Discussion: "Exploring Effective Methods for Simulating Damaged Structures With Geometric Variation: Toward Intelligent Failure Detection" (McAdams, D. A., Comella, D., and Tumer, I. Y., 2007, ASME J. Appl. Mech., 74, pp. 191–202)

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*The paper of McAdams et al. (ASME J. Appl. Mech. **74**, pp. 191–202) explored two different approaches for damage detection in vibrating beams having both manufacturing variations in geometry and crack damage. One of the approaches, however, has a significant error in its formulation. The effects of this error on the formulation and the analytical results are discussed.*
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In Ref. [1], the authors pursued two approaches for damage detection in the presence of geometric variations in cross section due to manufacturing tolerances. They exploit the fact that both the damage and the manufacturing tolerances produce spatial dependence in the beam's mass density and flexural rigidity, which in turn influences the modal vibration properties of the beam, specifically its natural frequencies. The consideration of such spatial variations has recently attracted attention in the structural health monitoring community, and other researchers [2,3] have proposed related approaches for identifying damage based on this concept, albeit without consideration of manufacturing tolerances.

One of the approaches used in Ref. [1] is a finite difference scheme for approximating the governing equation for Euler–Bernoulli beam vibration that allows for spatial variation of the mass density and flexural stiffness. For free vibration, the relevant equation is

$$\rho A(x)\frac{\partial^2 w}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w}{\partial x^2}\right]=0 \qquad (1)$$

where $w=w(x,t)$ is the transverse vibration response and $\rho A(x)$ and $EI(x)$ are the spatially dependent mass per unit length and flexural rigidity. This equation can be expanded by performing the indicated differentiation of the second term, resulting in

$$\rho A(x)\frac{\partial^2 w}{\partial t^2}+EI''(x)\frac{\partial^2 w}{\partial x^2}+2EI'(x)\frac{\partial^3 w}{\partial x^3}+EI(x)\frac{\partial^4 w}{\partial x^4}=0 \qquad (2)$$

where $(\cdot)'$ denotes an ordinary spatial derivative. However, Eq. (2) in Ref. [1] neglects the term $2EI'(x)\partial^3 w/\partial x^3$ due to the incorrect use of a "multiplication rule" for second derivatives. As a consequence, the authors' finite difference approximations for the free vibration equation (most importantly, Eqs. (14), (17), (26), and (27)) are incorrectly formulated, compromising the results of the analytical study performed by the authors that employs these equations.

A clear indication of the effect of this error can be seen in the results obtained by the authors for the shifts in natural frequencies induced by crack damage. They report in Ref. [1] that, while the first, third, and fifth vibration modes had natural frequencies that decreased (on average) in the presence of damage, the natural frequencies of the second and fourth modes increased. In light of the error made in the formulation of the finite difference equations, this issue can now be understood by considering the contributions of the missing term in Eq. (2) to the overall dynamics. Assuming that the perturbations of Eq. (2) arising from the geometric variations and the crack damage are suitably small, the mode shape $W_n(x)$ corresponding to the nth mode of vibration may be expressed as

$$W_n(x)=\sin\left(\frac{n\pi x}{L}\right)+W_{n,\mathrm{pert}}(x) \qquad (3)$$

where $W_{n,\mathrm{pert}}(x)$ represents the (presumably small) perturbations of the mode shape away from its "ideal" value in the absence of spatial variations. Letting $A_n(t)$ denote the corresponding time-dependent amplitude of vibration for the nth mode, we can observe that the term missing from the authors' formulation of the governing equation makes the following *approximate* contribution to the dynamics:

$$2EI'(x)\frac{\partial^3 w}{\partial x^3}\approx -2\left(\frac{n\pi}{L}\right)^3 A(t)EI'(x)\cos\left(\frac{n\pi x}{L}\right) \qquad (4)$$

(The *exact* contribution to the dynamics depends on the behavior of $W'''_{n,\mathrm{pert}}(x)$, which may or may not be negligible but is assumed to be smaller in magnitude than $-(n\pi/L)^3\cos(n\pi x/L)$.)

Now, in the authors' example, the damage location is taken to be $x=\frac{1}{2}L$, the midpoint of the beam. Evaluating Eq. (4) at this location, we obtain

$$2EI'\left(x=\frac{1}{2}L\right)\frac{\partial^3 w}{\partial x^3}\approx -2\left(\frac{n\pi}{L}\right)^3 A(t)EI'\left(x=\frac{1}{2}L\right)\cos\left(\frac{n\pi}{2}\right) \qquad (5)$$

Note that, for the first, third, and fifth modes of vibration, the right-hand side of Eq. (5) is zero, showing that the term missing from the authors' formulation of the dynamics makes very little contribution to the overall dynamics for these modes. (Of course, the term proportional to $W'''_{n,\mathrm{pert}}(x)$ will make a nonzero contribution, but this should be relatively small.) Thus, it is not unexpected that the authors' analysis should reproduce the anticipated behavior of the natural frequencies for these modes, as their choice of damage location effectively causes an important effect of their error to "disappear." (It is understood that this missing term will make other important contributions to the overall dynamics of these modes at other locations; thus, the previous statement should not be construed as saying the missing term makes no important contributions.) However, when one considers Eq. (5) for the second and fourth modes, it is apparent that the missing term's contribution to the overall dynamics of these modes will be quite noticeable; in fact, the right-hand side of Eq. (5) has its largest magnitude at the damage location. Hence, it is understandable that the natural frequencies of these modes do not shift appropriately, as a significant source of perturbation has been neglected.

It should be noted that the authors' second approach to the damage identification problem makes no use of Eq. (2), so there is no reason to believe that the results obtained via this approach have this source of error. The authors, however, are encouraged to revisit their finite difference analysis in light of the points raised by this discussion.

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