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M-theory from E8 FREE

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M-theory from E8

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Abstract. Bars, Sezgin and Nishino have proposed super Yang-Mills theory in $D = 12$ and beyond. Here we consider M-theory in signature (10,1) as a reduction of signature (10,2), using a projection of $e_{8(-24)}$ along b_2 . The projection gives a visual decomposition of $e_{8(-24)}$ that makes the vector of $\mathfrak{so}(10, 2)$ and its spinor manifest. Reduction along a time direction yields an $\mathfrak{so}(10, 1)$ invariant subspace that we associate with the spacetime of $D = 11$ M-theory.

INTRODUCTION

The study of super Yang-Mills beyond $D = 11$ with multiple time degrees of freedom was undertaken by Bars, Sezgin and Nishino [1, 2, 6]. Such higher dimensional Yang-Mills theories are especially of interest for quantum gravity in that $D = 10$ super Yang-Mills is used for a matrix formulation of M-theory [7].

It was noticed the algebraic structure of super Yang-Mills in $D = 10, 12, 14$ is captured by the exceptional Lie algebras $e_{6(-26)}$, $e_{7(-25)}$, and $e_{8(-24)}$, respectively [8]. Such Yang-Mills theories were dubbed *exceptional Yang-Mills theories* and defined to arbitrary dimension using the novel mathematical framework of *exceptional periodicity* [5] and its *magic star* projections along a_2 [3, 4, 5]. In this analysis, we use a projection of $e_{8(-24)}$ along b_2 to show the vector and spinor of M-theory in $D = 10 + 2$ can be recovered. Algebraically, this is tantamount to splitting the fundamental **56** of $e_{7(-25)}$ as a Freudenthal triple system into $\mathbf{12} \oplus \mathbf{32} \oplus \mathbf{12}$. By nullifying a timelike coordinate of the **12** one recovers an $\mathfrak{so}(10, 1)$ invariant subspace we associate with the total space of $D = 11$ M-theory.

M-THEORY

M-theory in $D = 10 + 1$ can be studied as a Yang-Mills theory starting with a 3-form $C_{(3)}$. Taking the electric field strength $F_{(4)} = dC_{(3)}$, gives a p -brane source called the M2-brane. Taking the Hodge dual of $F_{(4)}$ yields $\tilde{F}_{(7)}$ with 5-dimensional magnetic source called the M5-brane. The spinor is given by a **32**.

Consider the 5-grading of $e_{7(-25)}$:

$$e_{7(-25)} = \mathbf{1} \oplus \mathbf{32} \oplus (\mathfrak{so}(10, 2) \oplus \mathbf{1}) \oplus \mathbf{32} \oplus \mathbf{1}. \quad (1)$$

The 5-grading reveals an $\mathfrak{so}(10, 2)$ adjoint representation, as well as its **32** spinor, ingredients suitable for the study of M-theory. What is lacking is the **12** vector of $\mathfrak{so}(10, 2)$. In the next section, we show this can be recovered if we use the larger algebra $e_{8(-24)}$.

E8 AND M-THEORY

The structure of $D = 10 + 2$ Yang-Mills is almost encoded by the 5-grading of $e_{7(-25)}$. To complete the structure, one needs a **12** vector of $\mathfrak{so}(10, 2)$.

Consider the following decomposition of $e_{8(-24)}$, that makes the 5-grading of $e_{7(-25)}$ manifest:

$$e_{8(-24)} = \mathbf{1} \oplus (\mathbf{12} \oplus \mathbf{32} \oplus \mathbf{12}) \oplus [(\mathbf{1} \oplus \mathbf{32} \oplus (\mathfrak{so}(10, 2) \oplus \mathbf{1}) \oplus \mathbf{32} \oplus \mathbf{1}) \oplus \mathbf{1}] \oplus (\mathbf{12} \oplus \mathbf{32} \oplus \mathbf{12}) \oplus \mathbf{1}. \quad (2)$$

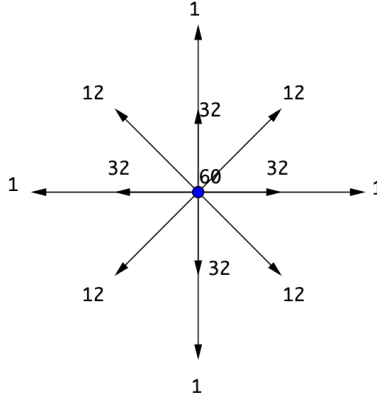


FIGURE 1. A projection of $e_{8(-24)}$ along a b_2 (Courtesy of P. Truini).

Figure 1 depicts a projection of $e_{8(-24)}$ along a b_2 , giving a visual depiction of the decomposition of $e_{8(-24)}$ with manifest $\mathfrak{so}(10, 2)$ adjoint, spinor and vector representations. The $\mathbf{12} \oplus \mathbf{32} \oplus \mathbf{12} = \mathbf{56}$ gives a decomposition of the fundamental of $e_{7(-25)}$ as a Freudenthal triple system (FTS) in terms of the $\mathbf{12}$ vector of $\mathfrak{so}(10, 2)$.

SPIN FACTORS AND YANG-MILLS

Through the lens of Jordan algebras, the $\mathbf{12}$ vector of $\mathfrak{so}(10, 2)$ can be expressed as:

$$\mathbf{12} = (J_2(\mathbb{O}) \oplus \mathbb{R}) \oplus \mathbb{R} \quad (3)$$

where the $J_2(\mathbb{O})$ spin factor part contributes a $(9, 1)$ signature when an element is written with lightcone coordinates on the diagonal. Embedded in $J_3(\mathbb{O})$, an \mathbb{R} is a scalar coordinate from the third diagonal entry of a 3×3 hermitian matrix. The additional copy of \mathbb{R} is the corresponding scalar diagonal of $J_3(\mathbb{O})$ embedded in its FTS, which is the fundamental $\mathbf{56}$ of $e_{7(-25)}$. In terms of signature, the third diagonal of $J_3(\mathbb{O})$ enhances the $(9, 1)$ signature to $(10, 1)$, and the additional real coordinate gives $(10, 2)$ signature.

One can descend to $(10, 1)$ by either nullifying a timelike coordinate in the $J_2(\mathbb{O})$ spin factor (which transforms as a $\mathbf{10}$ of $\mathfrak{so}(9, 1)$) or the corresponding diagonal coordinate of the FTS. As $J_2(\mathbb{O})$ can be used to give a matrix formulation of M-theory in $D = 10 + 1$ in the infinite momentum frame (where the third diagonal coordinate of $J_3(\mathbb{O})$ becomes a point at infinity), we associate the vector space $\mathbf{12} = (J_2(\mathbb{O}) \oplus \mathbb{R}) \oplus \mathbb{R}$ with the total space of M-theory in $D = 10 + 2$ signature. This is sufficient to show a matrix formulation of M-theory in $D = 10 + 1$ can be recovered from $e_{8(-24)}$ after a timelike coordinate reduction.

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