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Response to "Comment on 'Arbitrary amplitude ion-acoustic supersolitons in negative ion plasmas with two-temperature superthermal electrons'" [Phys. Plasmas 30, 044701 (2022)]

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Varghese *et al.* in their Comment reported that our results related to a recent paper entitled "Arbitrary amplitude ion-acoustic supersolitons in negative ion plasma with two-temperature superthermal electrons" are incorrect. In this connection, we would like to submit the following:

The first point of the Comment is that "The authors have considered a plasma consisting of two cold ion species, one of which is assumed to be positively charged while the other one can be either positive or negative." In reply of this, it may be noted that in the last paragraph of the left column on page no. 3, below Eq. (5) of Ref. 1, we have mentioned that the quantities n_1, n_2 and v_1, v_2 are the number densities and velocities of positive and negative ion species, respectively, which implies that Eqs. (1) and (2) are written for positive ion species (for n_1 and v_1) and Eqs. (3) and (4) are written for negative ion species (for n_2 and v_2). This can also be verified by Eq. (5), in which n_1 density is included taking positively charged, whereas n_2 density is considered as negatively charged. In the light of this, one can say that the present model is applicable only when one ion species is positive and the other ion species is negative. Therefore, the present theory cannot be applied to those cases in which both the species are positive or both are negative as mentioned by Varghese *et al.* in their Comment.[2] In fact in the present paper, ε_z represents the charge multiplicity of negative ion species and its value may be $\varepsilon_z = +1, +2$, etc. (only positive values). However, in the present paper, for numerical illustration in Figs. 1–7, we have considered $\varepsilon_z = +1$, which represents the presence of the second ion species with singly ionized negative ions. In the first paragraph of the right column on page no. 4 of our recent paper,[1] we have wrongly mentioned $\varepsilon_z = \pm 1$. It is actually $\varepsilon_z = +1$, and $\varepsilon_z = -1$ is not allowed in the recent paper. We are extremely sorry for this typological mistake due to which confusion arises about two ion species.

The second point of the Comment is that the investigation of the existence condition in Sec. III is incomplete and the resulting analysis is incorrect. Since we have already mentioned in the reply of the first Comment[2] that the present model is applicable to a multicomponent plasma consisting of one positive ion species and the other as negative ion species, one cannot consider both the ion species either positive or negative as considered in the Comment. In their Comment, Varghese *et al.*[2] mentioned that two additional equations should have been given after Eq. (14). In this connection, we would like to mention that after Eq. (14) only one equation will arise in the present model. We have already discussed about it in the right second paragraph of page no. 4 and this condition is also taken care while plotting Figs. 2 and 6. We agree that we have not provided its expression in the paper. For $\phi < 0$, we can obtain from the condition $V(\phi_c) \geq 0$ at the critical value of electrostatic potential $\phi_c = -\frac{M^2}{2\mu\varepsilon_z}$ and is given by

$$\frac{M^2\alpha}{\mu} + M^2\left(1 - \sqrt{\frac{1+\mu\varepsilon_z}{\mu\varepsilon_z}}\right) + f_c\left(1 - \left(1 + \frac{M^2}{\mu\varepsilon_z(2k_c - 3)}\right)^{-k_c+3/2}\right) + \frac{f_h}{\sigma}\left(1 - \left(1 + \frac{\sigma M^2}{\mu\varepsilon_z(2k_h - 3)}\right)^{-k_h+3/2}\right) \geq 0.$$

The next point of the Comment is that Figs. 2–7 in the paper[1] are completely incorrect in various ways. They have also mentioned that in part (a) of Figs. 2–7, the pseudopotential graphs are grievously wrong for the equilibrium conditions at $V(\phi) = 0$ at $\phi = 0$. We strongly disagree with this statement, and one can easily verify that at $\phi = 0$, $V(\phi) = 0$ and $V'(\phi) = 0$ as follows:

$$V(\phi, M) = M^2\left(1 - \sqrt{1 - \frac{2\phi}{M^2}}\right) + \frac{M^2\alpha}{\mu}\left(1 - \sqrt{1 + \frac{2\phi\mu\varepsilon_z}{M^2}}\right) + f_c\left(1 - \left(1 - \frac{\phi}{k_c - \frac{3}{2}}\right)^{\left(-k_c + \frac{3}{2}\right)}\right) + \frac{f_h}{\sigma}\left(1 - \left(1 - \frac{\sigma\phi}{k_h - \frac{3}{2}}\right)^{\left(-k_h + \frac{3}{2}\right)}\right). \quad (11)$$

By putting $\phi = 0$, it becomes

$$V(\phi, M)|_{\phi=0} = M^2(1-1) + \frac{M^2\alpha}{\mu}(1-1) + f_c(1-1) + \frac{f_h}{\sigma}(1-1) = 0,$$

which shows that $V(0, M) = 0$ at $\phi = 0$. Differentiating Eq. (11), we get

$$V'(\phi, M) = \left(1 - \frac{2\phi}{M^2}\right)^{-1/2} - \alpha\varepsilon_z\left(1 + \frac{2\phi\mu\varepsilon_z}{M^2}\right)^{-\frac{1}{2}} - f_c\left(1 - \frac{\phi}{k_c - \frac{3}{2}}\right)^{-k_c + \frac{1}{2}} - f_h\left(1 - \frac{\sigma\phi}{k_h - \frac{3}{2}}\right)^{-k_h + \frac{1}{2}}.$$

Now putting $\phi = 0$, it becomes

$$V'(0, M) = 1 - \alpha\varepsilon_z - f_c - f_h, \quad \text{where}$$
$$f_h = 1 - \alpha\varepsilon_z - f_c, \text{ i.e., } V'(0, M) = 0,$$

which again shows $V'(\phi) = 0$ at $\phi = 0$. One can also see that in Figs. 2–7, the condition $V''(0, M) \leq 0$ at $\phi = 0$. This clearly shows that all the soliton conditions are satisfied in Ref. 1 and the system will support large amplitude solitons for the given set of parameter values.

The next point addressed in the Comment is "assuming for a minute that these graphs were correct in reality only periodic nonlinear structures would be possible, hence localized forms i.e., 'solitons' or 'supersolitons' cannot exist." In reply of this, we submit that in the present paper using the Sagdeev pseudopotential technique, the arbitrary amplitude solitons are studied. On the other hand, periodic nonlinear structures are based on the reductive perturbation method and are valid for small amplitude theory. Therefore, on the basis of Ref. 1, one cannot predict that the system will support periodic nonlinear structures in place of localized solutions.

The next point addressed in the Comment is in part (b) of Figs. 2–7; it is clear that the different "soliton" forms depicted are only defined over a bounded interval of ξ, unlike soliton domains that are known to be unbounded. In reply of this Comment, we would like to mention that part (b) of Figs. 2–7 simply represents the corresponding soliton profiles of part (a) of Figs. 2–7. Such figures are frequently drawn by several authors[3–5] in the literature.

The next issue addressed in the Comment is that the hodographs in part (c) of these figures do not correspond at the origin of the coordinate axis to what is seen in parts (a) and (b) and thus are also not correct. We strongly disagree with this statement; if we enlarge part (c) of Figs. 2–7, in that case, one can easily verify that all these figures correspond at the origin of the coordinate axis. Here, we can also see that $\frac{d\phi}{d\xi} = 0$, both at the origin $\phi = 0$ and at the amplitude of soliton ϕ_m, which means that the different graphs correspond to the coordinate axis.

The last issue addressed in the Comment is that in part (b) of Figs. 2–7, the dotted curves refer to "supersolitons" solution(s), but in parts (a) and (c), they clearly correspond to "double layers." We again disagree with this statement. The dotted curve in parts (a) and (c) also represent supersoliton. In the case of double layer, the variation of the Sagdeev potential with potential shows that there exists only one potential well satisfying double-layer existence conditions. In other words, one can say that the Sagdeev pseudopotential for the double layers should have two finite consecutive roots for the real potential. Part (a) of figures clearly shows that it has three finite consecutive roots for the real potential. Assuming that the dotted curve of part (a) of Fig. 2 represents a double layer as suggested by Varghese *et al.*[2] in their Comment, in that case, it should satisfy the boundary conditions of double layers at the third root of ϕ, i.e., at $\phi = -1.154\,38$. However, the dotted curve of Fig. 2(a) clearly shows that at $\phi = -1.154\,38$, no double layer boundary conditions are satisfied. The second point is that if we see the profile of the dotted curve in Fig. 2(b), it does not support a double layer profile. It represents the same distorted profile of the supersoliton as mentioned in the literature by Rufai and co-workers[5–8] This implies that it supports supersoliton and not the double layer as mentioned by Varghese *et al.*[2] in their Comment. It may also be noted that in the literature, Rufai *et al.*,[5] Rufai,[6] Rufai *et al.*,[7] and Rufai *et al.*[8] also called such a structure as supersoliton.

Varghese *et al.*[2] in their Comment mentioned that another facet of ambiguity lies in Fig. 2(b), where the blue curve is inside the black one. In reply of this Comment, we would like to mention here that the comparison of the amplitudes of the corresponding curves is significant in place of the fact that which curve is inside or outside. One can easily verify from Fig. 2(b) that the amplitude for a solid curve is $\phi_m = -1.216\,28$ and for a black dotted line curve the amplitude is $\phi_m = -1.154\,38$ and these values are the same as in Figs. 2(a) and 2(c). This clearly shows that there is no facet of ambiguity in Fig. 2(b) as mentioned by Varghese *et al.*[2] in their Comment.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

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