RESEARCH ARTICLE | MARCH 24 2015

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AIP Advances 5, 037133 (2015)
https://doi.org/10.1063/1.4916364

Simulations for Maxwell fluid flow past a convectively heated exponentially stretching sheet with nanoparticles

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(Received 4 November 2014; accepted 17 March 2015; published online 24 March 2015)

This article addresses steady flow of Maxwell nanofluid induced by an exponentially stretching sheet subject to convective heating. The revised model of passively controlled wall nanoparticle volume fraction is taken into account. Numerical solutions of the arising non-linear boundary value problem (BVP) are obtained by using MATLAB built-in function bvp4c. Simulations are performed for various values of embedded parameters which include local Deborah number, Prandtl number, Biot number, Brownian motion parameter and thermophoresis parameter. The results are consistent with the previous studies in some limiting cases. It is found that velocity decreases and temperature increases when the local Deborah number is increased. Moreover the influence of Brownian diffusion on temperature and heat transfer rate is found to be insignificant. [http://dx.doi.org/10.1063/1.4916364]

I. INTRODUCTION

Many fluids of industrial importance such as multigrade oils, composite materials, blood, polymers, liquid detergents, fruit juices, printing inks and industrial suspensions exhibit shear-rate dependent viscosity and thus cannot be described by the classical model of Newtonian fluids. In view of flow diversity in nature, various models of non-Newtonian fluids have been proposed by researchers. The upper-convected Maxwell fluid is a class of visco-elastic fluid that can explain characteristics of fluid relaxation time. It excludes complicated effects of shear-dependent viscosity and thus allows one to emphasize the influence of fluid's elasticity on characteristics of its boundary layer. The boundary layer equations governing two-dimensional flow of upper-convected Maxwell fluid were first derived by Harris.[1] Sadeghy et al.[2] discussed flow over a moving flat plate, the so-called Sakiadis flow, considering Maxwell fluid. They derived local similarity solutions by four different approaches and concluded that velocity is a decreasing function of the local Deborah number. Sadeghy et al.[3] also studied stagnation-point flow of upper-convected Maxwell fluid using Chebyshev pseudo-spectral collocation-point method. Kumari and Nath[4] used finite difference method to compute numerical solutions of the boundary value problem arising in mixed convection stagnation-point flow of Maxwell fluid. Hayat et al.[5] obtained homotopy based series solution for stagnation-point flow of Maxwell fluid towards a stretching sheet. After these pioneering works, researchers have widely discussed two-and three-dimensional flows of Maxwell fluids in different geometries (see Aliakbar et al.,[6] Raftari and Yildirim,[7] Hayat et al.,[8,9] Mukhopadhyay,[10] Abel et al.,[11] Hayat et al.,[12] Shateyi[13] and Mushtaq et al.[14]).

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The pioneering study of Crane[15] on two-dimensional flow induced by a linearly stretching surface is relevant in diverse industrial processes such as polymer processing, cooling of metallic and rubber sheets, condensation process, crystal growth process etc. In such applications, the velocity of the extruded sheet may not be necessarily linear. Keeping this in view, Magyari and Keller[16] addressed boundary layer flow caused by an exponentially stretching surface. They also examined heat transfer characteristics by considering exponential surface temperature distribution. Elbashbeshy[17] extended this work to a porous stretching sheet. Viscoelastic effects in flow driven by an isothermal exponentially stretching surface were presented by Khan and Sanjayanand.[18] Radiation effects in viscous flow characterized by exponentially stretching surface were analytically investigated by Sajid and Hayat.[19] Recently, flow and heat transfer analyses concerning an exponentially stretching surface have been widely considered by many authors (see Bhattacharyya,[20] Mustafa et al.,[21,22] Liu et al.,[23] Abbas et al.,[24] Hussain et al.[25] etc.).

Enhanced thermal behavior of nanofluids could provide a basis for innovation in heat transfer intensification, which is of major importance for a number of industrial sectors including transportation, power generation, micro-manufacturing, thermal therapy for cancer treatment, chemical and metallurgical sectors, as well as heating, cooling, ventilation and air-conditioning.[26] Buongiorno[27] explored that significant improvement in heat transfer processes is the consequence of two main velocity slip effects namely Brownian motion and thermophoretic diffusion of nanoparticles. Kuznetsov and Nield[28] examined heat transfer characteristics in natural convective boundary layer flow of nanofluid past a vertical plate using Buongiorno's model. Nield and Kuznetsov[29] studied the Cheng–Minkowycz problem for natural convective flow past a stationary flat plate immersed in porous medium saturated by nanofluid. The fundamental work on steady boundary layer flow of nanofluid induced by linearly stretching surface was examined by Khan and Pop.[30] They found that thermal boundary layer thickness is an increasing function of both Brownian motion and thermophoresis parameters. Mustafa et al.[31] examined two-dimensional stagnation-point flow towards a stretching sheet, the so called Hiemenz flow, using nanofluid. In another investigation, Mustafa et al.[32] derived both numerical and homotopy solutions for stagnation-point flow of nanofluid caused by exponentially stretching surface. Unsteady flow of nanofluid caused by impulsively stretching plate was addressed by Mustafa et al.[33] Makinde et al.[34] explored the problem of Khan and Pop[30] by utilizing convective boundary condition. They observed that intense convective heating at the sheet results in larger temperature and rate of heat transfer from the sheet. During the past few years, flow and heat transfer characteristics in nanofluids have been substantially addressed by researchers (see Refs. 35–46).

To the best of our knowledge, flow of Maxwell nanofluid past an exponentially stretching surface has not been discussed in the literature. The present study investigates this problem by invoking convective surface boundary condition. Such consideration is important in the sense that base fluids in realistic processes exhibit visco-elastic properties. It has also experimentally proven that de-ionized water-nanofluids with poly-ethylene oxide or poly-vinyl-pyrrolidone as a dispersant significantly improved rheological properties in the fluid. Ethylene glycol-Al_2O_3, ethylene glycol-CuO and ethylene glycol-ZnO are the examples of visco-elastic nanofluids. Simulations in this study assume that mass flux of nanoparticles at the wall is zero. It is found the self similar solution of the problem is possible only when heat transfer coefficient (associated with the hot fluid below the surface) is proportional to $x^{(n-1)/2}$. The solutions have been computed by using the MATLAB built-in routine bvp4c and are found in agreement with previous studies in a limiting sense.

II. PROBLEM FORMULATION

Consider steady two-dimensional incompressible flow of Maxwell nanofluid driven by an exponentially stretching sheet aligned with the $x-$ axis as shown in Fig. 1. Let $U_w(x) = Ue^{x/L}$ be the velocity distribution across the sheet. The temperature at the sheet is controlled by convective process which is characterized by the heat transfer coefficient $h_f(x)$ and temperature of the hot fluid T_f below the surface. In accordance with Kuznetsov and Nield,[35] we assume zero nanoparticle mass flux at the wall. Let T_∞ be the ambient fluid temperature and C_∞ denotes the nanoparticle volume fraction outside the boundary layer. Using the velocity field $\vec{V} = [u(x, y), v(x, y), 0]$, the temperature

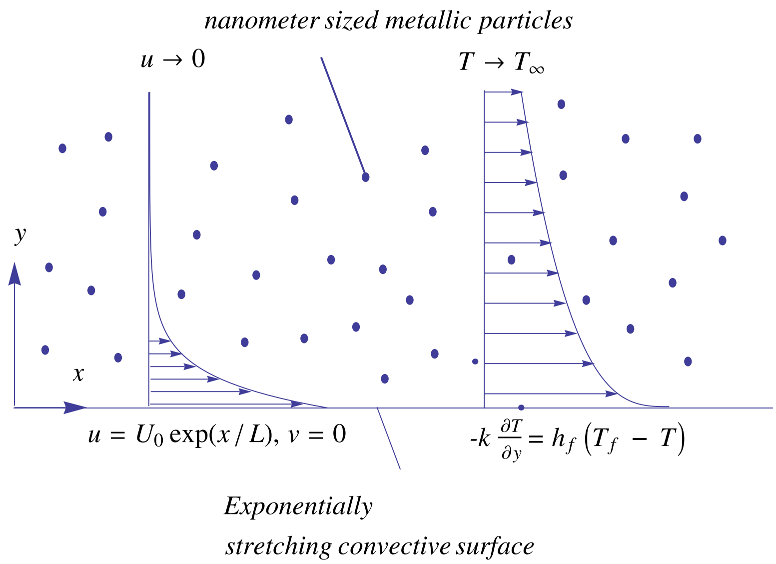

FIG. 1. Geometry of the problem.

distribution $T = T(x, y)$ and nanoparticle volume fraction distribution $C = C(x, y)$, the boundary layer equations governing the conservations of mass, momentum, energy and nanoparticle mass can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda_1\left[u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right] = \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

In Eqs. (1)-(4), u and v are the velocity components along the $x-$ and $y-$ directions respectively, ν is the kinematic viscosity, λ_1 is the relaxation time, T is the fluid temperature, C is the local nanoparticle volume fraction, α is the thermal diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and $\tau = (\rho c)_p/(\rho c)_f$ is the ratio of the effective heat capacity of the nanoparticle material to the effective heat capacity of the fluid. The boundary conditions in the present problem are

$$\begin{gathered} u = U_w(x) = U\exp(x/L), \quad v = 0, \quad -k\frac{\partial T}{\partial y} = h_f(T_f - T), \quad D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0 \text{ at } y = 0, \\ u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ as } y \to \infty, \end{gathered} \tag{5}$$

where k is the thermal conductivity and $h_f = he^{x/2L}$ is the heat transfer coefficient.

Using the following similarity transformations

$$\begin{gathered} \eta = \sqrt{\frac{U}{2\nu L}}e^{x/2L}y, \quad u = Ue^{x/L}f', \quad v = -\sqrt{\frac{\nu U}{2L}}e^{x/2L}(f + \eta f'), \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C}{C_\infty}, \end{gathered} \tag{6}$$

Eq. (1) is identically satisfied and Eqs. (2)–(5) become

$$f''' - 2f'^2 + ff'' + \lambda\left(3ff'f'' + \frac{\eta}{2}f'^2f'' - \frac{1}{2}f^2f''' - 2f'^3\right) = 0 \tag{7}$$

$$\frac{1}{Pr}\theta'' + f\theta' + Nb\theta'\phi' + Nt\theta'^2 = 0, \tag{8}$$

$$\phi'' + Scf\phi' + \frac{Nt}{Nb}\theta'' = 0, \tag{9}$$

$$\begin{gathered} f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -\gamma(1-\theta(0)), \quad Nb\phi'(0) + Nt\theta'(0) = 0, \\ f'(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0, \end{gathered} \tag{10}$$

where $\lambda = Re_x\lambda_1\nu/2L^2$ is the local Deborah number, $\gamma = h/k\sqrt{2\nu L/U}$ is the Biot number, $Nb = \tau D_B C_\infty/\nu$ is the Brownian motion parameter, $Nt = \tau D_T(T_f - T_\infty)/T_\infty\nu$ is the thermophoresis parameter, $Pr = \nu/\alpha$ is the Prandtl number and $Sc = \nu/D_B$ is the Schmidt number.

Physical quantity of interest in this study is the local Nusselt number Nu_x defined as

$$Nu_x = \frac{xq_w}{k\left(T_f - T_\infty\right)}, \tag{11}$$

where $q_w = -k\left(\partial T/\partial y\right)|_{y=0}$ is the wall heat flux. Now using dimensionless quantities (6) in Eq. (11), we obtain

$$Nu_x\text{Re}_x^{-1/2}\frac{2L}{x} = -\theta'(0) = Nur, \tag{12}$$

where $Re_x = 2Ue^{x/L}L/\nu$ is the local Reynolds number. It should be noted that reduced Sherwood number which is the dimensionless mass flux is identically zero.

III. NUMERICAL RESULTS AND DISCUSSION

Numerical solutions of the boundary value problems given in Eqs. (7)-(10) have been obtained by using the MATLAB built in function bvp4c. First of all we compare our results of $\theta'(0)$ with those of Magyari and Keller[16] and Abbas et al.[24] in limiting cases. The solutions are found in excellent agreement as can be seen from Table I. This gives us confidence that our results are accurate and more general than previously reported studies. In Table II, we provide a sample of our results for reduced Nusselt number $-\theta'(0)$ corresponding to different values of γ and λ with other parameters fixed. We found that stronger convective heating results in larger magnitude of reduced Nusselt number. However magnitude of reduced Nusselt number is a decreasing function of local Deborah number λ.

Fig. 2 shows effects of local Deborah number (λ) on the hydrodynamic boundary layer. Deborah number is a dimensionless variable that deals with the relaxation time i.e time taken by the fluid to obtain equilibrium in response to the applied stress. Fluids with small Deborah number display liquid-like behavior whereas large Deborah number corresponds to solid-like substances. An increase in λ corresponds to an increase in fluid viscosity which enhances resistance to flow

TABLE I. Comparison of values for $\theta'(0)$ with Magyari and Keller[16] and Abbas et al.[24] in the case of regular fluid.

	$\lambda = 0$		$\lambda = 0.5$	
Pr	Magyari and Keller [16]	Present	Abbas et al. [24]	Present
0.5	-0.330493	-0.330493	-0.175942	-0.301698
1	- 0.549643	- 0.549642	- 0.512337	- 0.512078
3	-1.122188	-1.122077	-1.074513	-1.074501
5	- 1.521243	- 1.521222	- 1.470621	- 1.47060
8	-1.991847	-1.991805	-1.939103	-1.93907
10	- 2.257429	- 2.257381	- 2.203874	- 2.20383

TABLE II. Values of reduced Nusselt number $-\theta'(0)$ for different values of γ and λ when $Pr = 10$, $Sc = 20$ and $Nt = Nb = 0.5$.

$\lambda \backslash \gamma$	0.1	0.5	1	2	5	10
0	0.095615282	0.39606233	0.62184538	0.8081829	0.91441581	0.94117195
0.5	0.095506377	0.39336962	0.61264612	0.78743758	0.88368911	0.90767901
1	0.095412876	0.39105294	0.60480027	0.7702038	0.85877696	0.88069131
1.5	0.095329259	0.3889778	0.59783014	0.75523798	0.83757471	0.85783446

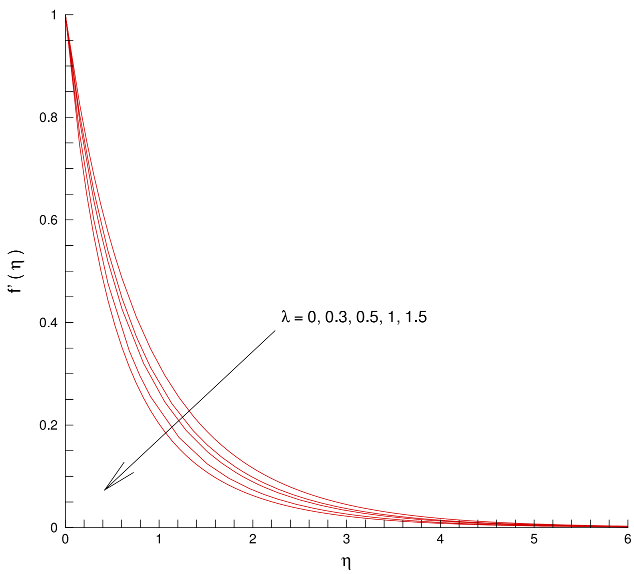

FIG. 2. Effect of λ on $f'(\eta)$.

and hence velocity decreases. Further, with an increase in λ, the profiles approach zero at smaller distances from the sheet indicating a reduction in boundary layer thickness.

Fig. 3 shows the influence of Biot number γ on temperature distribution. Biot number is defined as a ratio of convection heat transfer to the conduction heat transfer at the surface. It generally

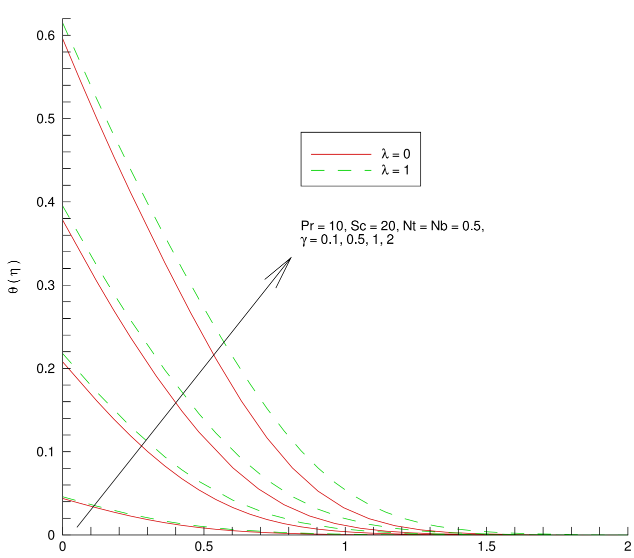

FIG. 3. Effect of γ on $\theta(\eta)$.

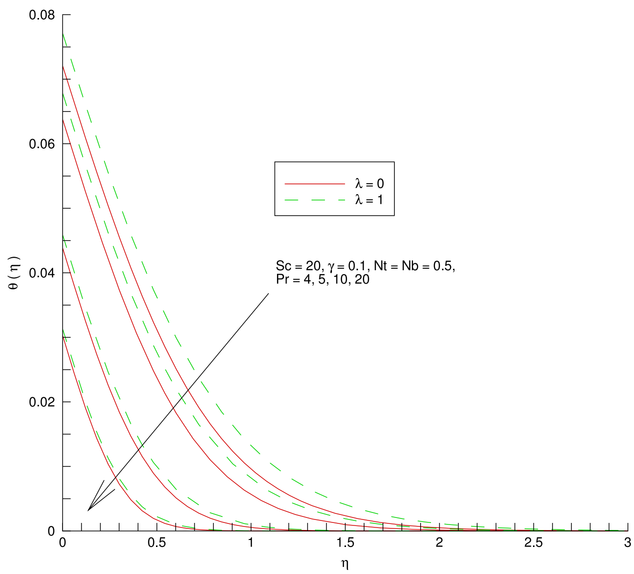

FIG. 4. Effect of Pr on $\theta(\eta)$.

depends on characteristic length of the surface, thermal conductivity of the surface and convective heat transfer coefficient of the hot fluid below the surface. A higher Biot number indicates less conductive substance such as plastic, paper, polymer etc. On the other hand, Biot number is small for higher conductive materials which include aluminum, iron, and steel etc. An increase in γ leads to higher surface temperature which results in larger penetration depth of temperature function.

Fig. 4 elucidates behavior of Prandtl number Pr on temperature θ. An increase in Pr reduces conduction and hence gives shorter penetration depth of temperature θ. Further the temperature profiles approach the free stream condition at large distances from the sheet when Pr is increased indicating an augmentation in thermal boundary layer thickness. In Fig. 5, we plot the temperature profiles corresponding to different values of thermophoresis parameter Nt. An increase in thermophoretic force allows nanoparticles to penetrate deeper into the fluid. As a consequence, temperature θ increases when therophoresis parameter Nt is increased.

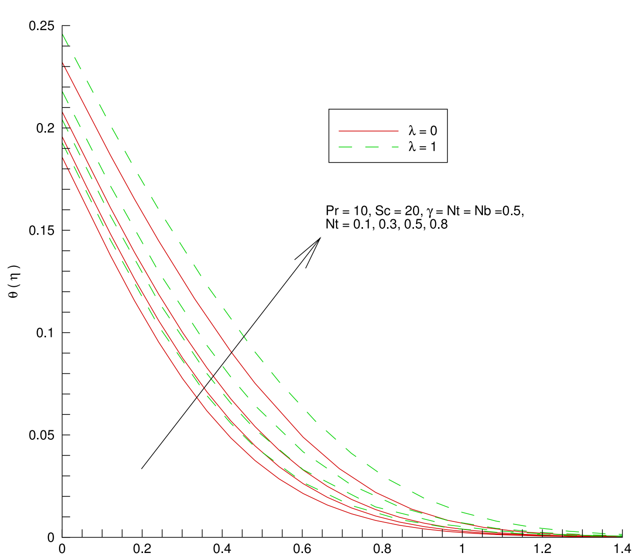

FIG. 5. Effect of Nt on $\theta(\eta)$.

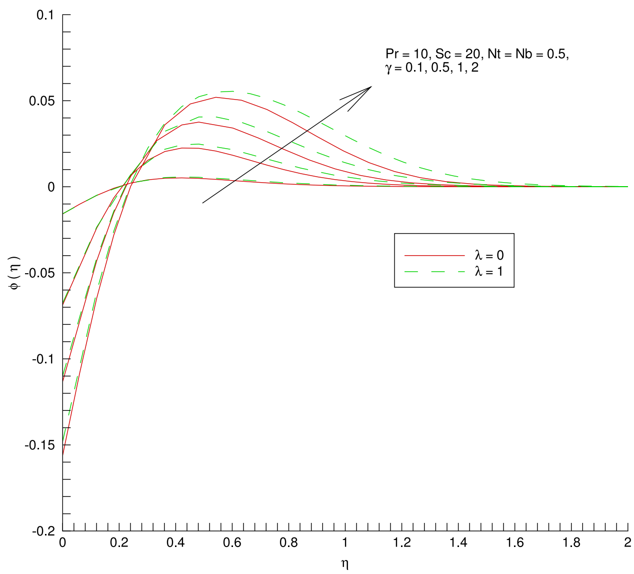

FIG. 6. Effect of γ on $\phi(\eta)$.

Fig. 6 depicts effects of Biot number γ on nanoparticle volume fraction ϕ. The higher surface temperature corresponding to larger Biot number energizes nanoparticles in the vicinity of the sheet. In order to release that additional energy, the nanoparticles travel away from the stretching wall. This results is larger penetration depth of nanoparticle volume fraction. The effect of Schmidt number Sc on the concentration profile is shown in Fig. 7. Schmidt number is a mass transfer analog of Prandtl number. Increase in Sc leads to decrease the Brownian diffusion coefficient which restricts the nanoparticles to infiltrate deeper into the fluid. For this reason, penetration depth of nanoparticle volume fraction decreases when Sc is increased. Fig. 8 gives the variation in nanoparticle volume fraction ϕ with an increase in thermophoresis parameter Nt. The bigger values of Nt coincide with

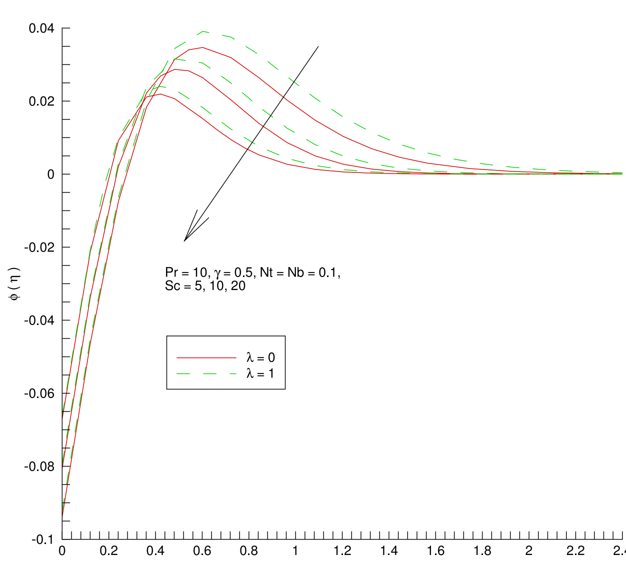

FIG. 7. Effect of Sc on $\phi(\eta)$.

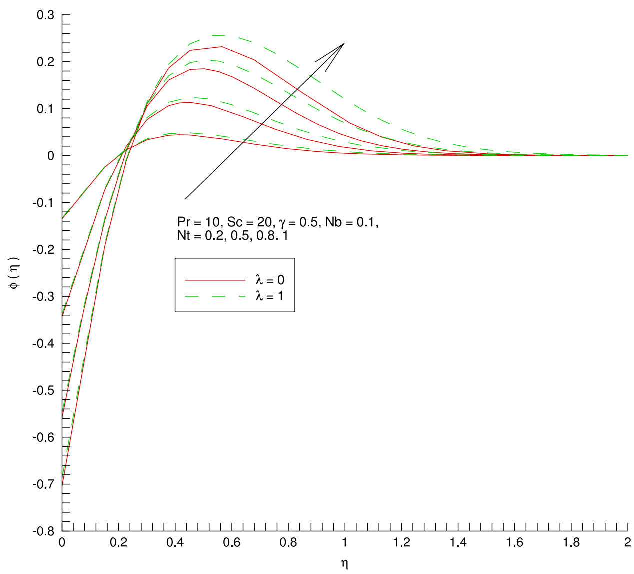

FIG. 8. Effect of Nt on $\phi(\eta)$.

the stronger thermophoretic diffusion which blows the nanoparticles away from the hot surface towards the cold ambient fluid. As a result the nanoparticle volume fraction is thicker for larger values of Nt. On the other hand, nanoparticle volume fraction ϕ is a decreasing function of Brownian motion parameter Nb (see Fig. 9).

Fig. 10 describes variation in reduced Nusselt number with an increase in Biot number γ. It can be seen that heat transfer rate at the sheet increases when γ is increased. Moreover $\theta'(0)$ approaches a constant finite value as $\gamma \to \infty$. This value of $\theta'(0)$ corresponds to the case of constant wall temperature in which $\theta(0) = 1$. Fig. 11 plots reduced Nusselt number versus Biot number γ for different values of λ. Heat transfer rate from the sheet is larger in the case of Newtonian fluid when compared with Maxwell fluid.

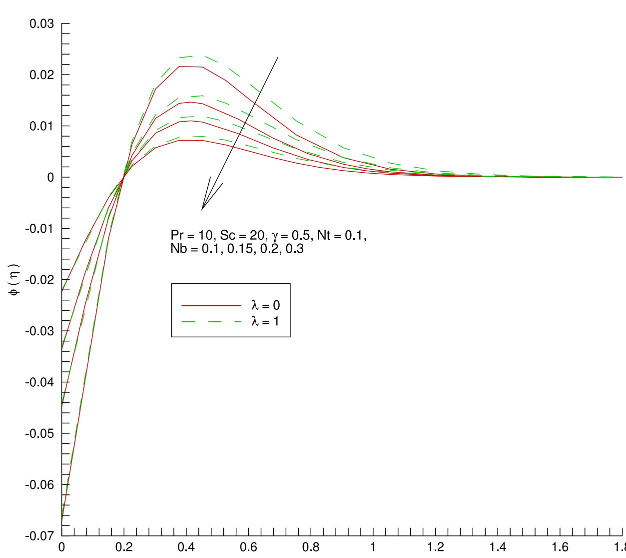

FIG. 9. Effect of Nb on $\phi(\eta)$.

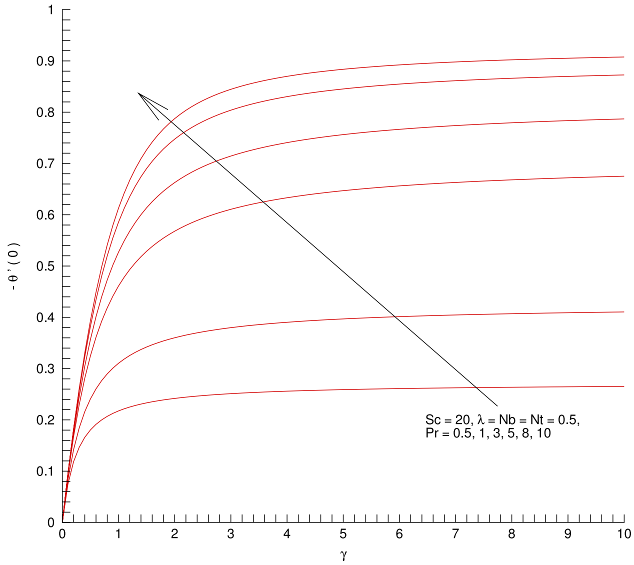

FIG. 10. Effects of Pr and λ on $-\theta'(0)$.

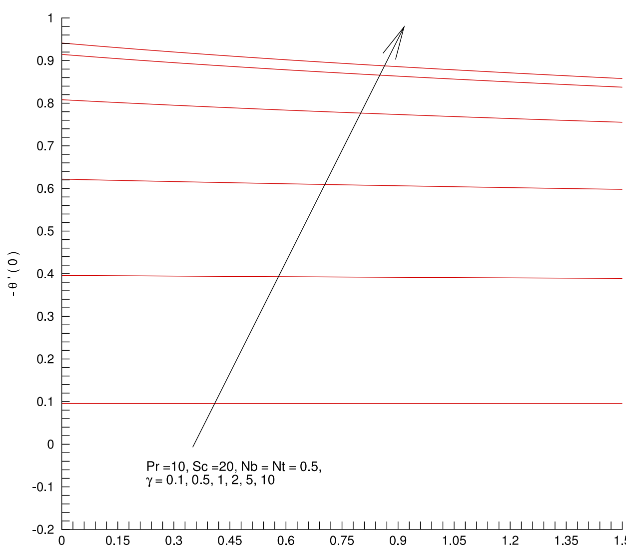

FIG. 11. Effects of γ and λ on $-\theta'(0)$.

IV. CONCLUSIONS

We investigated steady boundary layer flow of visco-elastic (Maxwell) fluid past an exponentially stretching sheet with nanoparticles. Brownian motion and thermophoresis effects were considered. Further the revised model of passively controlled wall nanoparticle volume fraction was adopted. Numerical solutions were obtained and these were found in agreement with the existing studies. Following are key points of this work.

1. Hydrodynamic boundary layer becomes thinner when local Deborah number λ is increased.
2. Temperature rises and nanoparticle volume fraction decreases when convective heating at the sheet is strengthened.
3. Reduced Nusselt number is a decreasing function of local Deborah number λ.
4. For sufficiently large values of Biot number γ, the results correspond to the case of constant wall temperature.

5. Impact of Brownian motion and thermophoretic diffusion on nanoparticle volume fraction is opposite in a qualitative sense.
6. Effect of Brownian motion on temperature and heat flux from the sheet is negligible.
7. The problem reduces to the case of Newtonian nanofluid by substituting $\lambda = 0$.

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