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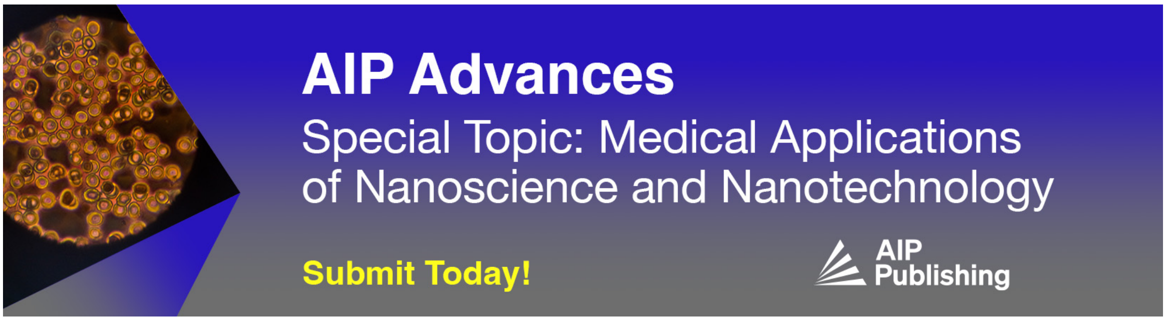

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An inverse problem of temperature estimation for the combination of the linear and nonlinear resistances

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The subject of the theoretical analysis presented in this paper is an analytical approach to the temperature estimation, as an inverse problem, for different thermistors – linear resistances structures: series and parallel ones, by the STFT - Special Trans Functions Theory (S.M. Perovich). The mathematical formulae genesis of both cases is given. Some numerical and graphical simulations in MATHEMATICA program have been realized. The estimated temperature intervals for strongly determined values of the equivalent resistances of the nonlinear structures are given, as well. *Copyright 2011 Author(s). This article is distributed under a Creative Commons Attribution 3.0 Unported License*. [doi:10.1063/1.3589909]

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I. INTRODUCTION

The main subject of the theoretical analysis presented here is an analytical approach to the thermistors – linear resistors series and parallel combinations analysis, preferably oriented to the ambient temperature estimation. This analytical approach is based on the Special Trans Functions Theory. Accordingly, within this section few introductory observations are given.

Thermistors are solid temperature sensors that behave like temperature-sensitive electrical resistors. They are type of resistors with resistance which is proportional to the temperature. Though, their name is a contraction of "thermal" and "resistors". Thermistors are widely used as inrush current limiters, temperature sensors, self-resetting over current protectors, and self-regulating heating elements. There are basically two broad types, negative temperature coefficient (NTC), used mostly in the temperature sensing and positive temperature coefficient (PTC), used mostly in electric current control. Because they are very small, thermistors are used inside many other devices as temperature sensing and correction elements, as well as, in specialty temperature sensing probes for science and technology.[1]

In this paper some different thermistors-linear resistances structures have been employed in unknown ambient temperature intervals estimation analytically, for given boundaries of the equivalent resistances of above noted structures, by the application of the STF Theory.[3–9]

Let us consider, now, both nonlinear resistances structures in more details.

II. NONLINEAR RESISTANCES STRUCTURES FOR SERIES CASE

Within this section, the problem of the ambient temperature estimation analytically, for the series case combination of K posistors (linear resistors) and M thermistors shall be presented.

Namely, for most linear resistors (posistors) the following empirical equation holds very closely:

$$R_m = R_{0m}\left(1 + \alpha_m\left(T - T_o\right)\right)$$

where, R_m is the resistance at temperature T; R_{0m} is the resistance at the reference temperatureT_0; T is the temperature, and α_m is the constant depending on material. Consequently, for K different posistors we have for equivalent resistance the following formulae

$$R_K = A_K + B_K T \tag{1}$$

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where

$$A_K = \sum_{m=0}^{K} R_{om}\left(1 - \alpha_m T_o\right); \quad B_K = \sum_{m=0}^{K} R_{om}\alpha_m.$$

The thermistor is a thermally sensitive variable resistor made of a ceramic like semi conducting material. Unlike metals, thermistors *respond negatively to temperature*. As the temperature rises, the thermistor resistance decreases. The temperature-resistance function for a thermistor is given by the relationship:

$$\mathrm{R_T} = \mathrm{R}_\infty \exp\left(\frac{\beta}{\mathrm{T}}\right) \tag{2}$$

Where, R_T is the resistance at temperature T, in [^{0}K]; R_∞ is the resistance at temperature $T = 0^0 K$, and β is a constant. The constant β depends on the type of the thermistor.

Thermistors are useful for compensating electrical circuits for changing ambient temperature largely because of the negative temperature characteristics possessed by most electrical components. Of course, environment (heat transfer condition and heat dynamics) is a major factor in an actual application, i.e. in nonlinear resistance structures analysis, because it is more important in sense of the physical aspects of the considering problem. On the other hand the high sensitivity possessed by thermistors permits application of very simple electrical circuits for measurement of temperature.

The nonlinear functional equations for thermistors-linear resistors structure, by equations (1) and (2), for K resistors and M thermistors, take the form:

$$R_u = A_K + B_K T + M \cdot R_\infty \exp\left(\frac{\beta}{T}\right). \tag{3}$$

After some simple modifications, from equation (3), we have the following nonlinear functional equation:

$$Ye^{-Y} - DY = Ce^{-Y}; \quad Y = \frac{\beta}{T} \tag{4}$$

where,

$$C = \frac{\beta B_K}{R_u - A_K}; \quad D = \frac{M R_\infty}{R_u - A_K}.$$

The nonlinear functional equation (4) has the analytical closed-form solutions under the condition:$1 > \mathrm{C} + \mathrm{D}\exp(1)$. Let us note that the nonlinear functional equation (4) is analytically solvable by application of the Special Trans Function Theory.[3–10] We remark that equation (4) has two real solutions ($\mathrm{Y} < 1, \mathrm{T} > \beta$ and $\mathrm{Y} > 1, \mathrm{T} < \beta$), but the subject matter of this paper is the first solution ($\mathrm{Y} < 1, \mathrm{T} > \beta$). The second solution, $\mathrm{Y} > 1, \mathrm{T} < \beta$, will be subject of one of the fore coming papers. Consequently, for $\mathrm{Y} < 1, \mathrm{T} > \beta$, the analytical closed-form solution of the equation (4) takes the form:

$$\mathrm{Y} = \lim_{\mathrm{x}\to\infty}\left[\ln\left(\frac{\mathrm{F(C, D, x+1)}}{\mathrm{F(C, D, x)}}\right)\right] \tag{5}$$

where,

$$\mathrm{F(C, D, x)} = (\mathrm{D}\varphi(\mathrm{C, D, x}) - \varphi(\mathrm{C, D, x-1})) \tag{6}$$

and,

$$\varphi(\mathrm{C, D, x}) = \sum_{\mathrm{n}=0}^{[\mathrm{x}]} \mathrm{n}! \sum_{\mathrm{k}=0}^{\mathrm{n}} \frac{(-1)^{\mathrm{k}}\mathrm{C}^{\mathrm{k}}(\mathrm{x}-\mathrm{n})^{\mathrm{k}}}{\mathrm{D}^{\mathrm{n}}\mathrm{k}!\mathrm{k}!(\mathrm{n}-\mathrm{k})!} \tag{7}$$

where [x] denotes the greatest integer less than or equal to x.

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A. Analytical closed-form solution of equation (4) for $Y < 1, T > \beta$

This section is organized as follows: an overview of the analytical closed form solutions is firstly presented, so the reader may inspect the formulae without previous rigorous concerning the derivation. The detailed derivation of the formulae is then presented. The outline of the derivation is based on the fact that the Special Trans Function Theory can be applied for arbitrary Thermistors - Linear Resistances Structures (TLRS) in a straightforward manner:

- determining the suitable equation for identification (EQID- equation for identification) of temperature nonlinear functional equations for TLRS in form of a partial differential equation;
- finding the analytical closed form solution to this chosen EQID;
- predicting the asymptotic solution of partial differential equation for identification, and finally,
- choosing the optimal equalization between unique solution and asymptotic solution.

As it was noted previously, the nonlinear functional equation for some nonlinear resistances structures takes the form (4). This nonlinear functional equation for $Y < 1,\ T > \beta$ has the analytical closed-form solution in the form:

$$Y = \mathrm{trans_T(C, D)} \tag{8}$$

where, $\mathrm{trans_T(C, D)}$ is a new special tran function defined as:

$$\mathrm{trans_T(C, D)} = \lim_{x\to\infty}\left[\ln\left(\frac{F(C, D, x+1)}{F(C, D, x)}\right)\right].$$

Or, more explicitly equation (8) takes the form

$$Y = trans_T(\alpha_m, R_{om}, \beta, R_\infty, M), \quad m = 1, \ldots K.$$

The practical numerical model for calculation of the transcendental number Y takes the form:

$$\langle Y\rangle_{[G]} = \langle trans_T(C, D)\rangle_G = \left\langle \ln\left(\frac{F(C, D, x+1)}{F(C, D, x)}\right)\right\rangle_{[G]} \tag{9}$$

where, $\langle Y\rangle_{[P]}$ denotes the numerical value of number Y given with $[G]$ accurate digits where the error function G is defined like: $G = \left|Ye^{-Y} - DY - Ce^{-Y}\right|$, and it satisfies the inequality: $G \le g_m$. Here g_m is an arbitrary small and positive, real number. Because the $\langle Y\rangle_{[P]}$ denotes the numerical value of number Y given with $[G]$ accurate digits, formulae (9) represents the numerical structures for transcendental number Y. The choice of x controls the number of accurate digits for the Y, and satisfies the error criteria. On the other hand, the number of accurate digits in the numerical structure of Y, is practically, determined by physical requirements for measurement ambient temperature level.

B. Equation for identification (EQID) in the form of the differential equation

The transcendental equation (4) can be identified with a differential equation of type:

$$F'(C, D, x-1) - DF'(C, D, x) - CF(C, D, x-1) = 0. \tag{10}$$

The differential equation (10) is analytically solvable using a Laplace transform. The Laplace transform to the differential equation (10) takes the form:

$$\Phi(C, D, s) = \frac{(e^{-s} - D)F(0)}{se^{-s} - Ds - Ce^{-s}}.$$

After a simple modification, the above equation takes the form:

$$\Phi(C, D, s) = \frac{(D - e^{-s})F(0)}{Ds}\sum_{n=0}^{\infty}\frac{e^{-ns}}{D^n}\sum_{k=0}^{n}\frac{(-1)^k C^k n!}{s^k (n-k)!k!}.$$

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By inverse Laplace transform approach,[2] from above equation, we have:

$$\mathrm{F(C, D, x)} = \left(\frac{\mathrm{F(0)}}{\mathrm{D}}\right)(\mathrm{D}\varphi(\mathrm{x}) - \varphi(\mathrm{x}-1))$$

where,

$$\varphi(\mathrm{C, D, x}) = \sum_{\mathrm{n}=0}^{[\mathrm{x}]} \frac{\mathrm{n}!}{\mathrm{D^n}} \sum_{\mathrm{k}=0}^{\mathrm{k}=\mathrm{n}} \frac{(-1)^{\mathrm{k}} \mathrm{C^k}\, (\mathrm{x}-\mathrm{n})^{\mathrm{k}}}{(\mathrm{n}-\mathrm{k})!\mathrm{k}!\mathrm{k}!}.$$

The above series is the unique analytical solution of the differential equation (10). Or, more explicitly, equation (10) takes the form:

$$\varphi(\mathrm{C, D, x}) = \sum_{\mathrm{n}=0}^{\infty} \frac{\mathrm{n}!}{\mathrm{D^n}} \sum_{\mathrm{k}=0}^{\mathrm{n}} \frac{(-1)^{\mathrm{k}} \mathrm{C^k} (\mathrm{x}-\mathrm{n})^{\mathrm{k}}}{\mathrm{k}!\mathrm{k}!(\mathrm{n}-\mathrm{k})!} \mathrm{H}(\mathrm{x}-\mathrm{n}) \tag{11}$$

where, $H(x-n)$ is the Heaviside's unit function defined as: $\mathrm{H(x-n)} = \begin{cases} 1 & \text{for } \mathrm{x} > \mathrm{n} \\ 0 & \text{for } \mathrm{x} < \mathrm{n}. \end{cases}$.

On the other hand, the asymptotic solution of the form:

$$\mathrm{F_{as}(C, D, x)} = \mathrm{F_{aso}} \exp(\mathrm{x}) \tag{12}$$

satisfies the differential equation (10), under the condition that Y satisfies the equation (4). According to the unique solution principle and function theory, we have:

$$\lim_{x\to\infty} F_{as}(C, D, x) = \lim_{x\to\infty} (F(C, D, x)). \tag{13}$$

C. General STFT scheme

From the prevision section it is clear that by applying the unique solution principle, we have that, analytical solution (6) is convergent to the asymptotic solution (12). Analogically, we have:

$$\mathrm{Y} = \lim_{\mathrm{N}\to\infty} \left\{ \ln\left[\frac{\mathrm{F(C, D, N+1)}}{\mathrm{F(C, D, N)}}\right]\right\} \tag{14}$$

where,

$$F(C, D, N) = \left(\frac{F(0)}{D}\right)(D\varphi(C, D, N) - \varphi(C, D, N-1)) \tag{15}$$

and,

$$\varphi(C, D, N) = \sum_{n=0}^{N} \frac{n!}{D^n} \sum_{k=0}^{n} \frac{(-1)^k C^k (N-n)^k}{k!k!(n-k)!} \tag{16}$$

where, N is a positive integer, when the error function defined as:

$$G \triangleq \left| Ye^{-Y} - DY - Ce^{-Y} \right|$$

satisfies inequality: $G \le g_m$, where, $\mathrm{g_m}$ is a real, positive arbitrary small number. By expression (14), we introduce the special trans function in form:

$$\mathrm{Y} = \mathrm{trans_T(C, D)} = \lim_{\mathrm{x}\to\infty}\left[\ln\left(\frac{\mathrm{F(C, D, N)}}{\mathrm{F(C, D, N-1)}}\right)\right]$$

or

$$Y = trans_T(C, D) = \lim_{x\to\infty}\left[\ln\left(\frac{D\varphi(C, D, N) - \varphi(C, D, N-1)}{D\varphi(C, D, N-1) - \varphi(C, D, N-2)}\right)\right].$$

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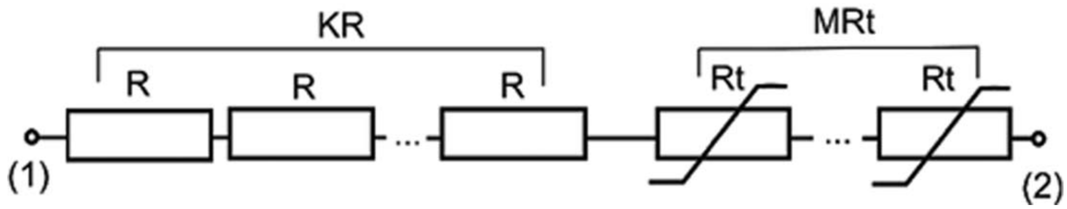

FIG. 1. The series of K linear resistances and M thermistors

More explicitly we have

$$Y=\lim_{x\to\infty}\left[\ln\left(\frac{D\sum_{n=0}^{N}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-n)^k}{k!k!(n-k)!}-\sum_{n=0}^{N-1}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-1-n)^k}{k!k!(n-k)!}}{D\sum_{n=0}^{N-1}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-1-n)^k}{k!k!(n-k)!}-\sum_{n=0}^{N-2}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-2-n)^k}{k!k!(n-k)!}}\right)\right]$$

In this way, the formula for unknown ambient temperature takes the form:

$$\mathrm{T}=\frac{\beta}{\mathrm{Y}}=\frac{\beta}{\mathrm{trans_T(C,\,D)}}. \tag{17}$$

Or, form

$$T=\frac{\beta}{\lim\limits_{x\to\infty}\left[\ln\left(\frac{D\sum_{n=0}^{N}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-n)^k}{k!k!(n-k)!}-\sum_{n=0}^{N-1}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-1-n)^k}{k!k!(n-k)!}}{D\sum_{n=0}^{N-1}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-1-n)^k}{k!k!(n-k)!}-\sum_{n=0}^{N-2}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-2-n)^k}{k!k!(n-k)!}}\right)\right]}$$

This is an analytical closed-form solution to the transcendental equation (4), for $\mathrm{T} > \beta$. For practical calculation the above equation takes the form

$$\langle T\rangle_{[G]}=\left\langle\frac{\beta}{\ln\left(\frac{D\sum_{n=0}^{N}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-n)^k}{k!k!(n-k)!}-\sum_{n=0}^{N-1}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-1-n)^k}{k!k!(n-k)!}}{D\sum_{n=0}^{N-1}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-1-n)^k}{k!k!(n-k)!}-\sum_{n=0}^{N-2}\frac{n!}{D^n}\sum_{k=0}^{n}\frac{(-1)^kC^k(N-2-n)^k}{k!k!(n-k)!}}\right)}\right\rangle. \tag{18}$$

Numerical results obtained from above formulae are obtained with high accuracy, controlled by integer N.

In the following section an appropriate numerical example has been given.

III. SOME NUMERICAL ANALYSIS OF FORMULAE (18)

Within this section, we used a series resistance combination of K linear resistors (posistors) and M thermistors in aim to determine temperature interval for determined variations of equivalent resistance (Figure 1).

For given all relevant circuit parameters, by the application of previously proposed and STF Theory we are in position to determine ambient temperature with high accuracy. Obtained numerical results by MATHEMATICA program are given in Table I. We remark that different values of integer K and M simulated the different parameters of thermistors and posistors. It is obvious nonsensitiv presentation but in sense of global analysis it is satisfactory. Of course, for posistors it is irrelevant because these resistances temperature functions are linear functions.

In the following table (Table II), some results of the numerical analysis are presented. These results show that, in this case, for relatively small ambient temperature variations, we have relatively large equivalent resistance changes. This is interesting, since it shows that the considered resistors structure can be used for temperature measurement.[11] On the other hand, the subject of the theoretical analysis in this paper is the case when the equivalent resistance of the series combination of posistor

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TABLE I. The simulation results for the linear thermometer combination

Ru [Ω]	Y	T [K]	G
7000	7.018223	569.945	7.569 E-08
7100	7.033041	568.744	7.371 E-08
7200	7.047633	567.566	7.178 E-08
7300	7.062009	566.411	6.993 E-08
7400	7.076201	565.275	6.814 E-08
7500	7.090186	564.160	6.649 E-08
7600	7.103979	563.065	6.482 E-08
7700	7.117597	561.987	6.323 E-08
7800	7.131008	560.930	6.171 E-08
7900	7.144349	559.883	6.029 E-08
8000	7.157331	558.868	5.886 E-08

TABLE II. The simulation results for the combination of two linear resistances and three thermistors

K (for M=2)	2	4	6	8	10	12	14
ΔT [K]	10.903	11.329	11.798	12.330	12.970	13.876	16.001
M (for K=3)	2	4	6	8	10	12	14
ΔT [K]	11.965	14.689	16.737	18.461	19.989	21.382	22.676

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and thermistor is changing due to the temperature increasing less than the resistances of posistor and thermistor being taken into account separately. Accordingly, the thermistors are commonly suitable for compensation of the environment temperature changes and its impacts to the signals in electrical circuits.[12] In other words, thermistors are useful as elements of electrical circuits for ambient temperature variations compensation that is mostly implied by the negative thermometers' temperature coefficients.

Also, some graphical presentations of evaluated ambient temperature(s) for nonlinear resistance structure of two thermistors and three posistors, for different values of α and β coefficients are given in Figures 2, 3, and 4.

IV. THE PARALLEL STRUCTURE OF THERMISTOR – LINEAR RESISTANCE

Analogy to the previous notation (1) let us present the linear resistance (posistor) in the parallel combination as:

$$\mathrm{R} = \mathrm{A} + \mathrm{BT}$$

where, $\mathrm{A} = \mathrm{R}_0 - \mathrm{R}_0 \cdot \alpha \mathrm{T}_0$, $\mathrm{B} = \alpha \cdot \mathrm{R}_0$, and R is the resistance at temperature T; R_0 is the resistance at the reference temperature; T is the temperature, and α is the constant depending on material.

The temperature dependence for a thermistor is given, in the same manner as in the previous section (2):

$$\mathrm{R}_\mathrm{T} = \mathrm{R}_\infty \exp\left(\frac{\beta}{\mathrm{T}}\right)$$

where, R_T is the resistance at temperature T, in [^{0}K]; R_∞ is the resistance at temperature $T = 0^0 K$, and β is a constant. As we noted in section II, the constant β depends on the thermistor type.

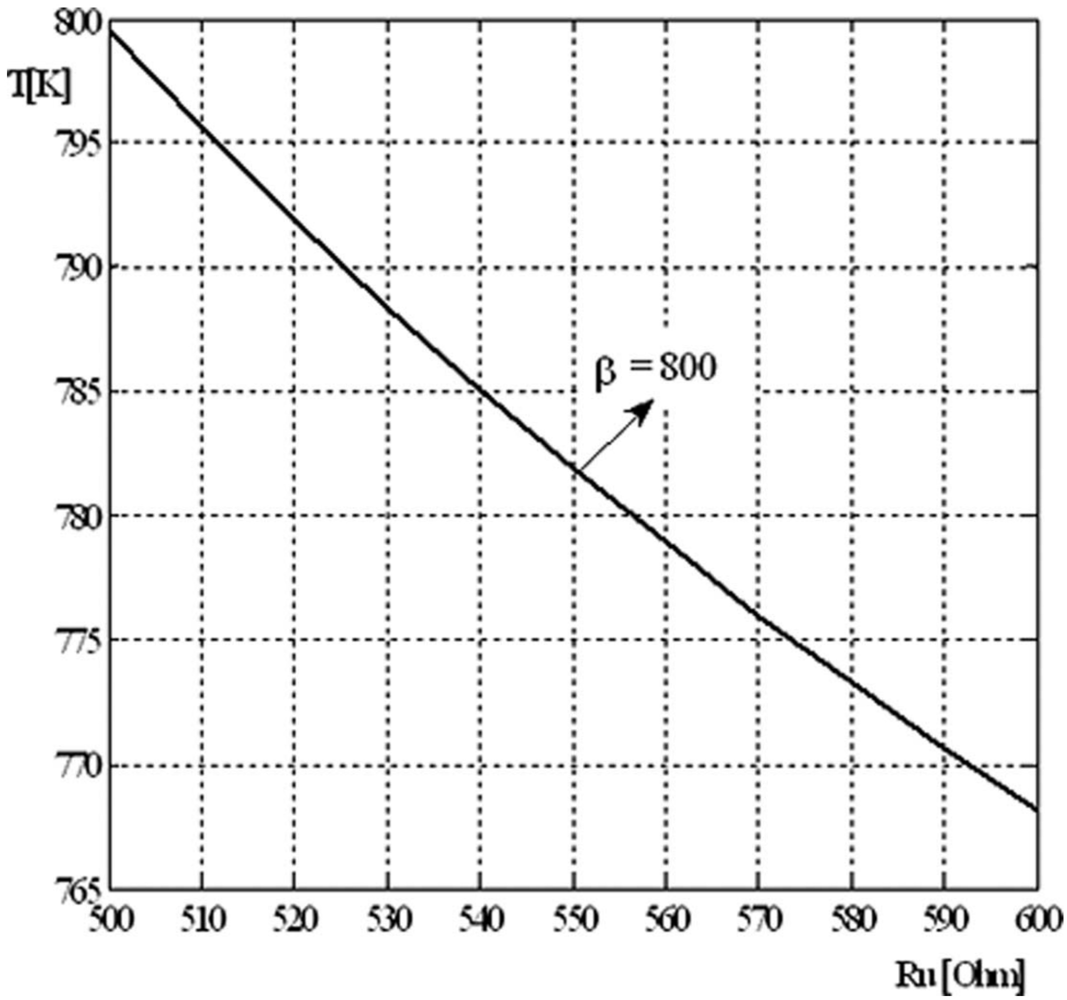

FIG. 2. Functional presentation of the form T=f (R_u)

The parallel thermistor - linear resistance (posistor) combination, by two latter equations takes the form:

$$\mathrm{Ru} = \frac{(\mathrm{A} + \mathrm{BT}) \cdot \mathrm{R}_{\infty} \exp\left(\frac{\beta}{\mathrm{T}}\right)}{\mathrm{A} + \mathrm{BT} + \mathrm{R}_{\infty} \exp\left(\frac{\beta}{\mathrm{T}}\right)}. \tag{19}$$

After a simple modification, from equation (19), we have the following nonlinear functional equation:

$$\Psi e^{-\Psi} + a\Psi - b = 0; \quad \Psi = \frac{\beta}{\mathrm{T}} + \frac{\beta \mathrm{B}}{\mathrm{A}} \tag{20}$$

where, $a = R_{\infty}(1 - \frac{A}{R_u}) \cdot e^{-\frac{\beta B}{A}}$; $b = \frac{R_{\infty}\beta B}{A} \cdot e^{-\frac{\beta B}{A}}$. We remark that equation (20) is an inverse problem.

Let us note that the nonlinear functional equation (20) is analytically solvable by application of the Special Trans Function Theory (STFT).[3–10] The analytical closed-form solution of the equation (20) takes the form:

$$\Psi = \lim_{x \to \infty} \left[\ln \left(\frac{\varphi(a, b, x+1)}{\varphi(a, b, x)} \right) \right] \tag{21}$$

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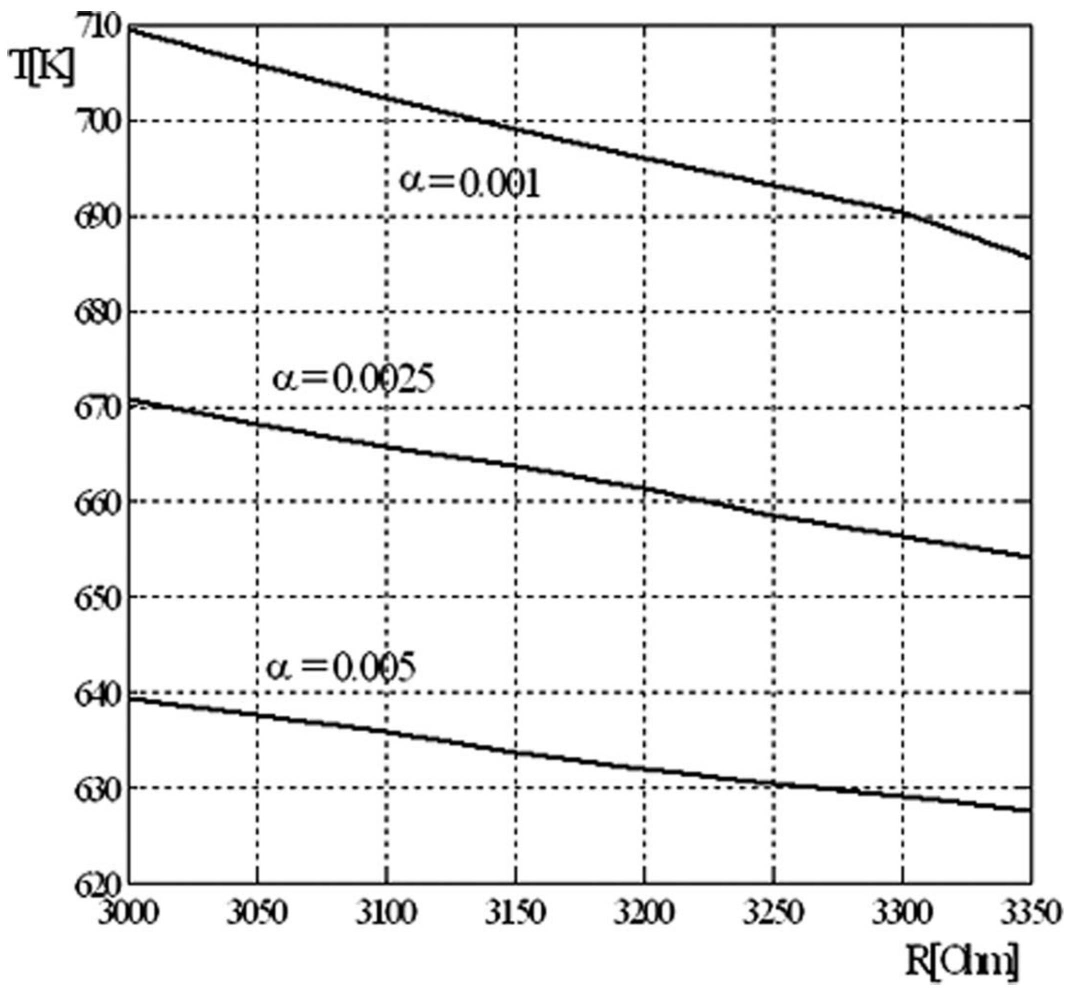

FIG. 3. The evaluated temperatures for different coefficients α

where,

$$\varphi(\mathrm{a, b, x}) = \frac{1}{\mathrm{a}}(\varphi_0(\mathrm{a, b, x} - 1) - \varphi_0(\mathrm{a, b, x})) \tag{22}$$

and,

$$\varphi_0(\mathrm{a, b, x}) = \sum_{\mathrm{n}=0}^{[\mathrm{x}]} \sum_{\mathrm{k}=0}^{\mathrm{n}} \frac{\mathrm{n!b^{n-k}}\,(-1)^{\mathrm{k}}\,(\mathrm{x-k})^{\mathrm{n+k}}}{(\mathrm{n-k})!\mathrm{k!a^{n+k}}\,(\mathrm{n+k})!} \tag{23}$$

where, [x] denotes the greatest integer less than or equal to x.

A. The analytical closed-form solution of equation (20)

The nonlinear functional equation for simple parallel resistances combination takes the form (20). This nonlinear functional equation has the analytical closed-form solution in the form:

$$\Psi = tran_{PR}(a, b) \tag{24}$$

where, $tran_{PR}(a, b)$ is a new special tran function defined as:

$$\mathrm{tran_{PR}(a, b)} = \lim_{\mathrm{x}\to\infty}\left[\ln\left(\frac{\varphi(\mathrm{a, b, x}+1)}{\varphi(\mathrm{a, b, x})}\right)\right].$$

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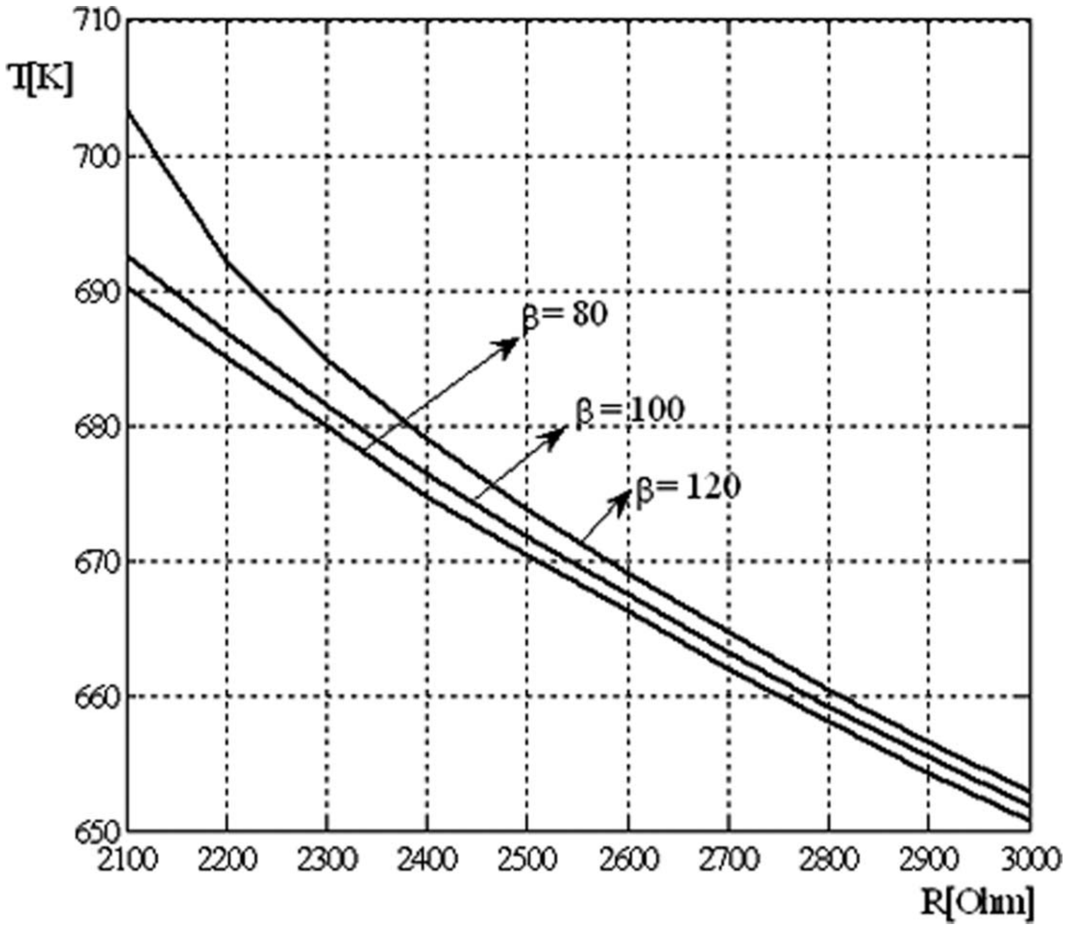

FIG. 4. The evaluated temperatures for different coefficients β

The practical numerical model for calculation of the transcendental number Ψ, takes the form:

$$\langle\Psi\rangle_{[G]} = \langle \ln\left(\frac{\varphi(a, b, x+1)}{\varphi(a, b, x)}\right)\rangle_{[G]} \tag{25}$$

where, $\langle\Psi\rangle_{[G]}$ is a value of the transcendental number Ψ given with [G] accurate digits, and G is error function, defined as:

$$G = \left|\Psi e^{-\Psi} + a\Psi - b\right| \leq \varepsilon$$

where, ε is an arbitrary small positive real number.

Consequently, formulae (25) and (23) represent the numerical structure of transcendental number ψ. The choice of x controls the number of accurate digits of ψ. On the other hand, the number of accurate digits in the numerical structure of ψ, is practically, determined by physical requirements for measurement ambient temperature level.

B. The differential equation as EQID

The transcendental equation (20) can be identified with a differential equation of type:

$$\varphi'(x-1) + a\varphi'(x) - b\varphi(x) = 0. \tag{26}$$

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The differential equation (26) is analytically solvable using a Laplace transform. Thus, the Laplace transform to the differential equation (26) takes the form:

$$\Phi\,(\mathrm{s}) = \Phi_0\,(\mathrm{s})\left(\mathrm{e}^{-\mathrm{s}} + \mathrm{a}\right), \text{ where}$$

$$\Phi_0(\mathrm{s}) = \frac{1}{\mathrm{as}}\sum_{\mathrm{n}=0}^{\infty}\sum_{\mathrm{k}=0}^{\mathrm{n}}\frac{\mathrm{n}!\mathrm{b}^{\mathrm{n}-\mathrm{k}}\,(-1)^{\mathrm{k}}\,\mathrm{e}^{-\mathrm{ks}}}{(\mathrm{n}-\mathrm{k})!\mathrm{k}!\mathrm{a}^{\mathrm{n}}\mathrm{s}^{\mathrm{n}-\mathrm{k}}}.$$

From obtained Laplace transform, in real domain, by inverse Laplace transform, we have:

$$\varphi_0\,(\mathrm{x}) = \frac{1}{\mathrm{a}}\sum_{\mathrm{n}=0}^{[\mathrm{x}]}\sum_{\mathrm{k}=0}^{\mathrm{n}}\frac{\mathrm{n}!\mathrm{b}^{\mathrm{n}-\mathrm{k}}\,(-1)^{\mathrm{k}}\,(\mathrm{x}-\mathrm{k})^{\mathrm{n}-\mathrm{k}}}{(\mathrm{n}-\mathrm{k})!\mathrm{k}!\mathrm{a}^{\mathrm{n}}\,(\mathrm{n}-\mathrm{k})!}$$

and,

$$\Psi = \lim_{\mathrm{x}\to\infty}\left(\ln\left(\frac{\varphi\,(\mathrm{x}+1)}{\varphi\,(\mathrm{x})}\right)\right) \tag{27}$$

where

$$\varphi(\mathrm{a},\mathrm{b},\mathrm{x}) = \frac{1}{\mathrm{a}}(\varphi_0(\mathrm{a},\mathrm{b},\mathrm{x}-1) + \mathrm{a}\varphi_0(\mathrm{a},\mathrm{b},\mathrm{x})). \tag{28}$$

C. General STFT scheme

By applying the unique solution principle, we have that, for large x, analytical solution (21) is convergent to the asymptotic solution of the form $\varphi_{as}(x) = \varphi_{aso}\exp(\psi x)$. Analogically, for x = N and equation (27), we have:

$$\Psi = \lim_{N\to\infty}\left\{\ln\left[\frac{\varphi(a,b,N+1)}{\varphi(a,b,N)}\right]\right\} \tag{29}$$

where,

$$\varphi(\mathrm{a},\mathrm{b},\mathrm{N}) = \frac{1}{\mathrm{a}}\sum_{\mathrm{n}=0}^{[\mathrm{N}]}\sum_{\mathrm{k}=0}^{\mathrm{n}}\frac{\mathrm{n}!\mathrm{b}^{\mathrm{n}-\mathrm{k}}\,(-1)^{\mathrm{k}}\,(\mathrm{N}-\mathrm{k})^{\mathrm{n}-\mathrm{k}}}{(\mathrm{n}-\mathrm{k})!\mathrm{k}!\mathrm{a}^{\mathrm{n}}\,(\mathrm{n}-\mathrm{k})!} \tag{30}$$

N is a positive integer, when the error function defined as:

$$G \overset{\triangle}{=} \left|\Psi e^{-\Psi} + a\Psi - b\right| \tag{31}$$

satisfies the inequality: $\mathrm{G}\,(\mathrm{a},\mathrm{b},\mathrm{x}) \leq \mathrm{g}_{\mathrm{m}}$, where g_{m} is a real, positive arbitrary small number.

In this way, the formula for ambient temperature takes the form:

$$\mathrm{T} = \frac{\beta}{\mathrm{tran}_{\mathrm{PR}}\,(\mathrm{a},\mathrm{b}) - \frac{\beta\mathrm{B}}{\mathrm{A}}}. \tag{32}$$

Finally, for practical calculation equation (32) takes the form

$$\langle T\rangle_{[G]} = \left\langle\frac{\beta}{tran_{PR}\,(a,b) - \frac{\beta B}{A}}\right\rangle_{[G]} \tag{33}$$

where $\langle T\rangle_{[G]}$ is value of temperature T given with [G] accurate digits. We remark that obtained temperature values are obtained with high accuracy, i.e., [G] is controlled by the integer N. More explicitly equation (33) takes the form

$$\langle \mathrm{T}\rangle_{[\mathrm{G}]} = \left\langle\frac{\beta}{\ln\left(\frac{1}{a}\sum\limits_{n=0}^{[N+1]}\sum\limits_{k=0}^{n}\frac{n!b^{n-k}(-1)^k(N+1-k)^{n-k}}{(n-k)!k!a^n(n-k)!}\right) - \ln\left(\frac{1}{a}\sum\limits_{n=0}^{[N]}\sum\limits_{k=0}^{n}\frac{n!b^{n-k}(-1)^k(N-k)^{n-k}}{(n-k)!k!a^n(n-k)!}\right) - \frac{\beta B}{A}}\right\rangle_{[\mathrm{G}]}.$$

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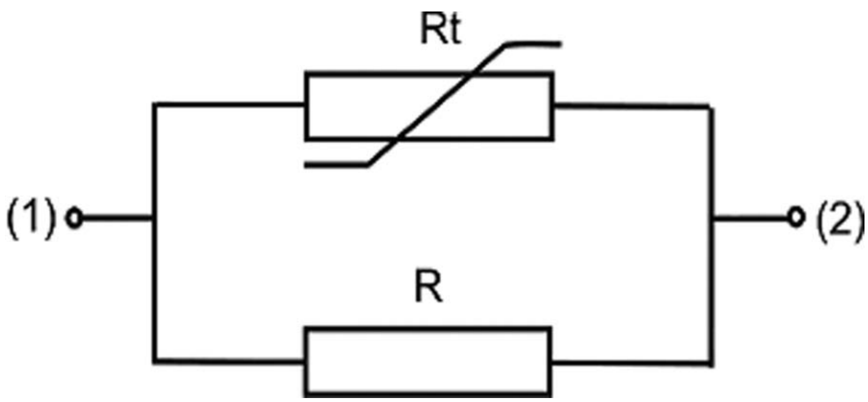

FIG. 5. The parallel structure of posistor - thermistor

TABLE III. The numerical simulation results for the parallel structure of posistor – thermistor

Ru [Ω]	Ψ	T [K]	G
500	22.827518	799.495	2.486 E-09
600	22.958546	768.157	1.830 E-09
700	22.930375	747.925	1.965 E-09
800	22.909756	733.780	2.065 E-09
900	22.894007	723.332	2.143 E-09
1000	22.881585	715.298	2.206 E-09
1100	22.871534	708.927	2.257 E-09
1200	22.863236	703.752	2.299 E-09
1300	22.856267	699.464	2.335 E-09
1400	22.850332	695.853	2.366 E-09
1500	22.845217	692.771	2.393 E-09

On the other hand, the equation (32) is the analytical closed-form solution of the transcendental equation (20).

V. THE NUMERICAL EXAMPLE FOR THE PARALLEL STRUCTURE OF POSISTOR - THERMISTOR

In this section, by the application of Special Trans Function Theory, we employed a numerical example for the parallel structure of one thermistor and one posistor (Figure 5), while the Ru variation has been given, as well as, the others, relevant for the circuit parameters.

By MATHEMATICA program, on the base of equation (33), we determined ambient temperature, as in the previous case of the series structures of posistors – thermistores, with high precision. The obtained numerical results are given in Table III.

In the next table (Table IV), some results of additional numerical analysis are presented. These results show that, in this case, for relatively small ambient temperature relative variations (expressed in %), we have relatively large equivalent resistance changes. The obtain results show the possibility of temperature measurement by this structure, with high accuracy, taking into account the accuracy of resistances measurements in the circuit, or the accuracy of the mathematical models of thermistor and posistor resistances. But, more rigorous analysis of this structure applicability in measuring temperature, in real conditions, shall be the subject of the forthcoming investigations.

Some graphical presentations for various parameter β and different intervals of Ru, are shown in Figure 6 and Figure 7, below.

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TABLE IV. Temperature changes (ΔT) due to changes of the equivalent resistance (ΔR_u) of the paralel thermistor-posistor combination

R_u [Ω]	$\Delta R_{ui} = R_u + \Delta_i, i = \overline{1,3}$			T	$T_{10\%}$	$T_{15\%}$	$T_{20\%}$	$\Delta T_{10\%}$	$\Delta T_{15\%}$	$\Delta T_{20\%}$
	$\Delta_1 = 10\%$	$\Delta_2 = 15\%$	$\Delta_3 = 20\%$							
500	550	575	600	799	781	774	768	0.18	0.25	0.31
750	825	863	900	740	730	726	723	0.10	0.14	0.17
1000	1100	1150	1200	715	708	706	703	0.07	0.09	0.12
2500	2750	2875	3000	676	674	673	672	0.02	0.03	0.04
5000	5500	5750	6000	664	663	663	662	0.01	0.01	0.02
7500	8250	8625	9000	660.964	660.294	660.004	659.738	0.06	0.09	0.01
10000	11000	11500	12000	659.127	658.629	658.413	658.216	0.005	0.007	0.009

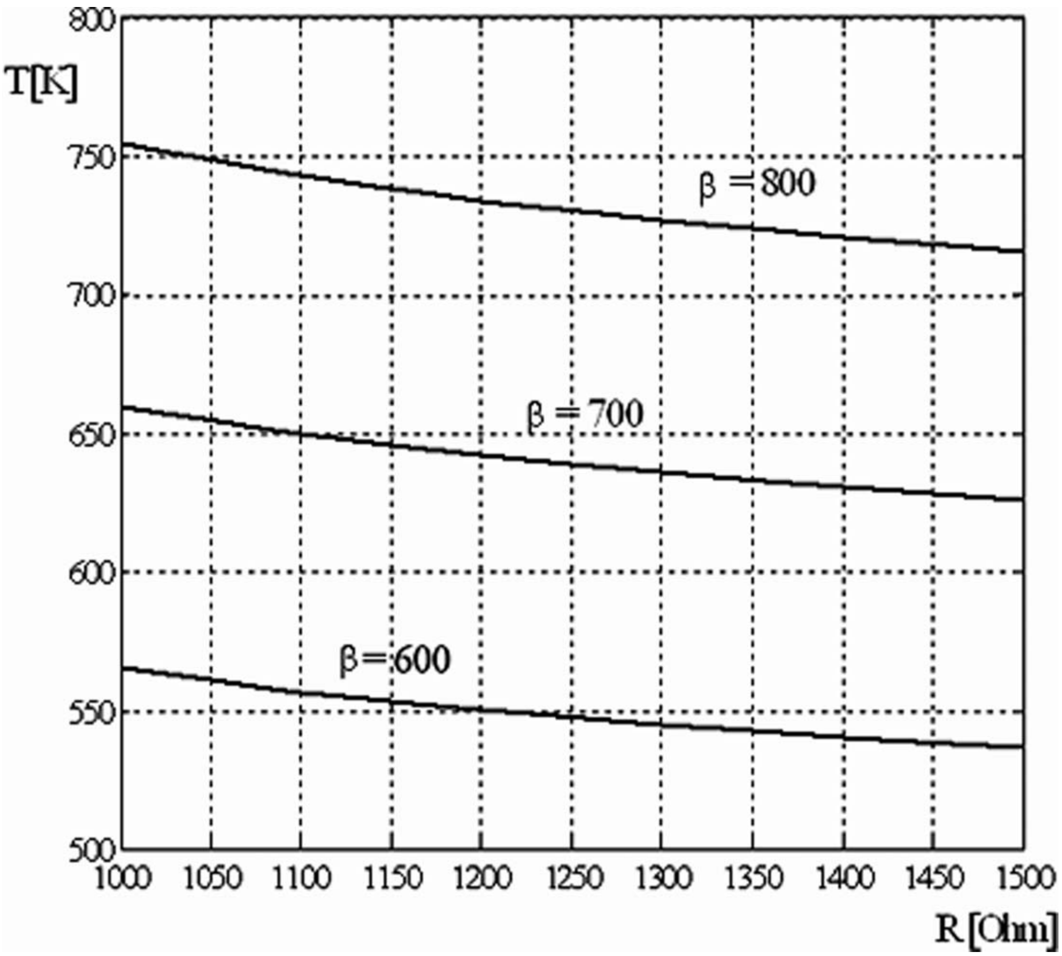

FIG. 6. The evaluated temperatures for different coefficient β and Ru intervals

VI. CONCLUSION

Within the common, classical analysis, the ambient temperature from nonlinear functional equations (4) and (20) are solvable by application of the approximation methods or various iterative procedures available in conventional numerical computation.

In this paper the thermistors – posistors (linear resistors) series and parallel combinations ambient temperatures are estimated analytically, in the closed form, by the application of the Special Trans Functions Theory (S.M. Perovich). In such way, the temperature is analytically obtainable

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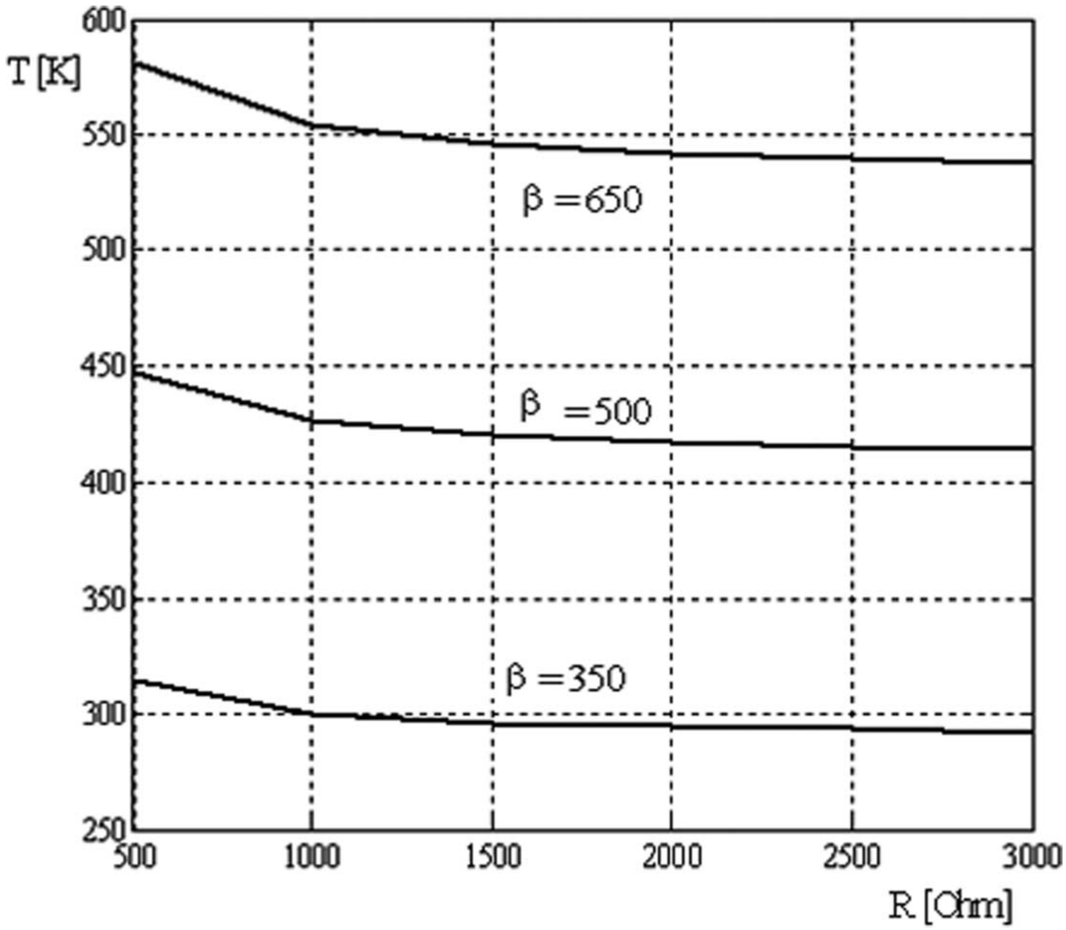

FIG. 7. The evaluated temperatures for different coefficient β and Ru intervals

by genesis of a new special tran function $trans_T(C, D)$, for the series resistors combination, i.e. by genesis of a new special tran function $tran_{PR}(a, b)$, for the parallel resistances combination. The numerical results obtained by application both special tran functions are of high accuracy. The STFT is the consistent theory applicable to the nonlinear functional equations for the nonlinear resistors structures analysis in the ambient temperature domain. In other words, the STFT is a novel and consistent theory for the evaluation of the ambient temperature for nonlinear- series and simple parallel resistors combinations.

Finally, it should be state that in the paper proposed STFT, should become a standard analytical method to the nonlinear functional equation for the ambient temperature intervals estimation (computation) for given intervals of equivalent resistances, to the broad temperature domain. Also, from the practical aspect, the presented method might be considered as a mathematical tool for compensation of ambient temperature impact to the signals in electrical circuits.

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