Journal of Mechanisms and Robotics

Editorial

21st Century Kinematics: Synthesis, Compliance, and Tensegrity

As we move into the second decade of the 21st century, we can identify three research trends that we can expect to persist into the future. They are the analysis and synthesis of (i) spatial mechanisms and robotic systems, (ii) compliant linkage systems, and (iii) tensegrity and cable-driven systems. In each case, we find that researchers are formulating and solving polynomial systems of total degrees that dwarf those associated with major kinematics problems of the previous century.

1 Analysis and Synthesis of Mechanisms and Robotic Systems

Early in the 2000s, Lee and Mavroidis [1] formulated the synthesis of a spatial serial chain as a generalized inverse kinematics problem, where, in order to position a serial chain in a required set of task positions, one computes the dimensions of the chain as well as its joint angles. About the same time, general polynomial continuation algorithms, such as *PHCpack* [2,3] and *POLSYS* [4], became available. Lee and Mavroidis used the PHCpack to solve 13 polynomials in 13 unknowns and obtained 36 sets of dimensions for spatial 3R chains that reach four required task positions. This system of polynomials had a total degree of 1,417,174 and took 33 days to solve on a computer workstation [5].

In 2004, Su et al. reported that a parallel version of the POLSYS code [6,7] obtained 42,625 solutions to the RRS design problem consisting of 11 polynomials in 11 unknowns for a total degree of 4,194,304. The computation time was 42 min on 128 nodes of the San Diego Supercomputer Center's Blue Horizon system.

Perez and McCarthy [8] and Perez-Gracia and McCarthy [9] used soma coordinates to formulate the design equations for general spatial serial chains and obtained 126 synthesis equations in 126 unknowns for a spatial 5R chain that reaches a set of 21 task positions (Fig. 1). This set of equations has been solved numerically to verify that they are correct but, so far, a complete solution has not been achieved.

Sommese and Wampler published their text [10] on the mathematical theory of polynomial continuation and its applications to systems that arise in engineering and science, which they termed "numerical algebraic geometry." This includes the polynomial continuation algorithm *HomLab*, which has recently been upgraded and called *Bertini* [11].

By 2008, Lee et al. [12] found that the performance of polynomial continuation algorithms had advanced to the point that on a Dell PC, they averaged 15 continuation paths per second for PHCpack and Bertini, and reached 150 paths per second for some problems using their *HOM4PS* package. At computation speeds of 1500 paths per second, Su's RRS synthesis problem could be achieved on a PC in 5 min.

Continued improvements in processing speed, parallel processing architectures, and algorithm efficiency have the potential to yield such rapid solution of very large polynomial systems, where in the numerical analysis and synthesis of complex spatial linkage systems can become practical and routine.

2 Analysis and Synthesis of Compliant Linkage Systems

In his speech of 1972, Freudenstein identified "a theory of kinematics of mechanisms with elasticity" as one of the nine areas for future research. Initial work in this area was limited to flexibility effects in the analysis of linkage movement [13] but this evolved in the late 1980s to yield a design theory for compliant linkages [13–15]. In the 1990s, compliant mechanisms found a wide range of applications including micromechanisms [16,17], and by 2001, Howell presented a complete design theory [18].

As the study of compliant mechanisms matured, it was shown that the flexibility in the links can be modeled by spring-loaded joints. This allowed kinematic loop equations to be combined with static equilibrium for the analysis and synthesis of these systems [19]. The result was the ability to design a linkage system with compliance that resists displacement from specific configurations [20]. Su and McCarthy formulated a system of polynomials for the synthesis of a four-bar compliant linkage with three specified equilibrium positions [21,22]. He obtained 12 polynomials with a total degree of 16,384, which was solved using polynomial continuation.

Current research directions in compliant linkage design include increasing the ease of the design process and extending it to spatial linkages. As an example, Hedge and Ananthasuresh described the sophistication of compliant mechanism synthesis methods and proposed a simplified approach for planar linkages [23] (Fig. 2). Similarly, Lusk and Howell [24] and Espinosa and Lusk [25] de-

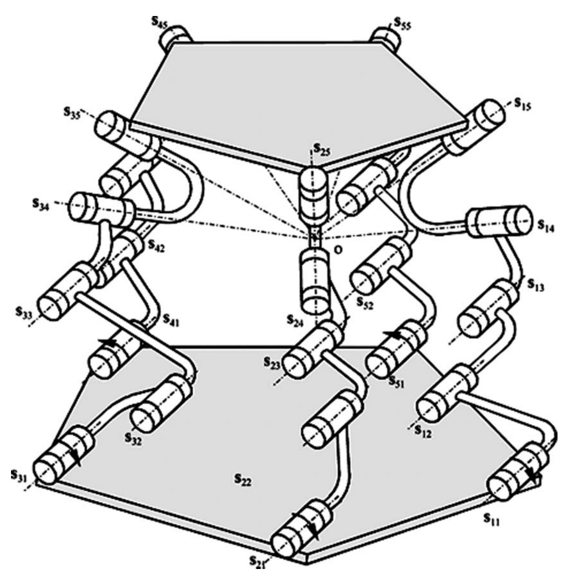

Fig. 1 A spatial one degree-of-freedom linkage formed from 5-5R chains

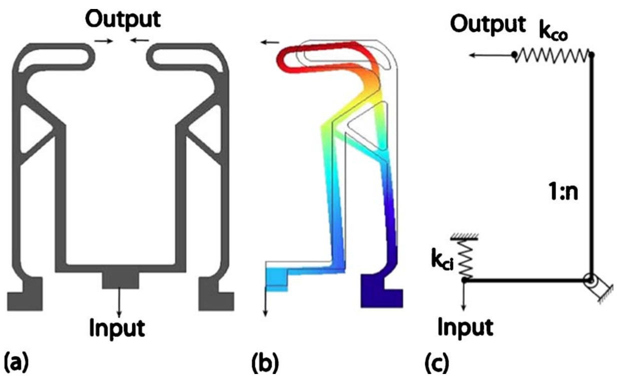

Fig. 2 A compliant mechanism designed for use as a gripper [23]

scribed the need for compliant linkages that are constructed in the plane by layered manufacturing but have the ability to move out of the plane for operation.

It is clear that computer algebra and polynomial continuation that serve the needs of design and analysis of mechanisms and robotics will also serve to advance the analysis and design of compliant linkage systems.

3 Analysis and Synthesis of Tensegrity and Cable-Driven Systems

In the early 1990s, Griffis and Duffy analyzed the stiffness of a Stewart platform supported by compliant legs [26], and by 2000, he formalized this into a theory of deployable tensegrity structures with elastic ties [27]. A tensegrity structure [28,29] is an assembly of compression elements, or struts, and extension elements, called ties, that form a stable structure.

Crane et al. [30] considered the equilibrium configurations of a set of struts and cable ties of a tensegrity system. His formulation yields 12 equations for a system of three struts and nine cables (Fig. 3). This formulation also applies to cable systems ranging from deployable structures [31] to cable manipulators [32,33]. In fact, Moon et al. [34] constructed spring-loaded actuators for a parallel mechanism to combine tensegrity and compliant mechanism design.

Recently, Jiang and Kumar formulated the problem of determining the equilibrium positions of objects supported from cables [35], which yielded nine polynomials in nine unknowns (Fig. 4). He reported that the complexity of the polynomials and associated resultants overwhelmed his computer algebra software.

The lightweight mechanical structures that become possible with a combination of actuators, cables, springs, and struts motivate the integration of spatial robots and compliant mechanisms into a new theory for the analysis and synthesis of tensegrity systems.

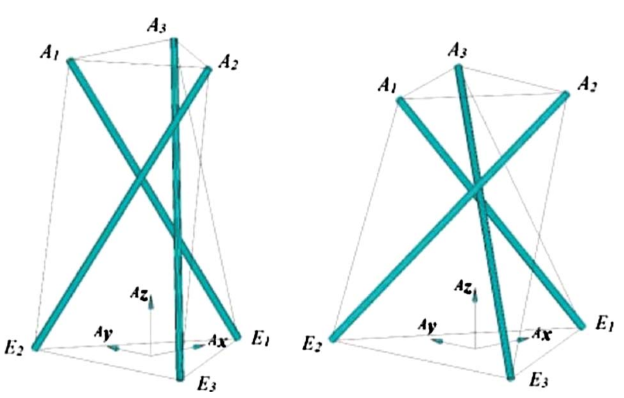

Fig. 3 A three-strut tensegrity system in its unloaded and loaded configurations [30]

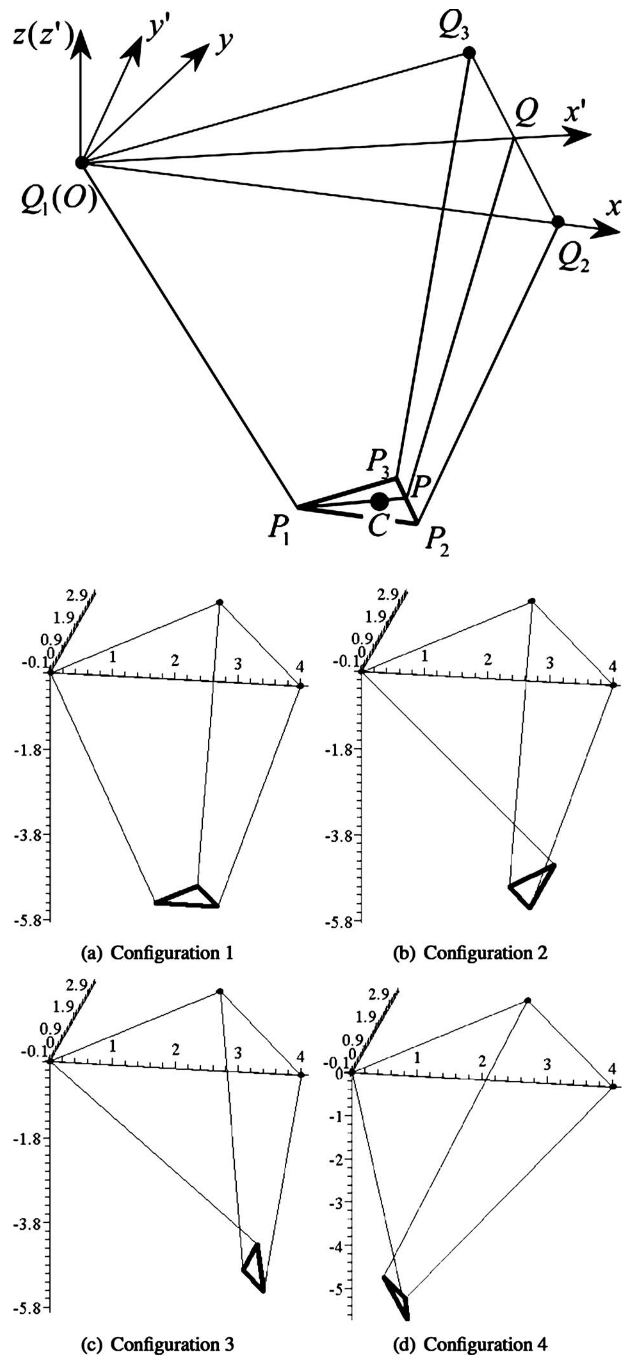

Fig. 4 Direct kinematic analysis of a cable system [35]

4 Conclusion

Solutions to the inverse kinematics of a general serial chain robot and to the direct kinematics of the general Stewart–Gough platform, which yielded polynomials of degree 16 and degree 40, respectively, were major advances in the last century. Less than 10 years later, researchers are deriving and solving polynomial systems for the analysis and design of robotic systems, compliant mechanisms, and tensegrity systems that have total degrees in millions. Advances in computational speed and effective algorithms to process these solutions promise new technologies and products well into the 21st century.

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