial (n_{n1} w_{n1})}{\partial \zeta} = 0, \quad (10)$$

$$\frac{\partial w_{p1}}{\partial \tau} - \lambda \frac{\partial w_{p2}}{\partial \zeta} + w_{p1} \frac{\partial w_{p1}}{\partial \zeta} = -\frac{\partial \phi_2}{\partial \zeta} + \eta_p \nabla^2 w_{p1}, \quad \frac{\partial w_{n1}}{\partial \tau} - \lambda \frac{\partial w_{n2}}{\partial \zeta} + w_{n1} \frac{\partial w_{n1}}{\partial \zeta} = \frac{\partial \phi_2}{\partial \zeta} + \eta_n \nabla^2 w_{n1}, \quad (11)$$

$$v_{p2} = -\frac{\lambda}{\Omega_c} \frac{\partial u_{p1}}{\partial \zeta}, \quad u_{p2} = \frac{\lambda}{\Omega_c} \frac{\partial v_{p1}}{\partial \zeta}, \quad v_{n2} = \frac{\lambda}{\Omega_c} \frac{\partial u_{n1}}{\partial \zeta} \quad \text{and} \quad u_{n2} = -\frac{\lambda}{\Omega_c} \frac{\partial v_{n1}}{\partial \zeta}. \quad (12)$$

By eliminating second order quantities from above equations, we obtain the ZKB equation in a magnetized dusty plasma as follows.

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} + C \frac{\partial}{\partial \xi} \left(\frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} \right) - D\nabla^2 \phi_1 = 0, \quad (13)$$

where

$$A = B\left[-2A_2\mu + 3\frac{(1+\nu)}{\lambda^4}\right], \quad B = \frac{\lambda^3}{2(1+\nu)}, \quad C = \frac{\lambda^3}{2(1+\nu)}\left[1 + \frac{1-\nu}{\Omega_c^2}\right] \quad \text{and} \quad D = \frac{\eta_p + \eta_n}{2}. \quad (14)$$

It is noticed that dissipative term appears only in the last term of Eq. (13) and for $\eta_p, \eta_n \to 0$, only solitary structures are formed. In our case, due to presence of dissipative term, some of the system energy is dissipated, which leads to the formation of shock waves. So the existence of shocks is considerably affected by the presence of dissipative coefficient. To obtain the solution of Eq. (13), we introduce the single variable transformation, $\chi = l\xi + m\eta + n\zeta - u\tau$ for co-moving frame, where l, m and n are the direction cosines between the propagation vector k, and u is the velocity of moving frame. By considering $\Phi(\chi) = \phi_1(\xi, \eta, \zeta, \tau)$, Eq. (13) leads to the ordinary differential equation as

$$-u\frac{d\Phi}{d\chi} + Al\Phi\frac{d\Phi}{d\chi} + lP\frac{d^3\Phi}{d\chi^3} - D\frac{d^2\Phi}{d\chi^2} = 0, \qquad P = Bl^2 + C(m^2 + n^2). \quad (15)$$

For the limiting case, i.e., for $D \to 0$, Eq. (15) reduces to well known ZK equation. Using the boundary conditions $\Phi \to 0$, $\frac{d\Phi}{d\chi}$, $\frac{d^2\Phi}{d\chi^2} \to 0$ at $\chi \to \pm\infty$, we get the solitary wave solution $\Phi = \Phi_0 sech^2\left(\frac{\chi}{w_m}\right)$, where $w_m = 2\sqrt{\frac{lP}{u}}$. By involving the dissipative and dispersion terms with implication of hyperbolic tangent (tanh) method [18] , we will obtain the exact travelling wave solution of Eq. (15). Introducing the new independent variable $W = \tanh(\rho\chi)$ in Eq. (15), we get

$$\begin{aligned} -u\rho(1-W^2)\frac{d\Phi}{dW} + Al\rho(1-W^2)\Phi\frac{d\Phi}{dW} + Fl\rho^3(1-W^2) \quad (16) \\ \times\frac{d}{dW}\left[(1-W^2)\frac{d}{dW}\left((1-W^2)\frac{d}{dW}\right)\right]\Phi - D\rho^2(1-W^2)\frac{d}{dW}\left[(1-W^2)\frac{d}{dW}\right]\Phi = 0 \end{aligned}$$

The solution is assumed to be in series form $\Phi(W) = f_0 + f_1 W + f_2 W^2$, where

$$f_0 = \frac{9}{25}\frac{D^2}{FAl^2}, \qquad f_1 = \mp\frac{6}{25}\frac{D^2}{FAl^2}, \quad f_2 = -\frac{3}{25}\frac{D^2}{FAl^2}, \qquad \rho = \pm\frac{D}{10Fl} \qquad \text{and} \qquad u = \frac{6D^2}{25Fl}. \quad (17)$$

So, the solution of Eq. (17) can be written as

$$\Phi(\chi) = \frac{3}{25}\frac{D^2}{FAl^2}\left[2 - 2\tanh\left(\frac{D}{10Fl}\chi\right) + sech^2\left(\frac{D}{10Fl}\chi\right)\right]. \quad (18)$$

PARAMETRIC ANALYSIS

It is noticed that the properties of wave potential is influenced by dispersive and dissipative effects which are present in the exact solution of ZKB equation. It can be seen that the polarity of nonlinear structures (i.e. shocks) depends upon the sign of the nonlinear coefficient A, as the other coefficients B, C and D are always positive, hence we plot the contour plot of A in the plane of $\delta_d - \kappa$ (see Fig. 1). It is found that in Fig. 1, region above the curve, i.e., for $A > 0$, positive potential shock structures are formed and in region below the curve, i.e., for $A < 0$, negative potential dust ion-acoustic shock structures are formed which propagate in the present system. By fixing the other parameters, the critical values of κ and δ_d are determined, to see the transition in polarity of dust ion-acoustic shock waves.

The effects of superthermality of electrons (via κ), dust concentration (via δ_d) and positive to negative ion number density ratio ν are displayed in Fig. 2. It is observed that these parameters have strong influence on shock wave profiles. The opposite polarity shocks are formed above or below critical values of κ and δ_d. From Fig. 2a, we analyse the effect of superthermality of electrons via κ on shock profile. Increase in κ shows that the amplitude and width of positive potential shock waves are reduced while the effect of κ on negative potential shock waves is in strong contrast to this. Figure 2b shows the effect of dust concentration δ_d on the characteristics of shock waves. It is observed that with the increase in dust concentration, amplitude as well as width of positive potential shock wave profile increases whereas, for negative potential shock waves, amplitude and width are decreased with increase in dust concentration. Figure 2c depicts the effect of ratio negative to positive ion number density (via ν) on shock wave profile. It is seen that the amplitude as well as width increases with the increase in ν.

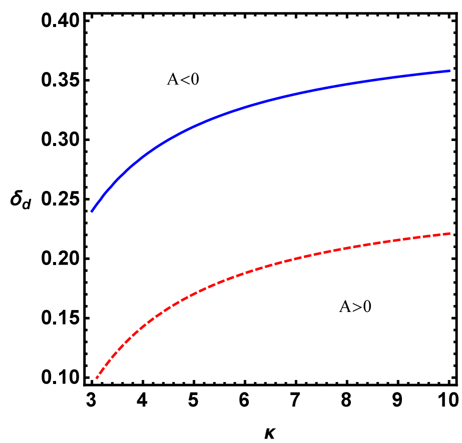


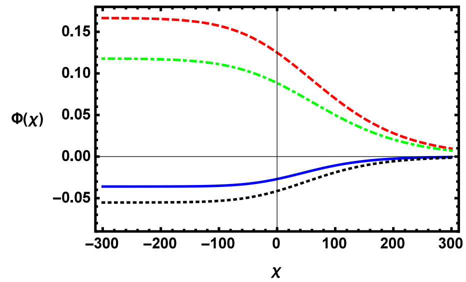
FIGURE 1: Contour plot of A for dust concentration (δ_d) vs superthermality of electrons κ for different values of ν with $\omega = 0.3$, $\Omega_c = 0.1$, $\eta_p = 0.2$, $\eta_n = 0.1$ and $l = 0.1$. $\nu = 0.1$ (solid (blue) curve) and $\nu = 0.2$ (dashed (red) curve).

CONCLUSIONS

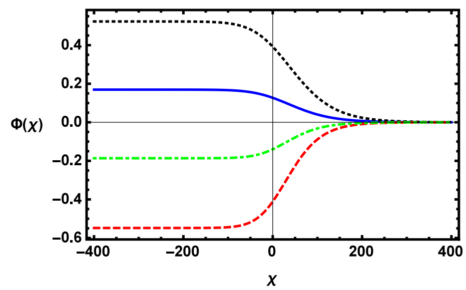
In the present investigation, we have studied the characteristics of dust ion acoustic shock waves in magnetized plasma consisting of two fluid positive and negative ions, immobile negatively charged dust and electrons obeying kappa distribution. By employing reductive perturbation technique, ZKB equation has been derived and the shock wave solution of ZKB equation is determined. The important results are summarized as follows:

- Both polarity positive and negative potential shock structures are formed on the basis of critical values of dust concentration δ_d and superthermality of electrons κ.
- Increase in dust concentration δ_d and ratio of positive to negative ion concentration ν, leads to increase (decrease) in amplitude and width of positive (negative) potential dust-ion-acoustic shock structures but decrease in superthermality of electrons leads to decrease (increase) in both amplitude and width of positive (negative) potential structures.

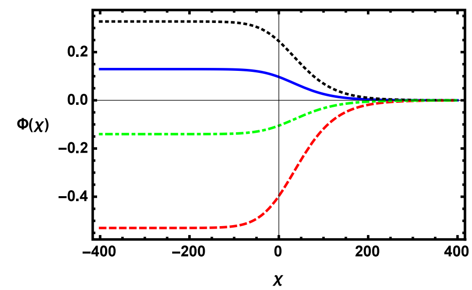
The present study may be useful to understand the characteristics of DIASWs in D and F regions of Earth's ionosphere.



(a) Rarefactive shocks for $\kappa = 3.25$ (solid (blue)), $\kappa = 3.75$ (dotted (black)) and compressive shocks for $\kappa = 5.5$ (dashed (red)), $\kappa = 5.75$ (dotdashed (green)).



(b) Compressive shocks for $\delta_d = 0.24$ (solid (blue)), $\delta_d = 0.25$ (dotted (black)) and rarefactive shocks for $\delta_d = 0.26$ (dashed (red)), $\delta_d = 0.27$ (dotdashed (green)).



(c) Compressive shocks for $\nu = 0.29$ (solid (blue)), $\nu = 0.30$ (dotted (black)) and rarefactive shocks for $\nu = 0.31$ (dashed (red)), $\nu = 0.32$ (dotdashed (green)).

FIGURE 2: The shock wave profile Φ is plotted against χ for different values of superthermality of electrons, dust concentration δ_d and positive to negative ion concentration ν and with fixed values of other parameters as given in caption of Fig. 1.

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