How to prevent intolerant agents from high segregation?

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Abstract

In the framework of *Agent-Based Complex Systems* we examine dynamics that lead individuals towards spatial *segregation*. Such systems are constituted of numerous entities, among which local interactions create global patterns which cannot be easily related to the properties of the constituent entities. In the 70's, Thomas C. Schelling showed that an important spatial segregation phenomenon may emerge at the global level, if it is based upon local preferences. Moreover, segregation may occur, even if it does not correspond to agent preferences. In real life preferences regarding segregation are influenced by individual contexts as well as social norms; in this paper we will propose a model which describes the dynamic evolution of individuals tolerance. We will introduce heterogeneity in agents' preferences and allow them to evolve over time. We will show that it is possible to dynamically get a distribution of tolerance over the agents with a low average and in the same time to deeply limit global aggregation. As the Schelling's model showed that individual *tolerance* can nevertheless induce global *aggregation*, this paper takes the opposite view showing that *intolerant* agents can avoid *segregation* in some extent.

Introduction

In his article Schelling (1971), Thomas Schelling developed a model of segregation and analysed how a simple preference not be a part of a minority in one's neighbourhood, without necessarily favouring dominance of one's own type, can generate small micro-shocks which have drastic consequences at the macro level. Aggregation happens through a chain reaction, even though the agents do not wish such an extreme situation. Agents interact only locally with their neighbours: every one agrees to stay in a neighbourhood with individuals that have the opposite type, only if there are enough individuals with the same type in the vicinity. This proportion is fixed by a threshold, denoted by the *tolerance* ratio.

More generally, the 'micromotives and macrobehaviour' problematic asks the question of the compliance between local micro-motives and the resulting macro-behaviour. Today, as problems become more and more complex, this problematic is more relevant than ever. In the fields of sociology, economics, ecology, energy, ..., each one has many *a priori* on the global consequence of his own actions. Most often, a person thinks in good faith that his action will produce faithful results for the community. For example, one can think that:

(a) *intolerant* behaviour lead to *high segregation*

(b) *tolerant* behaviour lead to *low segregation*

Let i (resp. $\bar{i}$) stands for individual intolerance (resp. tolerance) and S for a high level of global aggregation/segregation. Hypothesis (a) and (b) can be reformulated by the *micro to macro link* $[i \to S]$ and $[\bar{i} \to \bar{S}]$. The Schelling's model provides first an example for the expected case $[i \to S]$; but, as it shows that tolerance can nonetheless induce a significant level of segregation, it provides also an example for the paradoxical link $[\bar{i} \to S]$ where the macro-outcome is intuitively inconsistent with the preferences of the agents who generate it.

This paper shows that macro-segregation can be deeply limited despite the presence of intolerant agents; thus, it provides an example for the dual case $[i \to \bar{S}]$. In the model we propose, each agent has his own threshold of tolerance. At each time, for each agent, the tolerance is adapted using some meta-rules. As a consequence, the emergent state of the 'world' results from a spatio-temporal adaptive dynamics. The scientific question addressed in this work is an evolution of the Schelling Model, which consists in considering an adaptive micro level of tolerance and analysing its impact on the segregation phenomenon observed at the macro level.

This article is organized as follows. In section 2, we propose a generic model of satisfaction. Section 3 shows the global behaviour of models using the simple *Eulogy to Fleeing* rule. Section 4 examines the effects of introducing adaptive tolerance thresholds on the nature of frontier between patterns. Section 5 proposes the new model which allows to conciliate local intolerance and a low level of segregation. Finally, future works are listed and conclusions are drawn.

A generic model of satisfaction

The Schelling's checkerboard model of residential segregation has become one of the most cited and studied models in many domains as economics, sociology, complex systems science,... Pancs and Vriend (2003), Zhang (2004), Gerhold et al. (2008), Banos (2009). It is also one of the predecessor of agent-based computer models Rosser (1999). Taking inspiration from this model, we define a more generic model of satisfaction (GMS).

The GMS is similar to a 2-D cellular automata model: the 'world' includes numerous agents embedded on a toroidal grid. For each agent, the perception is centered on his local neighbourhood only, where the neighbourhood is constituted of the nearest cells surrounding him. We note $d_i(t)$ the social *degree* of the agent a_i at time t, that is the number of agents in its neighbourhood. Since some locations remain empty, the size of the neighbourhood is the maximum number of neighbours an agent can have. There are two types of agents and each agent has its own type. During a run the agent's type cannot change. The satisfaction of one agent is relative to the type of the agents in its own neighbourhood. For convenience we will denote by a color, *yellow* and *green*, the two possible types. Y (resp. G) represents the set of agents in the *yellow* type (resp. *green* type). Thus, the number of agents is $(\#Y + \#G)$ and at the global level, the basic hypothesis is $(\#Y = \#G)$. At each time t, for each agent a_i, $s_i(t)$ (resp. $o_i(t)$) represents the number of neighbours with the *same* type (resp. *opposite* type), so $s_i + o_i = d_i$.

From Thresholding to Satisfaction

For each agent a_i, we assume that there is some quantity measured by the variable Q_i in the range $[0, 1]$ which depends on s_i and o_i. At each time t, the value $requiredQ_i(t)$ is a number in the range $[0, 1]$ which denotes the threshold under which the agent is satisfied according to $Q_i(t)$. We define the local boolean indicator $satisfied$ as:

$$satisfied_i(t) = (Q_i(t) \leq requiredQ_i(t)) \tag{1}$$

Finally, we define the global indicator *satisfactionRatio* in the range $[0, 1]$ as:

$$satisfactionRatio(t) = \frac{\#\{satisfied_i(t) = true\}}{\#Y + \#G} \tag{2}$$

This is the ratio of satisfied agents at time t; if it is equal to 1, then all the agents are satisfied at time t.

Local rule

Once the static description of the model is specified, one must add rules that govern the dynamics of agents' movement. At each time, the motives of each agent are driven by its own satisfaction: an unsatisfied agent is motivated to move away to one vacant location. The gap between micro-motives and macro-behaviours is due to overlapping neighbourhoods: an agent who moves according to its own interest affects not only the neighbourhood it leaves and the one it arrives in, but also affects, in the long run, all the agents. In GMS we do not fix how an agent moves; this will be specified later when the model will be instantiated. One can only say that there are many ways for an unsatisfied agent to move to a vacant place.

An index to measure the degree of aggregation

To have some insight into the aggregation level, it is necessary to measure the global aggregate over the world. We reformulate measures proposed by Schelling, Carrington and Goffette-Nagot Schelling (1971), Carrington and Troske (1997), Goffette-Nagot et al. (2009). First, we define a global measure of *similarity* as:

$$s(t) = \frac{1}{\#Y + \#G} \sum_i (1 - Q_i(t)) \tag{3}$$

Then, we define the *aggregateIndex* by

$$aggregateIndex = \begin{cases} \frac{s - s_{rand}}{1 - s_{rand}} & \text{if s} \geq s_{rand} \\ \frac{s - s_{rand}}{s_{rand}} & \text{else} \end{cases} \tag{4}$$

where s_{rand} is the expected value of the measure s implied by a random allocation of the agents in the world. A null value for this index corresponds to an average random configuration. The maximum value of 1 corresponds to a configuration with two homogeneous patterns only.

The Schelling's model of segregation

The Schelling's model of segregation is a particular case for the generic model of satisfaction. In the following we are going to indicate its specificities.

How to compute satisfaction? In the Schelling model the quantity $Q_i(t)$ takes into account the proportion of neighbours of the opposite type; it is computed as the ratio between the number of neighbours having the opposite type and the social degree.

$$Q_i(t) = \begin{cases} \frac{o_i(t)}{d_i(t)} & \text{if } d_i(t) \neq 0 \\ 0 & \text{else} \end{cases} \tag{5}$$

For example, if a yellow agent a_i has three yellow neighbours and two green neighbours, $Q_i = \frac{2}{5}$. If there are no neighbours for the agent (i.e. if $d_i = 0$), $Q_i = 0$. If all neighbours have the same type (i.e. if $o_i = 0$ and $s_i \neq 0$), $Q_i = 0$. If all the neighbours are in the opposite type (i.e. if $s_i = 0$ and $o_i \neq 0$), $Q_i = 1$. As the initial spatial configuration is randomly chosen, the initial distribution of Q_i is binomial.

Table 1: Ratio between the number of neighbours of opposite type to the social degree: $Q_i = \frac{o_i}{o_i+s_i}$
Coloured values: agent a_i, will be satisfied if Q_i is under the *tolerance* threshold 0.37

s \ o	0	1	2	3	4	5	6	7	8
0	0	1	1	1	1	1	1	1	1
1	0	.500	.666	.750	.800	.833	.857	.875	
2	0	.333	.500	.600	.666	.714	.750		
3	0	.250	.400	.500	.571	.625			
4	0	.200	.333	.428	.500				
5	0	.166	.285	.375					
6	0	.142	.250						
7	0	.125							
8	0								

In this model, all the agents have the same threshold of tolerance: it is a constant value (noted $tolerance$) which is fixed before the run. So, at each time t, for each agent a_i, equation 1 becomes:

$$satisfied_i(t) = (Q_i(t) \leq tolerance) \quad (6)$$

The agents are said tolerant if the tolerance is greater than 0.5 ($0.5 \leq tolerance$) and intolerant otherwise. We use the *Moore* neighbourhood commonly employed in agent-based models. So the neighbours of an agent are those living in the eight nearest cells surrounding him and the degree d_i is a number between 0 and 8. For instance, if the *tolerance* threshold is 0.37, one particular agent a_i, at time t, will be satisfied if $Q_i(t)$ is under this value; this happens in the following eighteen cases: $(o_i = 0)$, $(s_i = 2, o_i = 1)$, $(s_i = 3, o_i = 1)$, $(s_i = 4, o_i = 1)$, $(s_i = 5, o_i = 1)$, $(s_i = 6, o_i = 1)$, $(s_i = 7, o_i = 1)$, $(s_i = 4, o_i = 2)$, $(s_i = 5, o_i = 2)$ and $(s_i = 6, o_i = 2)$ (see the coloured values on table 1). More, if there are exactly eight neighbours, i.e. $d_i = 8$, (see table 1, the diagonal line) such a tolerance means that the agents are intolerant and cannot suffer more than two opposite neighbours.

How do unsatisfied agents move away ? In standard Schelling's models agents move only to satisfy their own interest. This requires that agents must be able to access distant information in order to determine whether or not it will be satisfied in a new vacant cell. This kind of behaviour is characteristic among economical agents that seek to maximize their gain. Nonetheless such a behaviour come out of the idea of agents acting approximately rational, rather economically rational in terms of utility and breaks down the *principle of locality* (see Brownlee (2007)).

Global behaviour Regarding the micro-macro problematic, the Schelling model provides examples for the two cases: $[i \rightarrow S]$ and $[\bar{i} \rightarrow S]$ where i (resp. $\bar{i}$) stands for individual intolerance (resp. tolerance) and S stands for a high degree of global segregation. While the first case is the intuitive situation where micro-intolerance induces macro-segregation, the second case is more surprising as it shows that tolerant behaviours can nonetheless induce a global segregation.

The Schelling Model with the *Eulogy to Fleeing* rule

In standard Schelling's models agents aspire to satisfy their interests in the new places they move in. In this section, we rather assume a reaction from agent without real cognitive abilities expressed by the simple *Eulogy to Fleeing rule* (EF rule).

The *Eulogy to Fleeing* rule

The *Eulogy to Fleeing* rule is agreeing with the definition of the term *satisficing* proposed by Herbert A. Simon Simon (1956). "Satisficing describes the selection of a good enough solution, the selection of a decision that meets a minimum threshold or aspiration level, the selection of which occurs in the context of incomplete information or limited computation" Brownlee (2007).

The *EF* rule is defined as follows: *for each unsatisfied agent, a cell is randomly chosen 'all over the world' and the agent moves to this cell if and only if it is vacant.* So the agents may move at random towards a new location according to their preferences by allowing utility-increasing or utility-decreasing actions. Moves do generate new satisfied or unsatisfied agents by a chain reaction until an equilibrium is reached. At a time t, if all the agents are satisfied, the EF rule has no effect and then such a configuration is a *fixed point* for the dynamics.

This simple rule is more in the spirit of the complex system paradigm, and, as locally there is no seeking for immediate benefits, it is interesting to know its global consequences. Although it is easy to build some particular cases where the EF rule does not converge, in the following simulations this rule leads the system towards an equilibrium. Let's note that although similar rules based on a random choice of vacant locations are already proposed (Edmonds and Hales (2005), Izquierdo et al. (2009)), they do not look completely identical to the EF rule. In particular, with the EF rule, an unsatisfied agent may stay in place for a while if the randomly chosen locations are occupied.

Simulation and results

In this paper, all the simulations are realized in the *NetLogo*[1] multiagent programmable modeling environment Pham (2004), Wilensky (1999). For each simulation, the agent's features are updated in an asynchronous way and

[1] http://ccl.northwestern.edu/netlogo/

the global geographic parameters are fixed. The world is a square of locations horizontally and vertically wrapped. An agent with type 'yellow' (resp. green) is represented by a yellow (resp. green) square. A black square represents a vacant location. A simulation stops at convergence, when all the agents become satisfied.

The world is a grid-square composed of 10000 locations. This size is a good compromise between the necessity to have a large value to avoid small space effects and the convenience to have a small value to achieve short computation time. There are 1000 vacant locations, knowing that the *density rate* is 90% and 4500 agents in each type. We imposed a random initial configuration: in the cases studied below, the value of s_{rand} is indistinguishable from 0.5; thus initial configuration induces an $aggregateIndex$ closed to 0.

We conducted two types of experiment: in the first one, all the agents are *intolerant* and in the second they are *tolerant*.

Intolerant agents For this first experiment the *tolerance* is set to 0.37 (see table 1); so all the agents are intolerant. We can see in figure 1 the result of the agents spatio-temporal evolution at the end of a representative run: after 1150 steps all the agents are satisfied (i.e. $satisfactionRatio = 1$), the mean Q_i over the whole population (noted $\overline{Q}$) is 0.024 and the $aggregateIndex$ is 0.957. From 100 independent runs we obtain, a mean of 0.952 (0.0041)[2] for the $aggregateIndex$ and 0.024 (0.0022) respectively for $\overline{Q}$. We can observe the emergence of large spatial homogeneous patterns. Moreover the borderland between the patterns is almost build with every vacant location (black square). So patterns are isolated by a *no-man's-land* of vacant cells.

Tolerant agents Here, the goal is to show that segregation occurs even if no agent strictly prefers this. We set the *tolerance* to 0.63 (see table 1), so all the individuals are tolerant. In particular, if an agent has exactly eight neighbours, it can bear up to five opposite agents in its vicinity. Figure 2 gives an example of the evolution of the agents' locations during a representative run. At the end, after 228 steps, all the agents are satisfied, the mean Q_i over the whole population is 0.229 and the $aggregateIndex$ is 0.548. From 100 independent runs we obtain, a mean of 0.53 (0.0119) for the $aggregateIndex$ and 0.233 (0.0094) respectively for $\overline{Q}$. While spatial segregation is not an attribute of the rational individuals's behaviour, we can observe the emergence of many segregationist patterns, although they have a smaller size that in the previous case (see figure 1). More, vacant locations are scarce on borderline because with a high tolerance level vacant cells are not requisite to delimit segregationist patterns.

[2] standard deviation is shown in ()

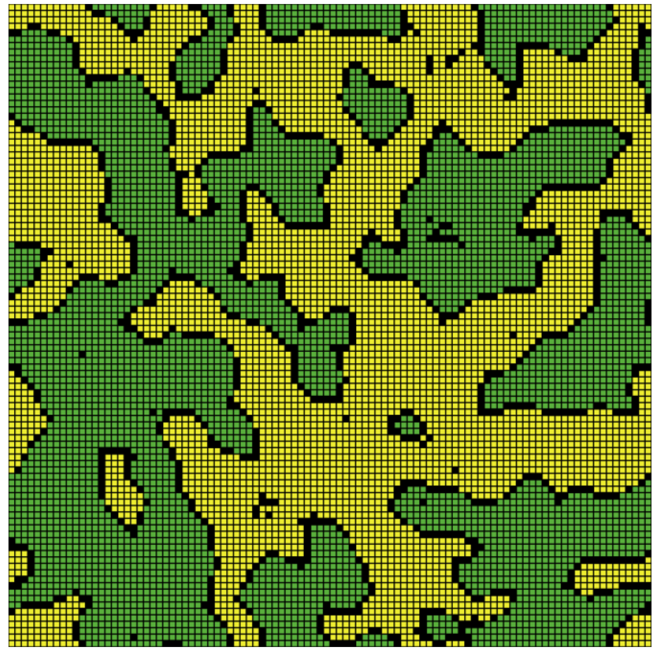

Figure 1: The Eulogy of Fleeing rule: $tolerance = 0.37$
View at convergence ($ticks = 1150$): $\overline{Q} = 0.024$
$aggregateIndex = 0.957$

Discussion

In this section, we have shown that in spite of the use of a more simple and realistic local rule, the model produces a comparable global behaviour than the classical Schelling model.

We have shown that both intolerant and tolerant local behaviours lead to the satisfaction of all the agents with the emergence of global segregationist patterns. Moreover, the gap between the $tolerance$ and the mean Q_i over the whole population is surprisingly large at the end of a run. In this way complex dynamics build much more liveable configurations than necessary. With intolerant agents, vacant places are required to form the frontiers and insulate agents in homogeneous patterns. In the next section, we propose to modify the model in order to insulate segregationist patterns without using vacant locations mainly.

From *no-man's-land* to *mediator-land*

Most often, in real life some individuals are tolerant whereas others are intolerant. In a model, there are two ways to take into account this fact: either fixing a distribution for the tolerance, or dynamically evolving tolerance to 'converge' toward a particular distribution. The first solution requires not only to choose one distribution: *uniform*, *normal*, *poisson*,... but also to fix its parameters: mean and standard deviation.

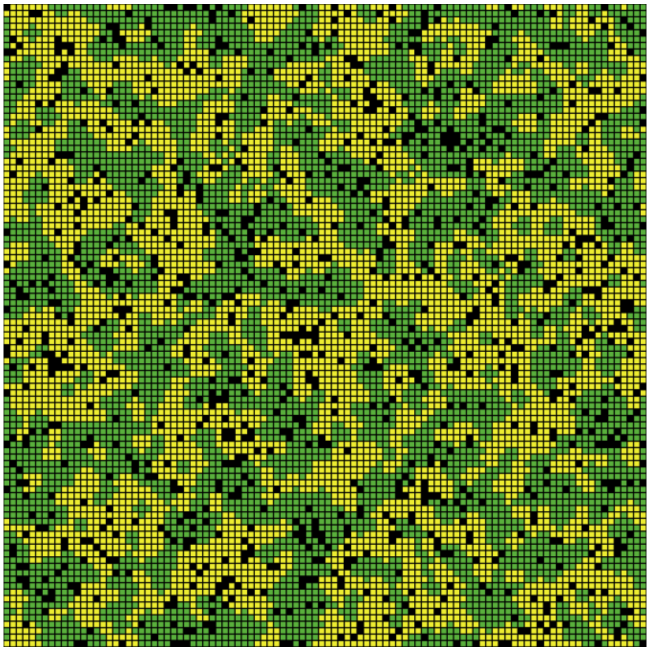

Figure 2: The Eulogy of Fleeing rule: $tolerance = 0.63$
View at convergence ($ticks = 228$): $\overline{Q} = 0.229$
$aggregateIndex = 0.548$

Adaptive local rule

As we have no *a priori* on a target level of tolerance, we choose to start from an intolerant configuration and to apply a local rule to gradually increase the tolerance. For instance, when a person is immersed in an unknown world, his first attempt will be to meet people which look like him; so initially, certainly with many apriority, such a person is gregarious or intolerant. Then, if his requirement is too high relatively to the environment, it will be difficult for him to find a fitting place; therefore a natural tendency will be to gradually reduce his stress by decreasing his gregariousness and/or increasing his tolerance.

In this new instance of the GMS, each agent has its own tolerance threshold. Furthermore, each individual threshold may vary over time. So, for each agent a_i, at each time t, the *satisfied* indicator (see equation 1) becomes:

$$satisfied_i(t) = (Q_i(t) \leq tolerance_i(t)) \qquad (7)$$

Initially, the tolerance of each agent is set to a very small value, therefore an agent is at first radically intolerant and so will be unsatisfied. At each time, for each unsatisfied agent, a cell is randomly chosen 'all over the world' in order to move in if it is vacant, otherwise, i.e. if the cell is already occupied, the agent stays put and adapts its own *tolerance* to the context by increasing its value with a small increment.

Simulation and results

For each agent, the $tolerance$ is initialised to 0.001 and we chose a small increment of 0.003. We can see on figure 3 the spatio-temporal evolution of the agents at the end of a representative run. After 663 steps all the agents are satisfied; the mean $tolerance$ over the population is 0.365, the mean Q_i over the population is 0.049 and the $aggregateIndex$ is 0.892. Even if dynamics are more complex than in Schelling's model, we can observe the emergence of spatial homogeneous patterns yet. On 100 runs we obtain, a mean of 0.919 (0.0120) for the $aggregateIndex$ and 0.360 (0.0047) respectively for the mean $tolerance$. So, on average, dynamics lead agents to remain intolerant and a high segregation emerges at the global level; once again this is an example for the case $[i \rightarrow S]$.

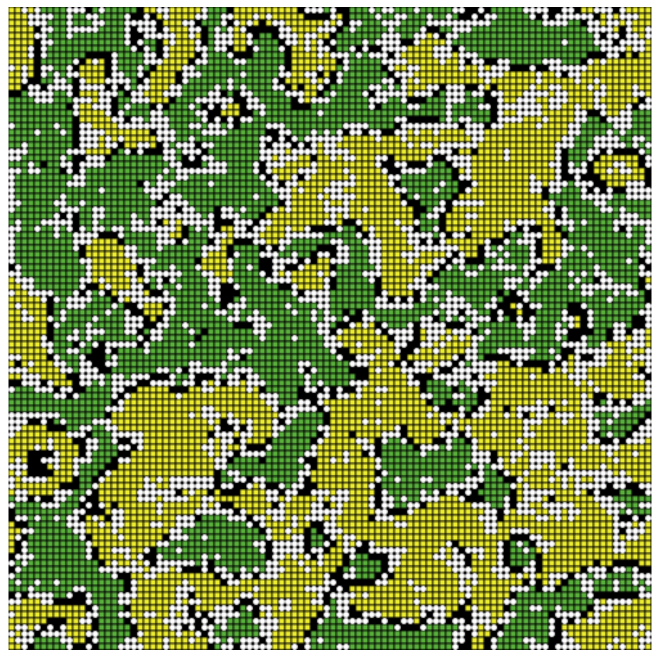

Figure 3: Dynamic tolerance
View at convergence ($ticks = 663$):
$aggregateIndex = 0.892$ mean $tolerance = 0.365$

We can observe that the frontier between homogeneous patterns is constituted both by vacant cells (black square) and by the most tolerant agents (white circle), i.e. agents with $tolerance \geq 0.39$; therefore, for a significant part, homogeneous regions are isolated by *places for mediation* where opposite agents may co-exist. We can note that there are also tolerant agents outside the *mediator-land*; this corresponds to *scoria*[3] in some areas where former conflicts have led to the local hegemony of one of the two types; thus data collected from the own tolerance of the agents allow to learn

[3]Scoria is the dross that remains after the smelting of metal from an ore

more about the past of the system.

Discussion

A first result is that dynamics leads the mean tolerance toward a relatively weak value (0.36); as a consequence, when all the agents become satisfied, they remain on average intolerant. The second result is that segregation is still high (0.919). The third result is that in a world where agents are on average intolerant there are some tolerant agents which play a crucial role in the spatial distribution. This can serve as a clue to extend the model toward more *mosaic-like* structure. *Type-mix* would be favoured by the existence of secluded agents amidst individuals having an opposite type. In the present model, this is impossible because agents are not tolerant enough to endure such a situation: we have to enhance the dynamics to allow tolerance to reach high values. On the contrary, the presence of *scoria* shows that one agent with high tolerance may be useful in a moment at a place then becomes superfluous later in the same location; so decrease the tolerance of satisfied agents may help to avoid such 'frozen region'. All this suggest us to manage two antagonist dynamics: increasing and decreasing the tolerance; so, we expect to significantly lower the level of segregation while maintaining a weak mean tolerance.

How to avoid high segregation ?

In this last section the goal is to respond to the question: *How intolerant agents can become satisfied without the emergence of macro segregation?*

In the new model we propose, there are two antagonist dynamics, the first one increases the tolerance of unsatisfied agents, whereas the second decreases the tolerance of satisfied agents. Initially, the agents have a weak tolerance and are thus radically intolerant and unsatisfied.

- An unsatisfied agent, can either move to a vacant place or else simply increase its tolerance (for details, see the previous section).

- Conversely, for a satisfied agent a_i, if the difference $delta$ between its $tolerance_i$ and the value of Q_i in the place it lives in is too high, its tolerance decreases.

In real life, when a person is no longer confronted with distressing circumstance, his ability to cope later in such a situation is reduced. This phenomenon can be explained by a mechanism of *forgetfulness*. In the model, an agent is satisfied if it is not faced to a large enough number of opposite agents. If over time such a lack of confrontation persists, then the agent gradually reduces his threshold of tolerance.

Parameter space exploration

There are two main parameters that control the dynamics of tolerance: the amount of increment inc and decrement dec. First we conduct a parameter space exploration in order to chose suitable values for the simulation.

In the context of complex systems, most often there are several parameters which together determine the global dynamics. In order to choose values for the parameters used in the simulations, we have first conducted an exploration of the *parameter space*. The objective to minimize both the mean $tolerance$ and the global $aggregateIndex$ is difficult because when the first one decreases, the second increases and conversely. Therefore, we conduct a tradeoff-analysis to identify compromise for which the two criteria are mutually satisfied in a *Pareto-optimal* sense. This is a typical multi-objective optimisation problem where the optimal solutions correspond to a set of compromises expresses by a *Pareto front* Dyer et al. (1992), Belton and Stewart (2002). In practice, the Pareto front is proposed to a human decision-maker who then chooses a solution according to his expertise.

For all the tests we perform, the parameter $delta$ is set to 0.1. We focus our effort on areas that lead to interesting regions where convergence occurs with low *tolerance* and low *segregation*: each test corresponds to one couple (inc, dec) in the range $[0.025, 0.040] \times [0, 0.030]$. There are 60 tests and, for each one, results are averaged over 100 runs. Each data point of the scatter plot (see figure 4) corresponds to a couple (inc, dec) and represents both the $aggregateIndex$ (y-value) and the mean $tolerance$ (x-value) obtained when all the agents are satisfied. We can observe that heightening the parameter dec (while inc remains constant) pushes the point solution to the left toward the *Pareto front*. Conversely, lowering the parameter inc (while dec is constant) moves up the point solution on one front. This analysis leads us to choose a particular point on the *Pareto-front* that represents a good compromise between both intolerance and low segregation. To conduct the following simulations, we choose the point corresponding to the parameter values $inc = 0.029$ and $dec = 0.017$ (See the arrow on figure 4).

Results

Initially, all the tolerances are set to 0.1. We can observe on figure 5 the spatial configuration at the end of a representative run when all the agents are satisfied: after 513 steps, the mean Q_i over the population is 0.306, the mean $tolerance$ over the population is 0.369 and the $aggregateIndex$ is 0.383. On 100 runs, we obtain on average an $aggregateIndex$ of 0.388 (0.0110) and a mean $tolerance$ of 0.370 (0.0048). The value for the $aggregateIndex$ (0.388) has to be compared with the ones obtained with the two previous models (0.957 and 0.919)

The frontier between homogeneous patterns is constituted by the most tolerant agents and there is no *scoria* inside the patterns. One observes that homogeneous areas are infiltrated by many secluded individuals: there are some niches which co-exist within a cohort of unlike agents; this is possible only because loners are very tolerant. In contrast with

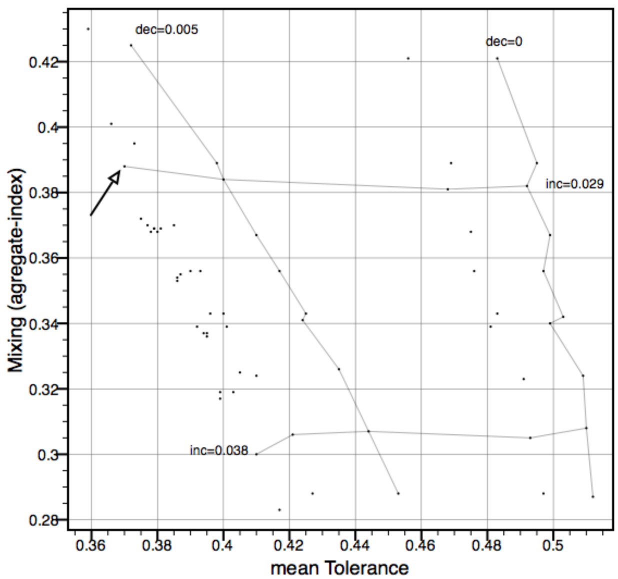

Figure 4: Parameter space analysis
Tolerance vs. Segregation

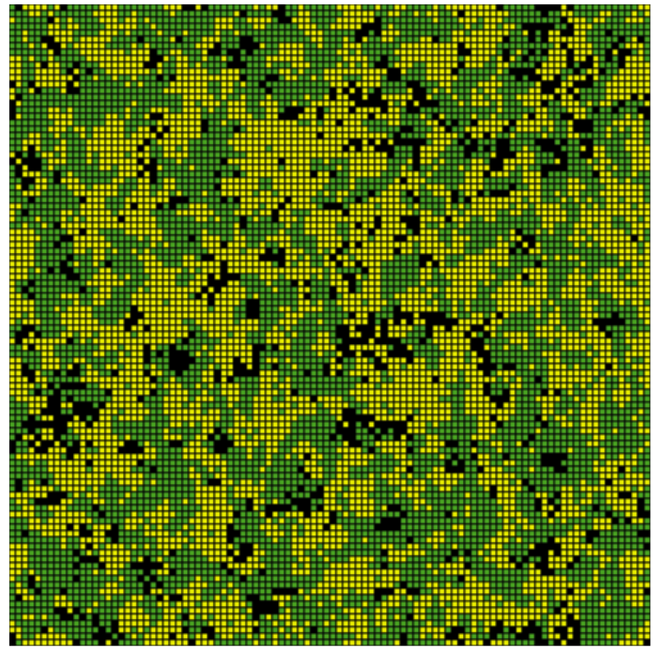

Figure 5: Intolerant agents avoid global segregation
View at convergence ($ticks = 513$): mean $tolerance = 0.369$, $aggregateIndex = 0.383$

the previous models, vacant locations don't play any role in isolating individuals from each other. The most important feature of this model is that it prevents intolerant agents from high segregation. As the Schelling's model provided an example for the case $[\bar{i} \rightarrow S]$, this model exemplify the $[i \rightarrow \bar{S}]$ micro-macro link.

Conclusion and future work

In this article, we have proposed to extend the Schelling's model considering that every individual has its own tolerance level. In a first step we have proposed a simple way to locally manage the tolerance; all that gives rise to the emergence of a new kind of border and inner *scoria* both made up of the most tolerant agents. In a second stage, we have introduced new dynamics that consists of combining two antagonist strengths. As a result of this confrontation, the agents are able to reach an equilibrium where they all are satisfied, rather intolerant, but where the aggregation level remains low. As, at our knowledge, there is no prior work on this topic, this result is a significant challenge to the analysis conducted by Schelling: it shows that one can avoid segregation if the tolerance level is adaptive, which is in our opinion a better assumption.

In future work, we will revisit those results by considering situations closer to reality. Beyond a simple world of agents embedded on an homogeneous toroidal-grid, we have to consider different types of network as for example neighbourhoods defined from a scale-free network. We have observed the emergence of very different type of frontiers: *no-man's-land*, *mediator-land* or in some extend *mixing*; thus, it might be interesting to study for a border, its composition, its spatial distribution, its volume, porosity, permeability,... and so to better understand its function: place of exchange and/or medium to isolate.

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