Discussion: "Effects of Various Parameters on Nanofluid Thermal Conductivity" (Jang, S. P., and Choi, S. D. S., 2007, ASME J. Heat Transfer, 129, pp. 617–623)

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Background Information

In a series of articles, Jang and Choi [1–3] listed and explained their effective thermal conductivity (k_{eff}) model for nanofluids. For example, in the 2004 article [1], they constructed a k_{eff} correlation for dilute liquid suspensions interestingly, based on kinetic *gas* theory as well as nanosize boundary-layer theory, the Kapitza resistance, and nanoparticle-induced convection. Three mechanisms contributing to k_{eff} were summed up, i.e., base-fluid and nanoparticle conductions as well as convection due to random motion of the liquid molecules. Thus, after an order-of-magnitude analysis, their effective thermal conductivity model of nanofluids reads

$$k_{\text{eff}} = k_f(1-\varphi) + k_{\text{nano}}\varphi + 3C_1\frac{d_f}{d_p}k_f\,\text{Re}_{d_p}^2\,\text{Pr}\,\varphi \tag{1}$$

where k_f is the thermal conductivity of the base fluid, φ is the particle volume fraction, $k_{\text{nano}}=k_p\beta$ is the thermal conductivity of suspended nanoparticles involving the Kapitza resistance, $C_1=6\times10^6$ is a constant (never explained or justified), d_f and d_p are the diameters of the base-fluid molecules and nanoparticles, respectively, Re_{d_p} is a "random" Reynolds number, and Pr is the Prandtl number. Specifically,

$$\text{Re}_{d_p} = \frac{\bar{C}_{\text{RM}}d_p}{\nu} \tag{2}$$

where $\bar{C}_{\text{RM}}$ is a random motion velocity and ν is the kinematic viscosity of the base fluid.

Jang and Choi [3] claimed that they were the first to propose Brownian motion induced nanoconvection as a key nanoscale mechanism governing the thermal behavior of nanofluids. However, they just added a random term to Eq. (1), actually quite small in magnitude for certain base liquids, although enhanced by the large factor 3 $C_1=18\times10^6$, while, independently, in the same year, Koo and Kleinstreuer [4] proposed their effective thermal conductivity model, based on *micromixing* induced by Brownian motion, followed by Prasher et al. [5] and others (see review by Jang and Choi [3]).

However, it should be noted that the validity of the different origins for the unusual thermal effect of nanofluids has been questioned (see Evans et al. [6] and Vladkov and Barrat [7], among others) as well as the actual k_{eff} increase as reported in experimental papers (see Venerus et al. [8] and Putnam et al. [9], among others). Controversies arose from using different experimental techniques (e.g., transient hot wire versus optical methods) and from phenomenological models relying more on empirical correlations rather than sound physics and benchmark experimental data.

In 2006, Jang and Choi [2] changed the thermal conductivity correlation slightly to

$$k_{\text{eff}} = k_f(1-\varphi) + k_p\beta\varphi + 3C_1\frac{d_f}{d_p}k_f\,\text{Re}_{d_p}^2\,\text{Pr} \tag{3}$$

where the volume fraction term φ is now missing in the last term. Most recently, Jang and Choi [3] tried to explain more clearly the modeling terms they had proposed. Their present thermal conductivity model reads

$$k_{\text{eff}} = k_f(1-\varphi) + k_p\beta\varphi + 3C_1\frac{d_f}{d_p}k_f\,\text{Re}_{d_p}^2\,\text{Pr}\,\varphi \tag{4}$$

Parameter Analysis

In the 2006 article [2], the random motion velocity, which is used to define the Reynolds number (see Eq. (2)), was defined as

$$\bar{C}_{\text{RM}} = \frac{2D_0}{l_f} \tag{5a}$$

while in the 2007 article [3], the authors changed the random motion velocity to

$$\bar{C}_{\text{RM}} = \frac{D_0}{l_f} \tag{5b}$$

where $D_0=\kappa_b T/3\pi\mu d_p$ is the diffusion coefficient given by Einstein [10], and l_f is the mean free path of the (liquid) base fluid. The mean free path of the base fluid is calculated from Kittel and Kroemer [11], which deals only with transport properties of ideal gases (see their Chap. 14):

$$l_f = \frac{3k_f}{\bar{c}\hat{C}_V} \tag{6a}$$

where $\bar{c}$ is the mean molecular velocity, and $\hat{C}_V$ is the heat capacity per unit volume. Although Eq. (6*a*) is certainly not applicable to liquids, the mean free path for (ideal) gases can also be written as

$$l_f = \frac{3k_f}{\rho c_v \bar{c}} \tag{6b}$$

with c_v being the thermal capacity at constant volume, where $\rho c_v \equiv \hat{C}_V$.

Model Comparisons

According to the parameters Jang and Choi [3] provided and the terms they explained, the effective thermal conductivities of CuO-water and Al_2O_3-water nanofluids were calculated and compared. Figures 1 and 2 provide comparisons of Jang and Choi's 2007 model [3] with the experimental data sets of Lee et al. [12] for CuO-water and Al_2O_3-water nanofluids, respectively. Two random motion velocities $\bar{C}_{\text{RM}}$ were compared, where the dashed line relates to Eq. (5*a*) while the solid line is based on Eq. (5*b*). Clearly, these comparisons do not match the results given by Jang and Choi [3] in their Fig. 2, unless new matching coefficients in the third term of Eq. (4) are applied. Specifically, the first two terms contribute very little, i.e., $\Sigma_{i=1}^2 \text{term}_i/k_f \approx 0.99$. Is the contribution of the particle's thermal conductivity really that small? Many researchers indicated that the higher thermal conductivity of the nanoparticles is a factor in enhancing the effective thermal conductivity (Hong et al. [13], Hwang et al. [14]). It has to be stressed that all the data comparisons are based on the thermal properties provided by Jang and Choi [3] in Table 1. However, thermal conductivity values found in the literature indicated

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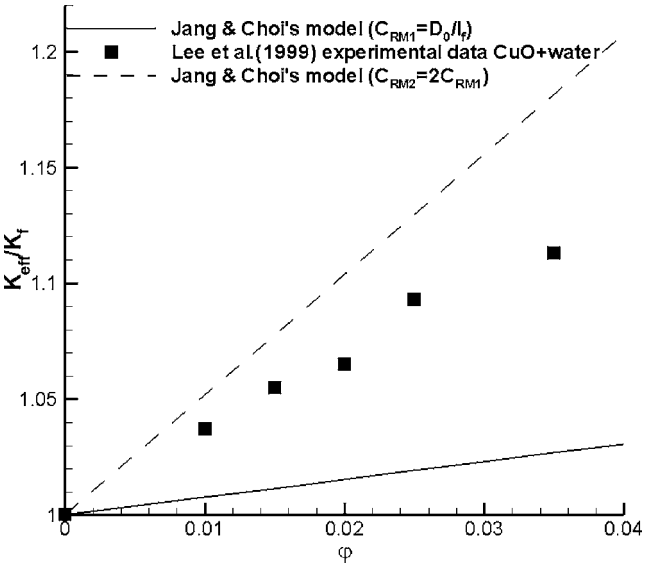

Fig. 1 Comparison of the experimental data (Lee et al. [12]) for CuO-water nanofluids with Jang and Choi's [3] model for different random motion velocity definitions

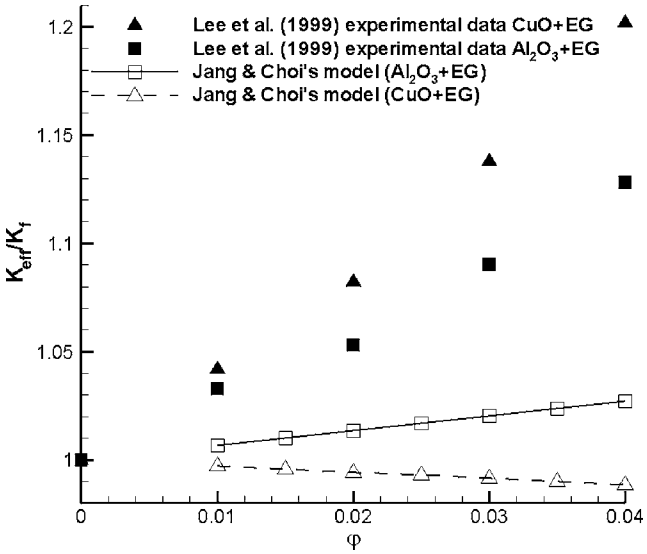

Fig. 3 Comparison of the experimental data (Lee et al. [12]) for Al_2O_3-EG and CuO-EG nanofluids with Jang and Choi's [3] model

32.9 W/m K for CuO (Wang et al. [15]) and, for Al_2O_3, a range of 18–35 W/m K depending on the purity, i.e., 94–99.5%.[1] When using the more reasonable particle thermal conductivity values in the model of Jang and Choi [3], only small differences were observed.

Now, in contrast to water, if the base fluid is changed to ethylene glycol (EG), the third term in Eq. (4) is suddenly of the order of 10^{-6}, i.e., it does not contribute to the effective thermal conductivity when compared to the first two terms (10^{-1} and 10^{-3}). The nondimensionalized effective thermal conductivity of CuO-EG nanofluids is about 0.99 for all volume fraction cases, while for Al_2O_3-EG nanofluids, k_{eff}/k_f is slightly higher at approximately 1.015. Both graphs are well below the experimental data of Lee et al. [12], as shown in Fig. 3. The larger EG viscosity provided a much smaller Reynolds number, which almost eliminates the third term.

For the experimental result of Das et al. [16], Jang and Choi [3] compared their model for Al_2O_3 particles with a volume fraction of 1% in their Fig. 7. Considering the temperature influence on the thermal characteristics of base fluid (water), Fig. 4 provides again an updated comparison. If we consider $\bar{C}_{\text{RM}}=2D_0/l_f$, indicated with the dashed curve, the model shows a good agreement in the lower temperature range; however, the model prediction fails when the temperature is higher than 40°C. Figure 5 shows the comparison of Jang and Choi's model with the experimental data of Das et al. [16] when the volume fraction is 4%. Clearly, their model does not match the experimental results well.

[1]http://www.accuratus.com/alumox.html

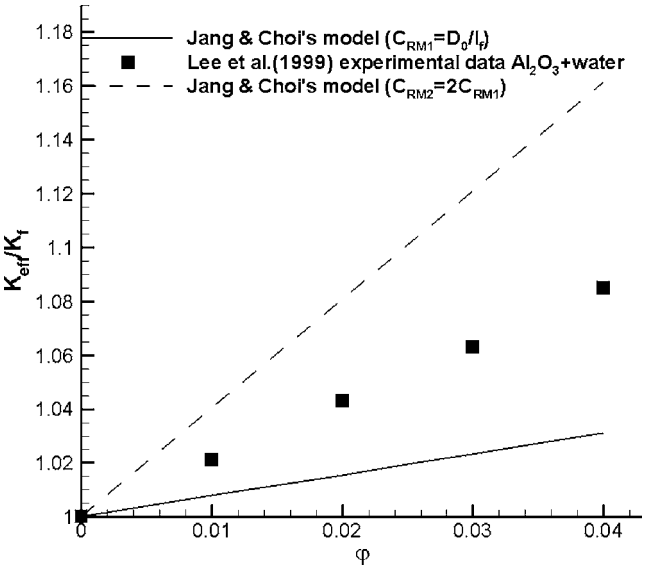

Fig. 2 Comparison of the experimental data (Lee et al. [12]) for Al_2O_3-water nanofluids with Jang and Choi's [3] model for different random motion velocity definitions

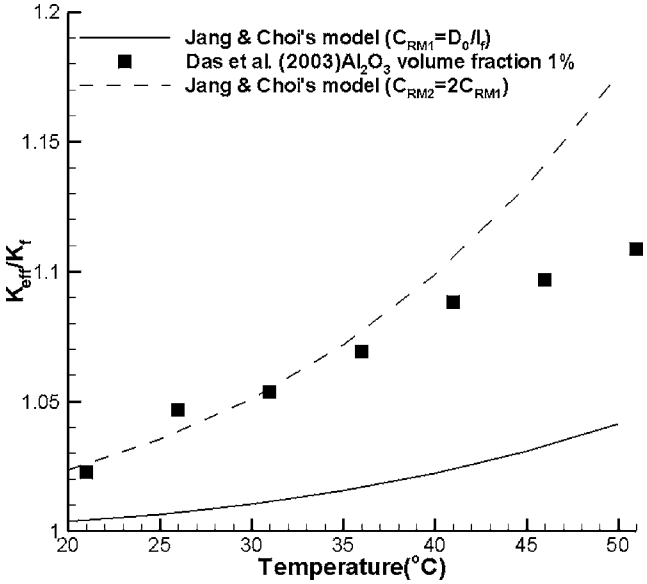

Fig. 4 Comparison of the experimental data (Das et al. [16]) for 1% Al_2O_3-water nanofluids with Jang and Choi's [3] model for different random motion velocity definitions

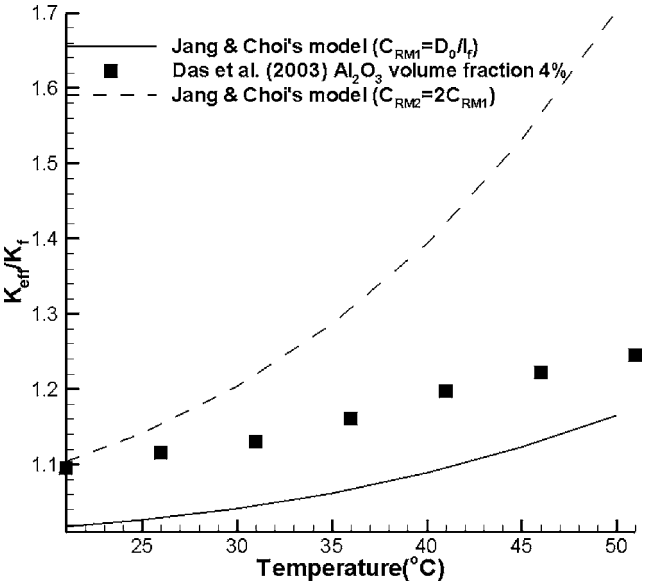

Fig. 5 Comparison of the experimental data (Das et al. [16]) for 4% Al_2O_3-water nanofluids with Jang and Choi's [3] model for different random motion velocity definitions

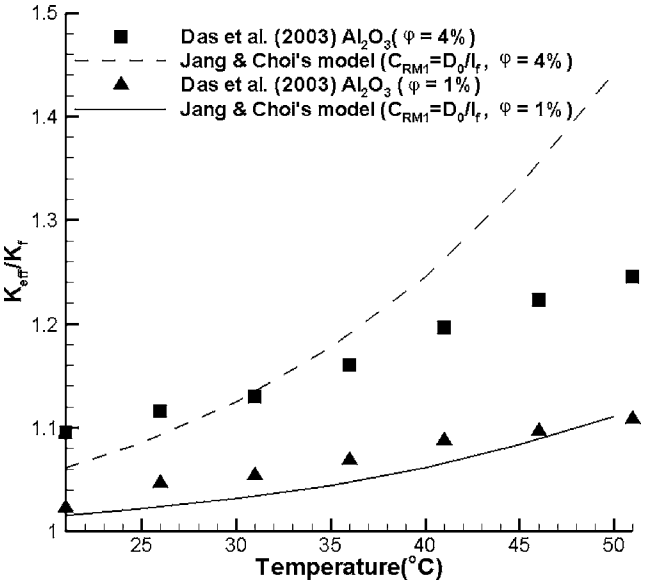

Fig. 7 Comparison of the experimental data (Das et al. [16]) for 1% and 4% Al_2O_3-water nanofluids with Jang and Choi's [3] model (new proportionality constant and new particle diameters are applied)

On June 14, 2007, Choi responded to the analysis presented so far. Specifically, he provided the following new information:

- Number-weighted diameters (24.4 nm for Al_2O_3 and 18.6 nm for CuO) were replaced with the area-weighted diameters (38.4 nm for Al_2O_3 and 23.6 nm for CuO),
- the random motion velocity $\bar{C}_{RM}=D_0/l_f$ was selected, and
- new proportionality constants, i.e., $C_1=7.2\times10^7$ for water and $C_1=3.2\times10^{11}$ for EG, were recommended.

Thus, employing the new information, Figs. 6 and 7 now replace Figs. 3–5, respectively. The Jang and Choi [3] model achieved a good match with the new numerical values for CuO-water nanofluids and Al_2O_3-water nanofluids (not shown). However, when using EG-based nanofluids, the model still cannot provide a good match even for the very large proportionality constant of $C_1=3.2\times10^{11}$ (see Fig. 6). When compared with the experimental data of Das et al. [16], as shown in Fig. 7 for a volume fraction of 1%, the model generates a decent data match, which is not the case when the volume fraction reaches 4%.

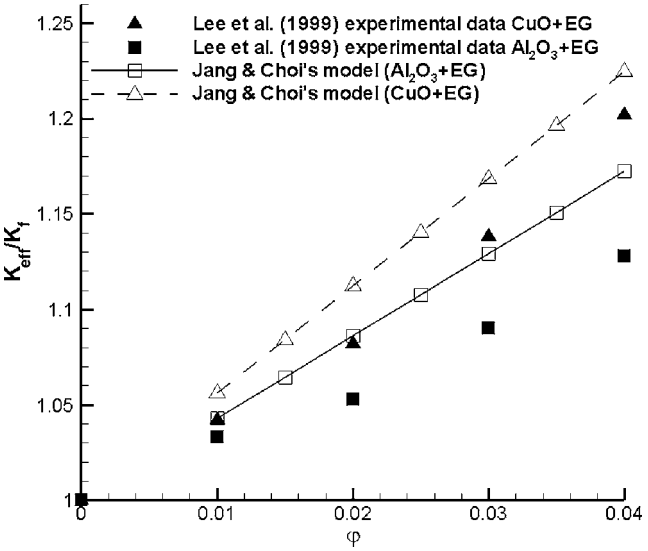

Fig. 6 Comparison of the experimental data (Lee et al. [12]) for Al_2O_3-EG and CuO-EG nanofluids with Jang and Choi's [3] model (new proportionality constant and new particle diameters are applied)

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