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Modeling of Mean-VaR Portfolio Optimization by Risk Tolerance When the Utility Function is Quadratic

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Abstract. The problems of investing in financial assets are to choose a combination of weighting a portfolio can be maximized return expectations and minimizing the risk. This paper discusses the modeling of Mean-VaR portfolio optimization by risk tolerance, when square-shaped utility functions. It is assumed that the asset return has a certain distribution, and the risk of the portfolio is measured using the Value-at-Risk (VaR). So, the process of optimization of the portfolio is done based on the model of Mean-VaR portfolio optimization model for the Mean-VaR done using matrix algebra approach, and the Lagrange multiplier method, as well as Khun-Tucker. The results of the modeling portfolio optimization is in the form of a weighting vector equations depends on the vector mean return vector assets, identities, and matrix covariance between return of assets, as well as a factor in risk tolerance. As an illustration of numeric, analyzed five shares traded on the stock market in Indonesia. Based on analysis of five stocks return data gained the vector of weight composition and graphics of efficient surface of portfolio. Vector composition weighting weights and efficient surface charts can be used as a guide for investors in decisions to invest.

INTRODUCTION

The investment is the planting of some funds which will hopefully give more results later in the day. Order to invest in the form of securities (securities) in General can be done via the capital market and the money market [9], [11]. But the level of profit earned in the capital markets in the form of securities, particularly stocks is greater than the profit rate in the money market which are embedded in the form of deposits [20]. In view of the risks, investment in the capital markets has a greater risk than the risk of an investment in the money market. Because of the risks and benefits in general have a positive relationship, that is, the greater the level of profit the greater the risk incurred anyway [7], [12].

Risk can be said to be a chance of the occurrence of the loss or destruction. More broadly, the risk can be defined as the possibility of the occurrence of undesirable results, or the opposite of the desired [7], [15]. In the financial industry in General, if you want to obtain greater results, will be exposed to greater risk anyway. Whereas refunds are profits of companies, individuals and institutions from the results of the policy of investments made [9].

So, the risk and rate of return is a condition experienced by companies, institutions, and individuals in investment decisions, i.e. either loss or gain in a given accounting period. Currently many developed calculation value risk in investing so that investors can know the value of risks and anticipate it early [16], [8].

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One form of measuring the value of risk that is often used is the Value-at-Risk (VaR). Value-at-Risk (VaR) is a statistical risk measurement tool that measures the maximum expected losses from an investment at the level of specific confidence and a specific time period in normal market conditions [5], [2].

Strategies commonly used in risky investment portfolio are shaping up. The nature of the establishment of the portfolio is to allocate funds on a range of alternative investment, thus obtained an optimal portfolio. Optimal portfolio can be found by using the model of Markowitz [3], [19].

To determine the optimal portfolio with this model, the first time it takes is to determine an efficient portfolio. For this model, the optimal portfolio of all is an efficient portfolio. Because every investor has a different curve is not the same, the optimal portfolio will be different for each investor [14], [13].

Investors who have a high risk tolerance will have a portfolio with a high return by paying the risk is also higher compared to investors who have a low tolerance for risk. If the assets are not risky in comparison, these assets can change the portfolio [17]. The risk tolerance with regard to the form of the utility function of each investor [12]. In the paper is examined on "Modeling of Mean-VaR Portfolio Optimization by Risk Tolerance When the Utility Function is Quadratic".

The goal is to formulate a model that can be used in the calculation of the optimal portfolio weights combination. As an illustration of numeric, analyzed some of the shares traded on the stock market in Indonesia.

MATHEMATICAL MODELS

In this mathematical models, first discussed about the investor's utility function which is the basis for lowering factor risk tolerance. Second, the investment portfolio modelling discussed about the Mean-VaR without the risk-free asset.

Utility Function of Investor

Individual investors generally have the equation of the curve or the different utility functions, according to the preferences of his attitude to a risk of investing. Based on the utility function that is owned by someone, investors risk aversion functions can be determined. The form of a utility function that is discussed here is the square-shaped.

Suppose W stated property (funds) which are owned to an investment. Suppose also that one investor has a utility function that squares shaped like the following [16], [22]:

$$U(W) = W - bW^2 \text{; with parameter coefficient } b > 0 .$$

The first and second derivatives of the utility function this is given as follows:

$$U'(W) = 1 - 2bW > 0 \text{ for } W < 1/2b \text{, and } U''(W) = -2b < 0$$

The function of risk aversion $\eta(W)$ for someone of such investors can be specified as:

$$\eta(W) = -\frac{U''(W)}{U'(W)} = -\frac{-2b}{1-2bW} = \frac{2b}{1-2bW}$$

Therefore, the factors of risk tolerance τ can be obtained as:

$$\tau(W) = \frac{1}{\eta(W)} = \frac{1-2bW}{2b}$$

If the initial fund is invested $W = W_0$, then factor into a risk tolerance:

$$\tau(W_0) = \frac{1}{\eta(W_0)} = \frac{1-2bW_0}{2b} \quad (1)$$

For the risk tolerance factors, then used to formulate the optimum portfolio based on Value-at-Risk, are discussed in the following.

Modeling the Mean-VaR Portfolio Optimization

In this section are discussed about an investment portfolio with discrete-time. Suppose return of assets *i*, where *i* = 1,2,…,*N* and *N* the number of assets in the portfolio. Portfolio return is the weighted average of the return of each asset in the portfolio [1]. If investors choose a portfolio with the vector of weights:

$$\mathbf{w}^T = (w_1, ..., w_N), \ \sum_{i=1}^{N} w_i = 1$$

where w_i the stated proportions (weights) of funds invested in assets *i*, then return the portfolio weighting vectors $\mathbf{w}$, r_p with given by [4]:

$$r_p = \sum_{i=1}^{N} w_i r_i \tag{2}$$

Based on equation (2), average (return expectations) portfolio μ_p is given by:

$$\mu_p = E[r_p] = \sum_{i=1}^{N} w_i E[r_i] = \sum_{i=1}^{N} w_i \mu_i \tag{3}$$

And portfolio variance σ_p^2 is given by:

$$\sigma_p^2 = Var(r_p) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov(r_i, r_j) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{4}$$

Where σ_{ij} states covariance between stocks i and j. Namely, given as:

$$\sigma_{ij} = Cov(r_i, r_j) = E[(r_i - \mu_i)(r_j - \mu_j)] = \rho_{ij}\sigma_i\sigma_j \tag{5}$$

Where ρ_{ij} the coefficient of correlation between the return of assets i and j, as well as $\sigma_i = \sqrt{\sigma_i^2}$ the standard deviation of the returned assets i [6].

Suppose given a covariance matrix $\mathbf{\Sigma}$ and identity matrix as follows:

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{pmatrix} \text{ and } \mathbf{I} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

where $\sigma_{ii} = \sigma_i^2$ with $i = 1, ..., N$.

Furthermore, the portfolio return expectations in the equation (3) can be expressed using the vector equation as:

$$\mu_p = E[r_p] = \boldsymbol{\mu}^T \mathbf{w} \tag{6}$$

and equation of variance (4) become:

$$\sigma_p^2 = Var(r_p) = \mathbf{w}^T \mathbf{\Sigma} \mathbf{w} \tag{7}$$

According to Goh et al. [10] and Tsai [21], risk measurement model of the Value-at-Risk for portfolios p formulated as $VaR_p = -W_0\{\mu_p + z_\alpha \sigma_p\}$. Refer to equation (6) and (7), the Value-at-Risk for the portfolio p can be expressed as:

$$VaR_p = -W_0\{\mathbf{w}^T \boldsymbol{\mu} + z_\alpha (\mathbf{w}^T \mathbf{\Sigma} \mathbf{w})^{1/2}\} \tag{8}$$

Where the sign (-) stated losses, W_0 the initial capital invested, and z_α percentile of the standard normal distribution when the given $(1-\alpha)\%$ level of significance.

Furthermore, we note the definition of efficient portfolio as follows:

Definition 1 (Qin [18]).

A portfolio p^* called (Mean – VaR) efficient if there is no portfolio p with $\mu_p \geq \mu_{p^*}$ and $VaR_p < VaR_{p^*}$.

Thus, if the risk of an investment portfolio is measured using the Value-at-Risk, then it is based on Markowitz's, the investment portfolio optimization problems will be resolved, is shaped [2]:

$$\text{Maximum } \{2\tau\mu_p - VaR_p\}$$
$$\text{Subject to } \sum_{i=1}^{N} w_i = 1$$

Suppose the initial capital invested amounting $W_0 = 0$ unit of money, then the objective function can be expressed as:

$$\text{Maximum } \{(2\tau+1)\mu_p + z_\alpha \sigma_p\}$$
$$\text{Subject to } \sum_{i=1}^{N} w_i = 1 \tag{9}$$

with τ factor in the risk tolerance of investors owned. Equation (9) can be expressed in the form of linear algebra to as:

$$\text{Maximum } \{(2\tau+1)\mathbf{w}^T\boldsymbol{\mu} + z_\alpha(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}\}$$
$$\text{Subject to } \mathbf{w}^T\mathbf{e} = 1 \tag{10}$$

The form of the equations (9) and (10), known as the investment portfolio optimization problems Mean-VaR.

To search for a solution to the question of optimization of investment portfolio of Mean-VaR, in the study developed a portfolio weighted optimization theorem by a factor of risk tolerance of the utility function quadratic as follows:

Theorem 1. If given an investment portfolio optimization problems Mean-VaR factor in risk tolerance as given in equation (10), then the solution is to determine the optimum weights are as follows:

(a) To factor risk tolerance is obtained from the quadratic utility function, the solution question of optimization investment portfolio weighting of Mean-VaR is:

$$\mathbf{w} = \frac{(\frac{1+b-2bW_0}{b})\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\boldsymbol{\Sigma}^{-1}\mathbf{e}}{(\frac{1+b-2bW_0}{b})\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}}$$

b) The Lagrange Multiplier λ can be calculated by using the formula of abc as follows

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ with the terms of } \lambda \geq 0 \quad ,$$

Where $a = \mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}$, $b = (\frac{1+b-2bW_0}{b})\{\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}\}$, and $c = (\frac{1+b-2bW_0}{b})\boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - z_\alpha^2$, with $\boldsymbol{\Sigma}^{-1}$ the inverse of the matrix $\boldsymbol{\Sigma}$ such that $\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma} = \boldsymbol{\Sigma}\boldsymbol{\Sigma}^{-1} = \mathbf{I}$

Proof:

Because of the covariance matrix $\boldsymbol{\Sigma}$ is semi- definite positive, the objective function in equation (10) is quadratic concave. Lagrange multiplier function from equation (10) is given by:

$$L(\mathbf{w},\lambda) = (2\tau+1)\boldsymbol{\mu}^T\mathbf{w} + z_\alpha(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2} + \lambda(\mathbf{w}^T\mathbf{e} - 1)$$

Kuhn-Tucker theorem uses, the terms of optimality are

$$\frac{\partial L}{\partial \mathbf{w}} = (2\tau+1)\boldsymbol{\mu} + \frac{1}{2}.2.z_\alpha \frac{\boldsymbol{\Sigma}\mathbf{w}}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}} + \lambda\mathbf{e} = 0 \tag{11}$$

and

$$\frac{\partial L}{\partial \lambda} = \mathbf{w}^T\mathbf{e} - 1 = 0 \tag{12}$$

Proof of Theorem 1(a)

Based on equation (11) can be obtained

$$\frac{z_\alpha \boldsymbol{\Sigma}\mathbf{w}}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}} = -\{(2\tau+1)\boldsymbol{\mu} + \lambda\mathbf{e}\} \tag{13}$$

If the equation (13) multiplied by $\boldsymbol{\Sigma}^{-1} / z_\alpha$, then obtained the following equation

$$\frac{\mathbf{w}}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}} = -\frac{\{(2\tau+1)\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\boldsymbol{\Sigma}^{-1}\mathbf{e}\}}{z_\alpha} \tag{14}$$

Equations (14), when multiplied by $\mathbf{e}^T$ the obtained equations such as

$$\frac{1}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}} = -\frac{\{(2\tau+1)\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}\}}{z_\alpha} \tag{15}$$

Substituting equation (15) into equation (14) and resolved will be retrieved the following weighting vector equation:

$$\mathbf{w} = \frac{(2\tau+1)\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\boldsymbol{\Sigma}^{-1}\mathbf{e}}{(2\tau+1)\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}} \tag{16}$$

Equation (4) is the solution of the question of portfolio optimization of Mean-VaR. Notation stating optimum weight solution to a value of a certain tolerance for risk factors.

Furthermore, because the function of risk tolerance is obtained from the quadratic utility function, then the based on equation (1), the equation of the vector of weights (16) can be expressed as:

$$\mathbf{w} = \frac{(\frac{1+b-2bW_0}{b})\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\boldsymbol{\Sigma}^{-1}\mathbf{e}}{(\frac{1+b-2bW_0}{b})\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}} \tag{17}$$

Equation (17) is a viable solution for the optimum weights of portfolio optimization problems Mean-VaR with a tolerance for risk factors derived from the utility function quadratic.

Proof of Theorem 1(b)

When the equation (16) multiplied by $\mathbf{w}^T$ the following equation is obtained:

$$\frac{z_\alpha \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}} = -\{(2\tau+1)\mathbf{w}^T\boldsymbol{\mu} + \lambda\mathbf{w}^T\mathbf{e}\}$$

or it can be written back into

$$z_\alpha(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2} = -\{(2\tau+1)\boldsymbol{\mu}^T\mathbf{w} + \lambda\} \tag{18}$$

Substituting equations (15) and (16) into the equation (18), obtained the following equation:

$$\frac{z_\alpha^2}{(2\tau+1)\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}} = (2\tau+1)\boldsymbol{\mu}^T\left(\frac{(2\tau+1)\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\boldsymbol{\Sigma}^{-1}\mathbf{e}}{(2\tau+1)\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}}\right) + \lambda$$

Or

$$z_\alpha^2 = (2\tau+1)^2 \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1}\mathbf{e} + \lambda\{(2\tau+1)\mathbf{e}^T \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda \mathbf{e}^T \boldsymbol{\Sigma}^{-1}\mathbf{e}\}$$

Or

$$(\mathbf{e}^T \boldsymbol{\Sigma}^{-1}\mathbf{e})\lambda^2 + \{(2\tau+1)\mathbf{e}^T \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1}\mathbf{e}\}\lambda + \{(2\tau+1)^2 \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - z_\alpha^2\} = 0 \quad (19)$$

Equation (19) is a quadratic equation in λ, so it can be calculated using the following abc formula

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{ for } \lambda > 0$$

Where $a = \mathbf{e}^T \boldsymbol{\Sigma}^{-1}\mathbf{e}$, $b = (\frac{1+b-2bW_0}{b})\{\mathbf{e}^T \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1}\mathbf{e}\}$ and $c = (\frac{1+b-2bW_0}{b})\boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - z_\alpha^2$ with $\boldsymbol{\Sigma}^{-1}$ the inverse of the matrix $\boldsymbol{\Sigma}$.

Based on the results of Theorem 1, If the vector $\mathbf{w}$ in substituting into equation (4) and (8), it will be retrieved, return expectations and Value-at-Risk optimum investment portfolio. When taken $b > 0$ give a real number value of risk tolerance $\tau \geq 0$ and substituting into equation (16) will produce a collection of vectors of real numbers of weights that are $\mathbf{w}^T \mathbf{e} = 1$ eligible. The weight vector bundles can be used to form the surface efficiently (efficient frontier) in the investment portfolio is concerned [14].

NUMERICAL ILLUSTRATION

Illustration of numeric is intended to demonstrate how the application of the model that has been formulated, to perform data analysis stocks traded on capital markets. Stock data were analyzed through the website http://www.finance.go.id//. The data consists of 5 (five) shares are selected, for during the period from 2 January 2013 until 4 June 2016. The data include stocks: TRUB, HDMT, BMRI, UNTR, and BBRI.

The value of the return of the mean five stocks respectively given in vector $\boldsymbol{\mu}^T$ = (0.022085, 0.003564, 0.001594, 0.020709, -0.000865). While the value of the variance return along with the value of the covariance between the return of the five stocks are given in the form of a covariance matrix $\boldsymbol{\Sigma}$ and its inverse $\boldsymbol{\Sigma}^{-1}$, as follows

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0.0012 & 0.0002 & 0.0003 & 0.0002 & 0.0001 \\ 0.0002 & 0.0011 & -0.0003 & -0.0001 & -0.0001 \\ 0.0003 & -0.0003 & 0.0014 & 0.0007 & 0.0007 \\ 0.0002 & -0.0001 & 0.0007 & 0.0013 & 0.0006 \\ 0.0001 & -0.0001 & 0.0007 & 0.0006 & 0.0012 \end{bmatrix} \quad \boldsymbol{\Sigma}^{-1} = \begin{bmatrix} 846.74 & -0.1625 & -0.1578 & -0.1019 & -0.1101 \\ -0.1625 & 931.97 & -0.0175 & 0.0320 & 0.0180 \\ -0.1578 & -0.0175 & 708.72 & -0.0381 & -0.4379 \\ -0.1019 & 0.0320 & -0.0381 & 748.50 & -0.3931 \\ -0.1101 & 0.0180 & -0.4379 & -0.3931 & 808.41 \end{bmatrix}$$

Because of the large number of stocks analyzed consists of five, then the vector unit defined as a $\mathbf{e}^T$= (1, 1, 1, 1, 1). Suppose also that the initial capital invested is W_0 = 1 unit of currency.

Furthermore, vector $\mathbf{e}^T$, vector $\boldsymbol{\mu}^T$ and inverse, covariance matrices $\boldsymbol{\Sigma}^{-1}$ jointly used for the calculation of the investment portfolio optimization process. Here it is assumed that in the transaction selling stock short sales are not allowed. The research of optimization is done by using a model of Mean-VaR optimization in the process. These values are constant coefficients determined in simulated sequence of smallest to largest value. Investment portfolio optimization process conducted with the help of software Matlab 7.0

For the process of optimization of investment portfolio of Mean-VaR done by referring to equations (14) and (11), as well as here defined value α = 5% so that the retrieved value $z_{5\%}$ =-1,645

The process of determining the magnitude of the coefficient constant $b > 0$ is determined randomly, such that the risk tolerance τ of the value 0.00 rose to 0.45. Portfolio optimization process results are given in Table-1. In table-1, increment the value of the risk tolerance of 0.00 increasing up to 0.45, which disclosed his ascension of 0.05.

TABLE 1. Portfolio Optimization Process Results

b	τ	Weights (w_i) for stocks					$\sum w_i$	$\hat{\mu}_p$	VaR_p	$\frac{\hat{\mu}_p}{VaR_p}$	$RVaR_p$
		TRUB	HDMT	BMRI	UNTR	BBRI					
10,000	0.00	0.32054	0.17441	0.11798	0.27265	0.11443	1.00	0.013436	0.014542	0.9240	0.74197497
20.00	0.05	0.33390	0.16768	0.11112	0.28319	0.10411	1.00	0.013923	0.014566	0.9559	0.77418646
10.00	0.10	0.34807	0.16055	0.10385	0.29436	0.09317	1.00	0.014440	0.014644	0.9861	0.80536739
6.67	0.15	0.36321	0.15292	0.09609	0.30630	0.08148	1.00	0.014992	0.014782	1.0142	0.83519145
5.00	0.20	0.37952	0.14470	0.08773	0.31916	0.06889	1.00	0.015587	0.014991	1.0398	0.86323794
4.00	0.25	0.39729	0.13575	0.07862	0.33318	0.05516	1.00	0.016235	0.015283	1.0623	0.88914480
3.33	0.30	0.41688	0.12589	0.06857	0.34862	0.04004	1.00	0.016950	0.015677	1.0812	0.91240671
2.86	0.35	0.43876	0.11487	0.05735	0.36588	0.02314	1.00	0.017748	0.016196	1.0958	0.93244011
2.50	0.40	0.46363	0.10234	0.04460	0.38549	0.00394	1.00	0.018655	0.016878	1.1053	0.94850101
2.44	**0.409**	**0.46850**	**0.09989**	**0.04210**	**0.38933**	**0.00018**	**1.00**	**0.018832**	**0.017021**	**1.1064**	0.95093120
2.22	0.45	0.49248	0.08781	0.02980	0.40824	**-0.0183**	1.0	0.019707	0.017770	1.1088	0.96009004

The value of the constant coefficients that satisfy the assumptions that short sales are not allowed is $2.44 \leq b \leq$ 10.000. Change the value b of a 10.000 decline until 2.44 here is done by randomly. Investment portfolio optimization process of Mean-VaR efficient portfolio charts also obtained such as Figure 1.

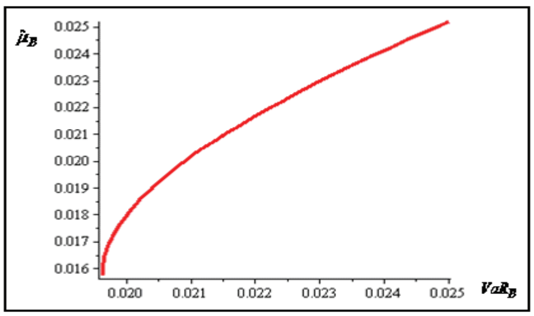

FIGURE 1. Graph of Mean-VaR Efficient Portfolio

In Figure 1, the minimum portfolio value at risk is VaR_p = 0.014542 and mean value $\hat{\mu}_p$ =0.013436 , occurs when the magnitude of the value of the constant coefficient b = 10.000. Portfolio minimum generated for combination weighting a portfolio as $\mathbf{w}^{Min}$ = (0.32054, 0.17441, 0.11798, 0.27265, 0.11443)

Whereas the maximum portfolio value at risk is VaR_p = 0.017021 and mean value $\hat{\mu}_p$ = 0.018832, occurs when the value of the constant coefficient VaR_p = 2.44. Maximum portfolio generated for combination weighting a portfolio as $\mathbf{w}^{Max}$ = (0.46850, 0.09989, 0.04210, 0.38933, 0.00018). On maximum portfolio is also the optimum portfolio of global, ratio VaR_p = 1.1064 is the largest. This is also demonstrated by the portfolio's performance as measured by the reward to Value-at-Risk $RVaR_p$ = 0.9509312 is the largest.

CONCLUSION

In this paper has done research on modeling of Mean-VaR portfolio optimization with the risk tolerance of the quadratic utility function. Based on research conducted the following conclusions to be drawn. First, the optimum solution of a model investment portfolio of Mean-VaR by risk tolerance is stated in the form of a weighting vector equation given as equations (11) and (14).

Second, based on the analysis of the five stock assets, to an investment portfolio of Mean-VaR portfolio weights combination obtained a global optimum is $\mathbf{w}^{Glob}$ = (0.46850, 0.09989, 0.04210, 0.38933, 0.00018), with expectations of portfolio return mean 0.018832 risk and Value-at-Risk of 0.017021, is reached when the value of the constant coefficient of b = 2.44.

It has a global portfolio of optimum ratio between mean against risk is the greatest. As well as having the performance measured using the reward to Value-at-Risk whose value is also the largest.

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