RESEARCH ARTICLE | MARCH 10 2015

Considerations on double porosity structure for micropolar bodies

M. Marin; S. Vlase ; M. Paun

AIP Advances 5, 037113 (2015)
https://doi.org/10.1063/1.4914912

AIP ADVANCES **5**, 037113 (2015)

Considerations on double porosity structure for micropolar bodies

M. Marin,[1] S. Vlase,[2,a] and M. Paun[1]
[1]*Department of Mathematics, TRANSILVANIA University of Braşov, 500036, Romania*
[2]*Department of Mechanics, TRANSILVANIA University of Braşov, 500036, Romania*

(Received 10 February 2015; accepted 2 March 2015; published online 10 March 2015)

In this study we want to extend the results of B. Straughan, which quite recently addressed the issue of double porosity structure for classical elastic bodies. In this case, the double porosity structure of the body is not influenced by the displacement field, which is not consistent with reality. For the mixed initial boundary value problem in the context of micropolar bodies with double porosity, we prove the existence of the solution, its uniqueness as well as some considerations on stability of solution. [http://dx.doi.org/10.1063/1.4914912]

I. INTRODUCTION

The problem of elastic bodies with double porosity was the subject of study for some papers more than fifty years ago, among the ftrst such works as Refs. 1 and 2. In recent years these studies have been intensified due to the involvement of double porosity in areas of great interest, such as the composition and behavior of bones[3] as well as some phenomena of geophysics.[4,5] According to a universally accepted definition of double porosity, such a body has a double porous structure: a macroporosity due to pores in the body and a microporosity due to fissures in the skeleton.[6–8] As is known, the theory of materials with voids or vacuous pores is the simplest extension of the classical theory of elasticity and was first proposed by Nunziato and Cowin.[9] In this theory the authors introduce an additional degree of freedom in order to develop the mechanical behavior of a body in which the skeletal material is elastic and interstices are voids of material. The intended applications of the theory are to geological materials like rocks and soil and to manufactured porous materials. The linear theory of elastic materials with voids was developed by Cowin and Nunziato in Ref. 10. Here the uniqueness and weak stability of solutions are also derived. The theory of bodies with voids has been extensively studied in many papers, such as Refs. 9–17.

II. BASIC EQUATIONS

An anisotropic elastic material is considered. Assume a such body that occupies a properly regular region B of three-dimensional Euclidian space R^3 bounded by a piecewise smooth surface ∂B and we denote the closure of B by $\overline{B}$. The boundary ∂B is smooth enough to apply the divergence theorem.

We use a fixed system of rectangular Cartesian axes Ox_i, (i = 1, 2, 3) and adopt Cartesian tensor notation. A superposed dot stands for the material time derivate while a comma followed by a subscript denotes partial derivatives with respect to the spatial coordinates. Einstein summation convention on repeated indices is used. Also, the spatial argument and the time argument of a function will be omitted when there is no likehood of confusion.

We will consider the mixed problem associated with the theory of double porosity structure for a thermoelastic micropolar body on the time interval I. The behavior of such a body is described

[a]Corresponding author: S.Vlase, email: svlase@unitbv.ro

by the displacement vector field u_i, the microrotation vector field $\boldsymbol{\varphi}_i$, a pressure due to pores and a pressure due to fissures. It is important to emphasize that in the case of equilibrium, both these pressures become dependent of the displacement vector field, in other words, the porosity structure of the body is influenced by the displacement vector field.

We denote by φ the volume fraction field corresponding to pores and by ψ the volume fraction field corresponding to fissures. The characteristics ε_{ij} and γ_{ij} of the strain are defined by means of the geometric equations:

$$\varepsilon_{ij} = u_{j,i} + \varepsilon_{jik}\,\boldsymbol{\varphi}_k, \quad \gamma_{ij} = \boldsymbol{\varphi}_{j,i} \tag{1}$$

where u_i are the components of the displacement vector field, $\boldsymbol{\varphi}_i$ are the components of the microrotation vector field and ε_{jik} is Ricci's tensor (the alternating symbol). Following the known procedureof Green and Rivlin, together with given motion, we can consider a second motion which differ from the given motion only by constant superposed rigid translation.

For the given motion, all characteristics of the body are unaltered by such superposed rigid translation velocity.

In the following, we restrict our considerations only to the case where the materials have a center of symmetry. If we suppose that the body in its reference configuration is free from stress and has zero intrinsic equilibrated body forces, body couples and entropy, the Helmholtz free energy can be written in the following form:

$$\begin{aligned}\Psi = &\frac{1}{2}A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + B_{ijmn}\varepsilon_{ij}\gamma_{mn} + A_{ij}\varepsilon_{ij}\varphi + B_{ij}\varepsilon_{ij}\psi - \alpha_{ij}\varepsilon_{ij}\theta +\\ &+ B_{mnij}\varepsilon_{ij}\gamma_{mn} + \frac{1}{2}C_{ijmn}\gamma_{ij}\gamma_{mn} + C_{ij}\varepsilon_{ij}\varphi + D_{ij}\gamma_{ij}\psi - \beta_{ij}\gamma_{ij}\theta +\\ &+ \frac{1}{2}a_{ij}\varphi_{,i}\varphi_{,j} + b_{ij}\varphi_{,i}\psi_{,j} + \frac{1}{2}c_{ij}\psi_{,i}\psi_{,j} + \frac{1}{2}\alpha_1\varphi^2 +\\ &+ \frac{1}{2}\alpha_2\psi^2 + \alpha_3\varphi\psi - \gamma_1\varphi\theta - \frac{1}{2}a\theta^2.\end{aligned} \tag{2}$$

Here the constitutive coeficients are given function which depend only on the spatial coordinates and are subject to the following symmetry relations

$$A_{ijmn} = A_{mnij}, \quad C_{ijmn} = C_{mnij}, \quad a_{ij} = a_{ji}, \quad c_{ij} = c_{ji} \tag{3}$$

Taking into account the free energy function, using a common method, we obtain the following constitutive equations:

$$\begin{aligned}t_{ij} &= A_{ijmn}\varepsilon_{mn} + B_{ijmn}\gamma_{mn} + A_{ij}\varphi + B_{ij}\psi - \alpha_{ij}\theta,\\ m_{ij} &= B_{ijmn}\varepsilon_{mn} + C_{ijmn}\gamma_{mn} + C_{ij}\varphi + D_{ij}\psi - \beta_{ij}\theta,\\ \sigma_i &= a_{ij}\varphi_{,j} + b_{ij}\psi_{,j}\\ \tau_i &= c_{ji}\varphi_{,j} + d_{ij}\psi_{,j}\\ \xi &= -A_{ij}\varepsilon_{ij} - B_{ij}\gamma_{ij} - \alpha_1\varphi - \alpha_3\psi + \gamma_1\theta,\\ \zeta &= -C_{ij}\varepsilon_{ij} - D_{ij}\gamma_{ij} - \alpha_3\varphi - \alpha_2\psi + \gamma_2\theta,\\ \rho_0\eta &= -\alpha_{ij}\varepsilon_{ij} + \beta_{ij}\gamma_{ij} + \gamma_1\varphi + \gamma_2\psi + a\theta,\\ q_i &= k_{ij}\theta_{,j}.\end{aligned} \tag{4}$$

The entropy inequality implies

$$k_{ij}\,\theta_{,i}\,\theta_{,j} \geq 0. \tag{5}$$

In the context of linear theory the equation of energy has the form:

$$\rho_0 T_0\dot{\eta} = q_{i,i} + \rho_0 S. \tag{6}$$

Taking into account the method used by Nunziato and Cowin in Ref. 3, the following fundamental equations are derived (see also, Ref. 9): -the equations of motion:

$$t_{ij,j} + \rho_0 F_i = \rho_0 \ddot{u}_i,$$
$$m_{ij,j} + \varepsilon_{ijk} t_{jk} + \rho_0 M_i = I_{ij} \ddot{\Phi}_j, \tag{7}$$

-the balances of the equilibrated forces:

$$\sigma_{i,i} + \xi + \rho_0 G = \kappa_1 \ddot{\varphi},$$
$$\tau_{i,i} + \varsigma + \rho_0 L = \kappa_2 \ddot{\psi}. \tag{8}$$

In the above equations we have used the following notations: ρ_0 - the constant mass density; η - the specific entropy; T_0 - the constant absolute temperature of the body in its reference state; I_{ij} - coeficients of microinertia; κ_1 and κ_2 are coeficients of equilibrated inertia; θ - the temperature variation measured from the reference temperature T_0; t_{ij} - the components of the stress tensor; m_{ij} - the components of the couple stress tensor; σ_i - the components of the equilibrated stress vector associated to pores; τ_i - the components of the equilibrated stress vector associated to fissures; F_i – the body force per unit mass; M_i - the body couple per unit mass; G-the extrinsic equilibrated body force per unit mass associated to pores; L – the extrinsic equilibrated body force per unit mass associated to fissures; q_i - the components of the heat flux vector; S – the heat supply per unit mass. To the above system of basic equations (4), (6)-(8) we add the following initial conditions:

$$\begin{aligned}
&u_i(x,0) = u_i^0(x), \quad \dot{u}_i(x,0) = u_i^1(x),\\
&\Phi_i(x,0) = \Phi_i^0(x), \quad \dot{\Phi}_i(x,0) = \dot{\Phi}_i^1(x),\\
&\varphi(x,0) = \varphi^0(x), \quad \varphi(x,0) = \varphi^1(x), \qquad x \in \bar{B}\\
&\psi(x,0) = \psi^0(x), \quad \psi(x,0) = \psi^1(x),\\
&\theta(x,0) = \theta^0(x),
\end{aligned} \tag{9}$$

and, in the case of first initial boundary value problem, the boundary conditions:

$$u_i(x,t) = \tilde{u}_i, \quad \Phi_i(x,t) = \tilde{\Phi}_i, \quad \varphi(x,t) = \tilde{\varphi},$$
$$\psi(x,t) = \tilde{\psi}, \qquad \theta(x,t) = \tilde{\theta}, \quad (x,t) \in \partial B\, x\, (t_0,t_1). \tag{10}$$

In this way, a solution of first initial boundary value problem, in the context of themoelastic micropolar bodies with double porosity, is the ordered array $(u_i, \boldsymbol{\varphi}_i, \varphi, \psi, \theta)$ which satisfy the equations (4), (6)-(8), the initial conditions (9) for all $x \in B$ and the boundary conditions (10) for all $(x,t) \in \partial B\; x(t_0,t_1)$.

In the case of the second initial boundary value problem, the boundary conditions (10) are replaced with the boundary conditions:

$$t_i(x,t) = \tilde{t}_i, \quad m_i(x,t) = \tilde{m}_i, \quad \sigma(x,t) = \tilde{\sigma},$$
$$\tau(x,t) = \tilde{\tau}, \quad q(x,t) = \tilde{q}, \quad (x,t) \in \partial B\, x\, (t_0,t_1) \tag{11}$$

where $\tilde{t}_i(x,t)$, $\tilde{m}_i(x,t)$, $\tilde{\sigma}(x,t)$, $\tilde{\tau}(x,t)$ and $\tilde{q}(x,t)$ are prescribed functions. In Eq. (11) we have the usual notations

$$t_i = t_{ji} n_j, \quad m_i = m_{ji} n_j, \quad \sigma = \sigma_i n_i, \quad \tau = \tau_i n_i, \quad q = q_i n_i,$$

where n_i are the components of the outward unit normal to the boundary surface.

III. BASIC RESULTS

In the following we need to impose the positivity of the thermal conductivity tensor k_{ij} . This means that there exists $k^0 > 0$ so that:

$$k_{ij}\eta_i\eta_j \geq k^0 \eta_i \eta_i \quad \text{for every vector } (\eta_i) \tag{12}$$

Also, in all what follows we shall use the following assumptions on the mass density ρ_0, the coeficients of microinertia I_{ij}, the coeficients of equilibrated inertia κ_1, κ_2 and the thermal capacity a:

$$\rho_0(x) \geq \rho^0 > 0, \quad I_{ij}(x) \geq I_{ij}^0 > 0, \quad \kappa_1(x) \geq \kappa_1^0 > 0, \quad \kappa_2(x) \geq \kappa_2^0 > 0, \quad a(x) \geq a^0 > 0 \quad (13)$$

These assumptions are in agreement with the usual restrictions imposed in the mechanics of continua.

We restrict our considerations only in the case in that in basic equations (6) -(8) the body force F_i, the body couple force M_i, the extrinsic equilibrated body forces G, L and heat supply S are absent.

Also, we shall assume that we have null boundary conditions (for first boundary value problem), that is

$$\begin{aligned} &u_i(x,t) = 0, \quad \Phi_i(x,t) = 0, \quad \varphi(x,t) = 0, \\ &\psi(x,t) = 0, \qquad \theta(x,t) = 0, \quad (x,t) \in \partial B\, x\, (0,t_1). \end{aligned} \quad (14)$$

The result in the next Theorem is an uniqueness result.

Theorem 1. *Suppose that conditions (12) and (13) hold. We also assume that the symmetry assumptions Eq. (3) are satisfied. Then the firrst initial boundary value problem, corresponding to the null loads and null boundary conditions, has at most one solution.*

Proof. Using the principle of conservation of energy, we have the following equality:

$$\begin{aligned} E(t) = &\int_B (\rho_0 \dot{u}_i \dot{u}_i + I_{ij}\, \dot{\Phi}_i \dot{\Phi}_j \quad + \kappa_1 \dot{\varphi}^2 + \kappa_2 \dot{\psi}^2 + a\theta^2) dV + \\ &+ \int_B (A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + 2B_{ijmn}\varepsilon_{ij}\gamma_{mn} + C_{ijmn}\gamma_{ij}\gamma_{mn} + a_{ij}\varphi_{,i}\,\varphi_{,j} + \\ &+ b_{ij}\varphi_{,i}\,\varphi_{,j} + c_{ij}\psi_{,i}\,\psi_{,j} + 2A_{ij}\varepsilon_{ij}\varphi + 2B_{ij}\varepsilon_{ij}\psi + \\ &+ 2C_{ij}\gamma_{ij}\varphi + 2D_{ij}\gamma_{ij}\psi + \alpha_1\varphi^2 + \alpha_2\psi^2 + 2\alpha_3\varphi\psi) dV + \\ &+ 2\int_0^t \int_B \frac{1}{T_0} k_{ij}\theta_{,i}\,\theta_{,j} = E(0). \end{aligned} \quad (15)$$

This law of energy conservation is satisfied for any solution of the above formulated problem. Using a suggestion given in the paper[10] of Ieşan and Quintanilla, we consider the function F(t) given by

$$\begin{aligned} F(t) = &\int_B (\rho_0 u_i u_i + I_{ij}\, \Phi_i \Phi_j \quad + \kappa_1 \varphi^2 + \kappa_2 \psi^2) dV + \\ &+2\int_0^t \int_B \frac{1}{T_0} k_{ij}\left(\Theta_{,i} + \Pi_{,i}\right)\left(\Theta_{,j} + \Pi_{,j}\right)\, dV ds \; + h(t+\omega)^2 \end{aligned} \quad (16)$$

where h and ω are two positive constants which will be subsequently determined. The function $\Theta(t)$ is given by:

$$\Theta(t) = \int_0^t \theta(s) ds.$$

Also, the function $\Pi(x,t)$ is the solution to the following boundary value problem:

$$\begin{aligned} &\frac{1}{T_0}(k_{ij}\Pi_{,i})_{,j} = \alpha_{ij}\varepsilon_{ij}^0 + \beta_{ij}\gamma_{ij}^0 + \gamma_1\varphi^0 + \gamma_2\psi^0 + a\theta^0, \\ &\Pi = 0 \quad \text{on } \partial B. \end{aligned} \quad (17)$$

It is clear that the equation $(17)_1$ is suggested by the energy equation (6) of which, by integration with respect to the time, leads us to equality:

$$\frac{1}{T_0}(k_{ij}\Pi_{,i})_{,j}ds - \alpha_{ij}\varepsilon_{ij} - \beta_{ij}\gamma_{ij} - \gamma_1\varphi - \gamma_2\psi - a\theta =$$
$$= -\alpha_{ij}\varepsilon_{ij}^0 - \beta_{ij}\gamma_{ij}^0 - \gamma_1\varphi^0 - \gamma_2\psi^0 - a\theta^0.$$

We calculate the derivative of the function F from Eq. (16):

$$\dot{F}(t) = \int_B (\rho_0 u_i u_i + I_{ij}\,\Phi_i\dot{\Phi}_j + \kappa_1\varphi\,\dot{\varphi} + \kappa_2\psi\,\dot{\psi})dV +$$
$$+ \int_0^t \int_B \frac{2}{T_0}k_{ij}\left(\Theta_{,i} + \Pi_{,i}\right)\left(\dot{\Theta}_{,j} + \dot{\Pi}_{,j}\right)dV ds \;+ 2h(t+\omega) - \int_B \frac{1}{T_0}k_{ij}\Pi_{,i}\Pi_{,j}\,dV. \quad (18)$$

Then the second order derivative is

$$\ddot{F}(t) = 2\int_B \left[\rho_0\left(u_i\ddot{u}_i + \dot{u}_i\dot{u}_i\right) + I_{ij}\left(\Phi_i\ddot{\Phi}_j + \dot{\Phi}_i\dot{\Phi}_j\right) + \kappa_1\left(\varphi\,\ddot{\varphi} + \dot{\varphi}^2\right) + \kappa_2\left(\psi\,\ddot{\psi} + \dot{\psi}^2\right)\right]dV +$$
$$+ \int_B \frac{2}{T_0}k_{ij}\left(\Theta_{,i} + \Pi_{,i}\right)\left(\dot{\Theta}_{,j} + \dot{\Pi}_{,j}\right)dV \;+ 2h \quad (19)$$

The inertial terms from Eq. (19) will be eliminated using the equations of motion (7), the balances of the equilibrated forces Eq. (8) and the equation of energy Eq. (6). Then we use the constitutive equations (4) and the divergence theorem so that the second order derivative $\ddot{F}(t)$ takes the form:

$$\ddot{F}(t) = 2\int_B \left[\rho_0\dot{u}_i\dot{u}_i + I_{ij}\dot{\Phi}_i\dot{\Phi}_j + \kappa_1\dot{\varphi}\,\dot{\varphi} + \kappa_2\dot{\psi}\,\dot{\psi}\right]dV -$$
$$-2\int_B (A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + C_{ijmn}\gamma_{ij}\gamma_{mn} + a_{ij}\varphi_{,i}\,\varphi_{,j} + c_{ij}\psi_{,i}\,\psi_{,j} + 2b_{ij}\varphi_{,i}\,\psi_{,j})dV -$$
$$-2\int_B (B_{ijmn}\varepsilon_{ij}\gamma_{mn} + A_{ij}\varepsilon_{ij}\varphi + B_{ij}\varepsilon_{ij}\psi + C_{ij}\gamma_{ij}\,\varphi + D_{ij}\gamma_{ij}\,\psi)dV -$$
$$-2\int_B (\alpha_1\varphi^2 + \alpha_1\varphi^2 + 2\alpha_3\varphi\psi)dV + 2h -$$
$$-2\int_B \frac{2}{T_0}\left[\left(k_{ij}\left(\Theta_{,i} + \Pi_{,i}\right)\right)_{,j} - \alpha_{ij}\varepsilon_{ij} - \beta_{ij}\gamma_{ij} - \gamma_1\varphi - \gamma_2\psi\right]\left(\dot{\Theta} + \dot{\Pi}\right)dV \quad (20)$$

For the last integral in Eq. (20) we can use the constitutive equation of entropy, η, so that we get

$$\ddot{F}(t) = 2\int_B \left[\rho_0\dot{u}_i\dot{u}_i + I_{ij}\dot{\Phi}_i\dot{\Phi}_j + \kappa_1\dot{\varphi}\,\dot{\varphi} + \kappa_2\dot{\psi}\,\dot{\psi}\right]dV -$$
$$-2\int_B (A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + C_{ijmn}\gamma_{ij}\gamma_{mn} + a_{ij}\varphi_{,i}\,\varphi_{,j} + c_{ij}\psi_{,i}\,\psi_{,j} + 2b_{ij}\varphi_{,i}\,\psi_{,j})dV -$$
$$-2\int_B (B_{ijmn}\varepsilon_{ij}\gamma_{mn} + A_{ij}\varepsilon_{ij}\varphi + B_{ij}\varepsilon_{ij}\psi + C_{ij}\gamma_{ij}\,\varphi + D_{ij}\gamma_{ij}\,\psi)dV -$$
$$-2\int_B (\alpha_1\varphi^2 + \alpha_1\varphi^2 + 2\alpha_3\varphi\psi)dV + 2h - 2\int_B a\left|\dot{\Theta} + \dot{\Pi}\right|^2 dV. \quad (21)$$

Now we will consider the energy conservation law (15) so we can write

$$\ddot{F}(t) = 4\int_B \left[\rho_0\dot{u}_i\dot{u}_i + I_{ij}\dot{\Phi}_i\dot{\Phi}_j + \kappa_1\dot{\varphi}\,\dot{\varphi} + \kappa_2\dot{\psi}\,\dot{\psi}\right]dV +$$
$$+4\int_B \frac{1}{T_0}k_{ij}\theta_{,i}\,\theta_{,j}dV - 2(E(0) - h). \quad (22)$$

By direct combination of the functions $F(t)$, $\dot{F}(t)$ and $\ddot{F}(t)$ we obtain:

$$F(t)\ddot{F}(t) - \left(\dot{F}(t) - \frac{2}{T_0}\int_B k_{ij}\Pi_{,i}\,\Pi_{,j}\,dV\right)^2 \geq 2(h + E(0))F(t) \quad (23)$$

If we consider the null initial data, from Eq. (17) we deduce that:

$$\frac{2}{T_0}\int_B k_{ij}\Pi_{,i}\,\Pi_{,j}\,dV = 0 \tag{24}$$

Also, we denote by $\mathcal{F}$ (t) the function which corresponds to $F(t)$ in the case $h = \omega = 0$. This particularity together with Eq. (24) and Eq. (23) lead to the conclusion:

$$\mathcal{F}(t)\ddot{\mathcal{F}}(t) - \dot{\mathcal{F}}^2(t) \geq 0$$

This last inequality through integration ensures us that

$$\mathcal{F}(t) \leq \mathcal{F}(0)^{1-t/t1}\,\mathcal{F}(t_1)^{t/t1}, \quad 0 \leq t \leq t_1$$

from which we conclude that

$$\mathcal{F}(t) = 0, \quad 0 \leq t \leq t_1$$

which guarantees the uniqueness of the solution and the proof of the Theorem 1 is complete. ■

The following result proves that the solution of the first boundary value problem in our context is unstable.

Theorem 2. *Suppose that conditions from Eq.(12) and Eq.(13) hold. If the symmetry assumptions Eq.(3) are satisfied and $E(0) < 0$, then the solution of the first initial boundary value problem is asymptotically unbounded.*

Proof. Because of the assumption $E(0) < 0$ we are able to choose ω large enough to have satisfied the condition:

$$F(0) > \nu$$

which leads to

$$F(t) \geq \frac{F(0)\dot{F}(0)}{\dot{F}(0)-\nu}\exp\left(\frac{\dot{F}(0)-\nu}{F(0)}\right) - \frac{\nu F(0)}{\dot{F}(0)-\nu} \tag{25}$$

Inequality (25) proves the exponential growth of solutions, ie if t tends to infinity, the solutions become unbounded since they are bounded inferiorly by an exponential function. ■

In the following we intend to state and prove a result of existence of the solution for mixed problem formulated above. For this purpose we use a semigroup of operators properly constructed, by which the mixed initial boundary value problem will be transformed into an abstract evolution equation in a Hilbert space, conveniently built.

Using a suggestion given by Helmholtz free energy Eq.(2), we consider the expression U defined by

$$\begin{aligned} 2U = {} & A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + 2B_{ijmn}\varepsilon_{ij}\gamma_{mn} + 2A_{ij}\varepsilon_{ij}\varphi + 2B_{ij}\varepsilon_{ij}\psi + \\ & + C_{ijmn}\gamma_{ij}\gamma_{mn} + C_{ij}\gamma_{ij}\varphi + D_{ij}\gamma_{ij}\psi + a_{ij}\varphi_{,i}\varphi_{,j} + \\ & + 2b_{ij}\varphi_{,i}\psi_{,j} + c_{ij}\psi_{,i}\psi_{,j} + \alpha_1\varphi^2 + \alpha_2\psi^2 + 2\alpha_3\varphi\psi \end{aligned} \tag{26}$$

and assume that U is a positive definite quadratic form, i.e., there is a positive constant C_0 such that

$$U \geq C_0\left(\varepsilon_{ij}\varepsilon_{ij} + \gamma_{ij}\gamma_{ij} + \varphi_{,i}\varphi_{,i} + \psi_{,i}\psi_{,i} + \varphi^2 + \psi^2\right). \tag{27}$$

Condition (27) appears as a natural condition imposed for reasons of stability of the solution for the dynamic problem in our context. Indeed, in Theorem 2 we proved that if condition (27) fails, then the solution of mixed initial boundary value problem becomes unbounded when t tends to infinity.

Along with the usual Hilbert spaces $W_0^{1,2}$, $\mathbf{W}_0^{1,2}$, L^2 and $\mathbf{L}^2$ we consider the set $\mathcal{Z}$ defined by:

$$\mathcal{Z} = \{(\boldsymbol{u},\boldsymbol{v},\boldsymbol{\Phi},\boldsymbol{\eta},\varphi,\varsigma,\psi,\chi,\theta) : \boldsymbol{u} \in \mathbf{W}_0^{1,2}, \boldsymbol{v} \in \mathbf{L}^2, \boldsymbol{\varphi} \in \mathbf{W}_0^{1,2}, \boldsymbol{\eta} \in \mathbf{L}^2, \varphi,\psi \in W_0^{1,2}, \varsigma,\chi,\theta \in L^2\} \tag{28}$$

In the above, the bold notations are for vector field three dimensional, that is:

$$\mathbf{W}_0^{1,2} = \left[W_0^{1,2}\right]^3, \quad \left[\mathbf{L}^2\right]^3 = \left[L^2\right]^3.$$

With a suggestion given by the equations of motion (7) and the balances of the equilibrated forces Eq. (8), we consider the operators:

$$\begin{aligned}
&M_i^1\,\mathbf{u} = (A_{ijmn}\varepsilon_{mn})_{,j}, \quad M_i^2 = (B_{ijmn}\gamma_{mn}),j, \quad N_i\varphi = (A_{ij}\varphi)_{,j}, \quad H_i\,\psi = (B_{ij}\psi)_{,j},\\
&M_i^{1^*}\mathbf{u} = (B_{mnij}\varepsilon_{mn})_{,j} + \varepsilon_{ijk}A_{jkmn}\varepsilon_{mn}, \quad M_i^{2^*}\boldsymbol{\Phi} = (C_{ijmn}\gamma_{mn})_{,j} + \varepsilon_{ijk}B_{jkmn}\gamma_{mn},\\
&P_i\theta = (-\alpha_{ij}\theta)_{,j}, N_i^*\varphi = (C_{ij}\varphi)_{,j} - \varepsilon_{ijk}A_{jk}\varphi,\\
&H_i^*\psi = (D_{ij}\psi)_{,j} - \varepsilon_{ijk}B_{jk}\psi,\; P_i^*\theta = (-\beta_{ij}\theta)_{,j} - \varepsilon_{ijk}\alpha_{jk}\theta,\\
&R\,\mathbf{u} = -A_{ij}\varepsilon_{ij}, \quad V\boldsymbol{\Phi} = -B_{ij}\gamma_{ij}, \quad S\varphi = (a_{ij}\varphi_{,j})_{,i} - \alpha_1\varphi,\\
&T\psi = (b_{ij}\psi_{,j})_{,i} - \alpha_3\psi, U\theta = \gamma_1\theta, \quad R^*\mathbf{u} = -C_{ij}\varepsilon_{ij}, \quad V^*\boldsymbol{\Phi} = -D_{ij}\gamma_{ij},\\
&W\varphi = (c_{ij}\varphi_{,j})_{,i} - \alpha_3\varphi, \quad X\psi = (d_{ij}\psi_{,j})_{,i} - \alpha_2\psi,\\
&Y\,\theta = \gamma_2\theta, \quad Q\mathbf{v} = -\alpha_{ij}e_{ij}, \quad Q^*\boldsymbol{\eta} = -\beta_{ij}f_{ij},\\
&L\varsigma = -\gamma_1\varsigma, \quad G\chi = -\gamma_2\chi, \quad Y^*\theta = (k_{ij}\theta_{,i})_{,j},
\end{aligned} \tag{29}$$

where we used the notations:

$$\mathbf{u} = (u_i), \quad \boldsymbol{\Phi} = (\boldsymbol{\Phi}_i), \quad \mathbf{v} = (v_i), \quad \boldsymbol{\eta} = (\eta_i).$$

The operators defined above will be considered as components of a matrix operator which we will denote by $\mathcal{T}$ and which is defined on the set $\mathcal{Z}$ introduced in Eq. (28).

Now, with the help of operator $\mathcal{T}$, we can transform the mixed initial boundary value problem in the context of micropolar bodies with double porosity into an abstract evolution equation, as follows:

$$\frac{dw}{dt} = \mathcal{T}w + \mathcal{G}(t), w(0) = w_0 \tag{30}$$

Here we used the following notations

$$\mathcal{G}(t) = ((F_i), (M_i), G, L, S), \quad w_0 = \left((u_i^0), (u_i^1), (\boldsymbol{\Phi}_i^0), (\boldsymbol{\Phi}_i^1), \varphi^0, \varphi^1, \psi^0, \psi^1, \theta^0\right).$$

Now we want to equip the set $\mathcal{Z}$ by a structure of Hilbert space. So, for two arbitrary elements $w, w' \in \mathcal{Z}$,

$$w = ((u_i), (v_i), (\boldsymbol{\Phi}_i), (\eta_i), \varphi, \varsigma, \psi, \chi, \theta), \quad w' = \left((u_i'), (v_i'), (\boldsymbol{\Phi}_i'), (\eta_i'), \varphi', \varsigma', \psi', \chi', \theta'\right)$$

we define the inner product $\langle w, w'\rangle$ by:

$$\langle w, w'\rangle = \int_B \left(\rho v_i v_i' + I_{ij}\eta_i\eta_i' + \kappa_1\varsigma\varsigma' + \kappa_2\chi\chi' + a\theta\theta' + 2U^*\right)dV \tag{31}$$

where U^* has the expression (see Eq. (26))

$$\begin{aligned}
2U^* &= A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}' + B_{ijmn}(\varepsilon_{ij}'\gamma_{mn} + \varepsilon_{ij}\gamma_{mn}') + A_{ij}\left(\varepsilon_{ij}'\varphi + \varepsilon_{ij}\varphi'\right) +\\
&+ B_{ij}\left(\varepsilon_{ij}'\psi + \varepsilon_{ij}\psi\right) + C_{ijmn}\gamma_{ij}\gamma_{mn}' + C_{ij}\left(\gamma_{ij}'\varphi + \gamma_{ij}\varphi'\right) +\\
&+ D_{ij}\left(\gamma_{ij}'\psi + \gamma_{ij}\psi'\right) + a_{ij}\varphi_{,i}\varphi_{,j}' + b_{ij}\left(\varphi_{,i}'\psi_{,j} + \varphi_{,i}\psi_{,j}'\right) +\\
&+ c_{ij}\psi_{,i}\psi_{,j}' + \alpha_1\varphi\varphi' + \alpha_2\psi\psi' + 2\alpha_3(\varphi'\psi + \varphi\psi').
\end{aligned}$$

As is known, the inner product Eq.(31) induces a norm on space $\mathcal{Z}$, as follows:

$$\|w\| = \int_B \left(\rho v_i v_i + I_{ij}\eta_i\eta_i' + \kappa_1\varsigma^2 + \kappa_2\chi^2 + a\theta^2 + 2U^*\right)dV \tag{32}$$

where the bilinear form U have the expression given in Eq. (26).

Assume that the domain $\mathcal{D}$ of the operator $\mathcal{T}$ is:

$$\mathbf{W}_0^{1,2} \cap \mathbf{W}^{2,2}\text{ x }\mathbf{W}^{1,2}\; x\; W_0^{1,2} \cap W^{2,2} x W_0^{1,2} x W_0^{1,2} \cap W^{2,2} x W_0^{1,2} x W^{2,2}.$$

Now we consider the expressions (39) of the operators which are part of the matrix operator $\mathcal{T}$ so that we get the inequality:

$$\langle \mathcal{T} w, w \rangle = -\frac{1}{2T_0} \int_B k_{ij} \theta_{,i} \, \theta_{,j} dV \leq 0. \tag{33}$$

I have included in the following theorem a result of existence and uniqueness.

Theorem 3. *Assume be satisfied the relations (3), (12), (13), and (27). If $\mathcal{G}(t)$ is a continuous differentiable function and $w_0 \in \mathcal{D}$, then the abstract problem Eq.(30) has a solution $w(t)$ which is a continuous function and it is unique.*

To proove this important result, we need two auxiliary results included in the following two Propositions.

Proposition 1. There is a solution $w = (\boldsymbol{u}, \boldsymbol{v}, \mathbf{\Phi}, \boldsymbol{\eta}, \varphi, \varsigma, \psi, \chi, \theta)$ of the equation

$$\mathcal{T} \mathrm{w} = \mathcal{F} \tag{34}$$

where $\mathcal{F} = (f_1; f_2; f_3; f_4; f_5; f_6; f_7; f_8; f_9) \in \mathcal{Z}$.

Proof. First, we write equation (34) on components, so that we obtain the following system of equations:

$$\begin{aligned}
&\mathbf{v} = \mathbf{f}_1\\
&\mathbf{M}^1\mathbf{u} + \mathbf{M}^2\boldsymbol{\Phi} + \mathbf{N}\varphi + \mathbf{H}\psi + \mathbf{P}\theta = \mathbf{f}_2\\
&\boldsymbol{\eta} = \mathbf{f}_3\\
&\mathbf{M}^{1^*}\mathbf{u} + \mathbf{M}^{2^*}\boldsymbol{\Phi} + \mathbf{N}^*\varphi + \mathbf{H}^*\psi + \mathbf{P}^*\theta = \mathbf{f}_4\\
&\varsigma = f_5\\
&R\mathbf{u} + V\boldsymbol{\Phi} + S\varphi + T\psi + U\theta = f_6
\end{aligned} \tag{35}$$

and, also, the system of equations:

$$\begin{aligned}
\chi &= f_7\\
R^*\mathbf{u} + V^*\boldsymbol{\Phi} + W\varphi + X\psi + Y\theta &= f_8\\
Q\mathbf{v} + Q^*\boldsymbol{\eta} + L\varsigma + G\chi + \mathrm{Y}^*\theta &= \mathrm{g}_9.
\end{aligned} \tag{36}$$

From the systems of equations (35) and (36) we deduce that $\mathbf{v}, \boldsymbol{\eta} \in \mathbf{W}_0^{1,2}$ and $\varsigma,\ \chi \in W_0^{1,2}$. Also, considering the equations $(35)_1$, $(35)_3$, $(35)_5$, $(36)_1$ and $(36)_3$, we obtain:

$$Y^*\theta = g_9 - Q\mathbf{f}_1 - Q^*\mathbf{f}_3 - Lf_5 - gf_7. \tag{37}$$

The equation (37) has a solution $\theta \in W_0^{1,2}$, because the right-hand side of this equation is a function which belongs to $W^{-1,2}$. Therefore, in what follows we will assume that θ is a known function and we can rewrite the system (35) and (36) in the form:

$$\begin{aligned}
\mathbf{M}^1\mathbf{u} + \mathbf{M}^2\boldsymbol{\Phi} + \mathbf{N}\varphi + \mathbf{H}\psi &= \mathbf{f}_2 - \mathbf{P}\theta\\
\mathbf{M}^{1^*}\mathbf{u} + \mathbf{M}^{2^*}\boldsymbol{\Phi} + \mathbf{N}\varphi + \mathbf{H}\psi &= \mathbf{f}_4 - \mathbf{P}^*\theta\\
R\mathbf{u} + \mathbf{V}\boldsymbol{\Phi} + S\varphi + T\psi &= f_6 - U\theta\\
R^*\mathbf{u} + V^*\boldsymbol{\Phi} + W\varphi + X\psi &= f_8 - Y\theta.
\end{aligned} \tag{38}$$

Now we will try to find a solution $(\mathbf{u}, \Phi, \varphi, \psi)$ of the system (38). For this purpose we use bilinear form $\mathcal{B}$ defined by:

$$\begin{aligned}
\mathcal{B}[(\mathbf{u},\boldsymbol{\Phi},\varphi,\psi),(\mathbf{u},\boldsymbol{\Phi},\varphi,\psi)] = \int_B [&(\mathbf{M}^1\mathbf{u} + \mathbf{M}^2\boldsymbol{\Phi} + \mathbf{N}\varphi + \mathbf{H}\psi)\,\mathbf{u}^* + \left(\mathbf{M}^{1^*}\mathbf{u} + \mathbf{M}^{2^*}\boldsymbol{\Phi} + \mathbf{N}\varphi + \mathbf{H}\psi\right)\boldsymbol{\Phi}^* +\\
&+ (R\,\mathbf{u} + \mathbf{V}\boldsymbol{\Phi} + \mathbf{S}\varphi + \mathbf{T}\psi)\varphi^* + (R^*\mathbf{u} + \mathbf{V}^*\boldsymbol{\Phi} + \mathbf{W}\varphi + \mathbf{X}\psi)\psi^*]dV.
\end{aligned} \tag{39}$$

Due to the regularity conditions imposed to the domain B and its boundary ∂B, we can use the divergence theorem so that the integral of the right-hand side of relation (39) reduces to an integral over the surface. Thus we deduce that $\mathcal{B}$ is a bounded bilinear form de_ned in $\mathbf{W}^{1,2}$. In addition, considering that U is a positive definite quadratic form, i.e. it satisfies the condition (27), we are led to the conclusion that $\mathcal{B}$ is a coercive bilinear form. It means that are satisfied the conditions of known Lax-Milgram Theorem[19] with which we deduce the existence of a solution $(\mathbf{u}, \boldsymbol{\Phi}, \varphi, \psi)$ of the system (38).

Finally, we obtain that the equation (34) has a solution in the domain D, which concludes the proof of Proposition 1. ∎

Remark 1. If we denote by $\rho(\mathcal{T})$ the resolvent of the operator $\mathcal{T}$, then Liu and Zheng[18] have shown that the statement of proposition 1 can be rephrased as $0 \in \rho(\mathcal{T})$.

Proposition 2. Assume be satisfied the relations (3), (12), (13), and (27). Then the operator $\mathcal{T}$ is the generator of a C^0- semigroup of contractions in the Hilbert space $\mathcal{Z}$.

Proof. If we consider the components of the operator T, we deduce that it is dissipative. Also, the effective domain $\mathcal{D}$ of the operator $\mathcal{T}$ is dense in the Hilbert space $\mathcal{Z}$. According to the remark above, we have that $0 \in \rho(\mathcal{T})$. So, we met all the conditions of known Lumer-Phillips Theorem,[20] which ensures that $\mathcal{T}$ is the generator of a C^0 - semigroup of contractions and such, Proposition 2 is proved. ∎

Remark 2. If we take into account the fact that $w_0 \in \mathcal{D}$, the effective domain of the operator $\mathcal{T}$, from Propositions 1 and 2 we obtain the conclusion of Theorem 3.

As usual, with the help of C^0 - semigroup of contractions we can deduce the continuous dependence of solutions with respect to initial data and, also, in relation to the loads.

Furthermore, in case of the homogeneous problem corresponding to problem (30) we have the following result:

Theorem 4. *Suppose that the relations (3), (12), and (13) and (27) hold. If the internal energy is positive definite, then the solutions of homogeneous problem Eq.(30) are stable.*

Proof. Taking into account the conservation law (25) and the fact that the solution $w(t)$ satisfies the relation (32), where $w = ((u_i), (v_i), (\mathbf{\Phi}_i), (\eta_i), \varphi, \varsigma, \psi, \chi, \theta)$ we deduce that in the case of the homogeneous problem we have

$$\|w(t)\| \le E(0), \quad \forall t \ge 0.$$

This gives the stability of the solutions. ∎

IV. CONCLUSIONS

As usual, with the help of C^0 - semigroup of contractions we can deduce the continuous dependence of solutions with respect to initial data and, also, in relation to the loads. Since we have already proved the existence and uniqueness of the solution, the continuous dependence of solutions, ensures that the problem is well posed. We must emphasize that these results were possible only because the internal energy is positive. Also, in the last part of our study can be seen as the positive definiteness of the internal energy guarantees the stability of the solutions of the homogeneous problem.

[1] G. I. Barenblatt and I. P. Zheltov, "On the basic equations of seepage of homogeneous liquids in fissured rock," Akad. Nauk SSSR **132**, 545 (1960).
[2] R. Wilson and E. Aifantis, "On the theory of consolidation with double porosity," Int. J. Eng. Sci. **20**, 1009 (1982).
[3] S. C. Cowin, "Bone poroelasticity," J. Biomech. **32**, 217 (1999).

[4] N. Khalili and A. P. S. Selvadurai, "A fully coupled constitutive model for thermohydro-mechanical analysis in elastic media with double porosity," Geophys Res. Lett. **30**, 2268 (2003).
[5] S. R. Pride and J. G. Berryman, "Linear dynamics of double-porosity dual-permeability materials-I," Phys. Rev. E **68**, 366 (2003).
[6] Y. Zhao and M. Chen, "Fully coupled dual-porosity model for anisotropic formations," Int. J. Rock Mech. Min. Sci. **43**, 1128 (2006).
[7] B. Straughan, "Stability and uniqueness in double porosity elasticity," Int. J. Eng. Sci. **65**, 1 (2013).
[8] D. Iesan and R. Quintanilla, "On a theory of thermoelastic materials with double porosity structure," J. Thermal Stresses **37**, 1017 (2014).
[9] J. W. Nunziato and S. C. Cowin, "A nonlinear theory of elastic materials with voids," Arch. Rational Mech. Anal. **72**, 175 (1979).
[10] S.C. Cowin and J.W. Nunziato, "Linear elastic materials with voids," J. Elasticity **13**, 125 (1983).
[11] D. Iesan, *Thermoelastic Models of Continua* (Kluwer Academic, Dordrecht, The Netherlands, 2004).
[12] K.A. Ames and L.E. Payne, "Stabilizing solutions of the equations of dynamical linear thermoelasticity backward in time," Stability Appl. Anal. Contin. Media **1**, 243 (1991).
[13] G. Iovane and F. Passarella, "Saint-Venant's principle in dynamic porous thermoelastic media with memory for heat flux," J. Therm. Stresses **27**, 983 (2004).
[14] M. Marin, "Some estimates on vibrations in thermoelasticity of dipolar," bodies, Journal of Vibration and Control **16**, 33 (2010).
[15] M. Marin and G. Stan, "Weak solutions in Elasticity of dipolar bodies with stretch," Carpathian Journal of Mathematics **29**, 33 (2013).
[16] M. Marin, R.P. Agarwal, and S.R. Mahmoud, "Nonsimple material problems addressed by the Lagrange's identity," Boundary Value Problems (2013) Art. no. 135.
[17] M. Marin and O. Florea, "On temporal behavior of solutions in thermoelasticity of porous micropolar bodies," Ann. Sci. Univ. Ovidius **22**, 169 (2014).
[18] Z. Liu and S. Zheng, "Semigroups associated with dissipative systems," *Research Notes in Mathematics* (Chapman & Hall/CRC, Boca Raton, FL, 1999), Vol. 398.
[19] R.A. Adams, *Sobolev Spaces* (Academic Press, New York, 1975).
[20] A. Pazy, *Semigroups of linear operators and applications* (Springer, New York, Berlin, 1983).