Discussion: "Yield Function for Solder Elastoviscoplastic Modeling" (Dube, M., and Kundu, T., 2005, ASME J. Electron. Packag., 127, pp. 147–156)

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This Discussion is submitted with the understanding that the authors would respond only to the technical issues with appropriate proofs and not address unrelated items and attack others' work.

1. The authors have employed their *new* yield function, Eq. (12), to plot the yield surfaces both for the compressive and tensile yields in Fig. 3, by using *the same* material parameters for both compression and tension behavior. Authors state that the yield function is valid for continuous yield in all directions and for continuous yield in both compression and tension. However, the compression and tension responses of materials usually are different, so the parameters should be different; hence, the yield behaviors will be different and cannot be continuous both for compression and tension. Also, the behavior under different stress paths (e.g., compression and tension) will be different, and the yield surfaces will be different in the in the $J_1 - \sqrt{J_{2D}}$ space, Fig. 3; for instance, the intercepts proportional to the compressive and tensile *strengths* along the $\sqrt{J_{2D}}$-axis (Fig. 3) will be different, which would introduce a discontinuity between the surfaces for compression and tension. Hence, the authors' claim that their function, Eq. (12), provides continuous yield in all directions and under both tension and compression (page 150, paragraph after Eq. (12)) may not be valid. Only in the case when the behaviors under all stress paths, e.g., tension and compression, are *identical*, which is indeed rare, the function may be continuous over both zones, Fig. 3.

   In view of the above, the authors' claim that they have developed the new function to correct the deficiency of the HISS function (in the tensile zone) appears to be unfounded, because their function will not be continuous for the tensile yield if different and realistic parameters were used for compression and tension.

   Suggested Response: *Authors can prove their claim by showing plots of continuous yield using their function for typical stress paths, e.g., compression and tension, by using parameters from laboratory tests data for a material whose behavior is different under both paths* ***OR*** *they could state in their response or rebuttal that their function can provide continuous yield only when the material behavior is the same for all stress paths, i.e., in all directions.*
2. The statement that "HISS is not a true continuous yield function [19][1]" on page 149 (line 2 after Eq. 7) may not be correct; because the HISS function is *continuous for the loading and stress space it is defined for*. For example, it is continuous for the compressive yield for geologic materials; indeed, as explained below, it is not valid and not continuous for the tensile yield. The authors' statement also contradicts their earlier statement on page 148 (paragraph 1, section 2.3), which says that "The HISS yield function represents a unification of the above trends, including the various prior functions as special cases" (*e.g., critical state, CAP, Mohr-Coulomb, Drucker Prager)*; the authors have cited Desai [1] for this statement. Then they themselves say, "The HISS-δ_0 yield function has been used extensively for solder as well as a wide range of other materials, *to provide continuous yield response*." Note also that the authors have categorized (page 148, paragraph 1, section 2.3) the critical state and CAP functions as "continuous yield functions." *How can the HISS function unify other continuous yield functions, contain them as special cases, and provide continuous yield response for a wide range of materials, and then end up not continuous*?

   For predominant compressive loading materials such as geologic, the HISS model is valid and continuous only for the compressive yield, because the parameters determined from compression tests are used. If the parameters for other stress paths such as tension were used, the yield surfaces for each can be different. This is analogous to the well-known and classical Mohr-Coulomb model (for geologic materials) where the strength envelope is valid for compression and not for tension because the tensile strength is different from the compressive strength. Similarly, for predominant tension loading materials such as metals and solders, the HISS model is valid and continuous for tensile yield, and is not valid for compressive yield, because the parameters from tension tests are commonly used (Chia [50], Desai et al. [48]). If parameters from other types of tests, e.g., shear and compression, etc., were used (with certain assumptions), they (models) should be validated as described in item No. 4 below.

   Because the yield function (HISS) for geologic materials is not valid for the tensile zone, very often a different model is used when the material experiences tensile condition during (finite element) computations, e.g., Desai and coworkers (see references cited in Desai [1]) have used the model based on the "stress transfer" approach proposed by Zienkiewicz et al. (1968),[2] and Scarpas et al. (1997) have used the Hoffman model. Hence, the issues such as the unconstrained thermal loading in the tensile zone raised by the authors are irrelevant and unwarranted.

   Suggested Response: *Authors could withdraw statement like "HISS is not a true continuous yield function" because HISS is continuous for the stress space it is defined for, e.g., for compressive for geologic or tensile for metallic materials* ***OR*** *authors should provide proof that it (HISS) is not continuous in the compressive stress space, for geologic materials.*
3. Roscoe et al. (1958) are credited with the first identification of the *continuous yield behavior,* i.e., plastic or irreversible deformation can occur almost from the start of loading for the soils tested by them. As the title, "Description of Stress-Strain Curves by Three Parameters," in the paper by Ramberg Osgood (1943) suggests, the Ramberg-Osgood equation used by Rafanelli is a mathematical function to fit the curve passing through a number of data points, Fig. 2 in Rafanelli [3]. The use of such a function would result into a piecewise linear or nonlinear elastic model (Desai and Siriwardane, 1984). Thus, the Rafanelli model is based on the theory of elasticity and does *not include plastic or irreversible deformations and continuous yield behavior*. Furthermore, it does not involve the use of the yield and hardening functions required for the theory of plasticity. Hence, the citation by the authors to Rafanelli [3], page 148 (column 1, line 22), for the continuous-yield behavior can be erroneous. Roscoe

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[1] References in the square brackets are those cited in the paper, Dube and Kundu (2005).

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[2] References with the year in the parentheses are cited in this Discussion and are listed in the Reference section of this paper.

et al. (1958) should have been cited for the development of continuous yield, and Chia [50] and Desai et al. [48] for its modification and implementation in the HISS/DSC model for solders in electronic packaging. Desai and coworkers (Desai [1]) have used the continuous yield behavior based on Roscoe et al's work with the HISS/DSC models for geologic and other materials; the authors appear to have also used the *same concept* from Roscoe et al.; for this reason, they could use the yield function, Eq. 12, and the hardening function, Eq. 15, in their plasticity formulation. The Rafanelli model does not use the theory of plasticity, and therefore, there can be no rationale to define continuous yield behavior, as it is known in the profession, from the Rafanelli model because continuous yield behavior requires that almost every point on the stress-strain response is a yield point with plastic strain.
Suggested Response: *Provide proof that Ramberg Osgood model used by Rafanelli is "continuous yield plasticity" model* ***OR*** *state that it was not proper to assign continuous yield behavior to Rafanelli (3); in fact, the continuous yield behavior was developed by Roscoe et al. (1958).*

4. It is a normal requirement that a (new) constitutive model should be validated at the specimen level with respect to (1) the test data used for finding its parameters, which is termed as Level 1 or curve fitting validation, (2) with respect to *independent* tests not used to find the parameters (Level 2), and (3) for the boundary value problem(s) with comparison of predictions with measurements for chip-substrate packages using solution (finite element) procedures, which is termed the Level 3 validation. Level 1 validation is desirable, but Levels 2 and/or 3 are essential. Moreover, it is required to validate a (new) model with respect to significant factors such as stress paths (e.g., tension, compression, shear, and hydrostatic), volume change, and confining stress that influence the behavior of the material under consideration. Such validations are required for the reliability of the model and the parameters. The authors have performed essentially Level 1 or curve fitting validation, for the tests under the (shear) stress path from which the parameters were determined. Such an elementary (curve fitting) validation may not be sufficient and may not prove the validity of the proposed *new* model. It may be noted the research by Desai and coworkers involving the use of the HISS/DSC models, validations have been provided by following the above criteria, e.g., Wang (2001), from which the authors have adopted some parameters, has presented validations for stress-strain responses and predictions of measured behavior in a boundary value problem involving a TSOP package, by using the HISS/DSC model.
   Suggested Response: *Provide validations (for the authors' model) with respect to some independent test data that were not used to find the parameters. Such data are available from tests performed by Wang (2001) that have been used by the authors, and test data on solders with similar compositions are also available in the literature,* ***OR*** *state that this paper contains only limited validations with respect to tests used for finding the parameters. Additional independent validations will be performed in future research.*
5. The authors have concluded on page 155 (second paragraph under Conclusions) that "Trapezoidal cycles indicate that the elastoplastic formulation is limited to relatively simple loading paths with constant strain rate." However, in the paper, they have analyzed *only* the monotonic (shear) loading path and not any trapezoidal cycles. Hence, an explanation is needed how they arrived at this conclusion when they have not used trapezoidal loading cycles *for the work presented in the paper.*
   Suggested Response: *Include analysis based on trapezoidal cycles to support the statements presented in the paper* ***OR*** *state that additional research would be required to support this conclusion.*

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NOTE: *The following data are provided as suggested by a Reviewer; hence, no response by Authors is needed.*

Parameters for HISS Plasticity Model. A reviewer has asked for a table with the "correct" HISS parameters used and published in previous papers by Desai and coworkers. The discusser believes that such a table may not be relevant and warranted for the Discussion that addresses specific topics such as potential inaccuracies and unsupported claims in the proposed new yield function by Dube and Kundu (2005). In fact, it may divert the attention of the reader from the specific and relevant topics raised in the

Table 1 HISS parameters for given temperature and strain rate

From Chia (1994), Desai, et al. (1997).	From Wang (2001), Desai, et al. (2004)
Determined by using test data for Pb37/Sn63 solder reported by Reimer (1990) and Skiport, et al. (1992). Parameters are reported for two strain rate $\dot{\varepsilon} = 0.002/\mathrm{s}$ and $0.0002/\mathrm{s}$ and four different temperatures 208, 273, 348, 373 K. Typical parameters for T = 273 K and $\dot{\varepsilon} = 0.002$ are as follows:	Determined from test data obtained by using the test device (TMDIC) for four different temperatures (−20, 25, 75, and 125 °C) and three different strain rates (2.78×10^{-4}, 2.78×10^{-3}, and 2.78×10^{-2} 1/s). They are presented graphically in the above paper. Typical parameters for T = 25 °C and $\dot{\varepsilon} = 2.78 \times 10^{-3}/\mathrm{s}$ are given below, based on the parameters given in Wang (2001).
E (GPa) = 24.0	E (GPa) = 13.0
$\nu = 0.395$	$\nu = 0.40$
$\gamma = 0.00082$	$\gamma = 0.00112$
$\beta = 0.00$	$\beta = 0.0$
n = 2.10	n = 2.09
R(MPa) = 288	R(MPa) = 680.0
$a_1 = 2.93 \times 10^{-6}$	$n_1 = 8.8 \times 10^{-6}$
$a_1 = 0.553$	$n_1 = 0.94$

Discussion. However, Table 1 with the following comments is provided to address the reviewer's comment:

The parameters for any constitutive model, for example, a solder, can depend on many factors such as the composition, number of tests available at a given time, type (uniaxial, shear; biaxial, multiaxial) of tests available, reliability of test data, and sometimes modifications in the procedures for finding the parameters. For example, parameters for a model could change as the research advances over years. Moreover, some parameters may be sensitive to available, number and type of tests, e.g., a_1 in the HISS model. Also, the variability (from 12.4 to 43.4 GPa for Pb40/Sn60 solder) in the Young's modulus has been reported in the literature (Knecht and Fox, 1990; Basaran, 1994).

A major proof of realistic and reliable parameters for a constitutive model is the validation by comparing the model predictions with measurements at the specimen level for the test data from which the parameters are determined, for independent test not used for finding the parameters, and/or at the boundary value problem level, in which the model is implemented in a computer (finite element) procedure and predictions are compared with measurements of laboratory simulated and/or field testing for practical problems. In almost all research on constitutive models in electronic packaging and civil engineering reported by Desai and coworkers, such validations have been performed. Typical research when parameters were determined for the Pb/Sn solder is described below.

In the research for modeling and testing for solders (Pb/Sn) by Desai and coworkers, main developments for the parameters for HISS/DSC model are: (1) use by Chia (1994) of available (uniaxial) data reported by Reimer (1990), Skipor et al. (1992), Pan (1991) and Solomon (1985, 1992), and reported by Desai et al. (1997); and (2) use by Wang (2001) of test data performed with a new test device and reported by Desai et al. (2004). In both cases, the parameters were determined from uniaxial, shear, and creep test data for different temperatures and strain rates, in Desai et al. (2004), variations for temperature and strain rates are presented graphically. Other investigators (e.g., Basaran et al., 1998; Desai et al., 1998; Whitenack et al., 2007) have used the parameters reported in Desai et al. (1997, 2004), sometimes with modifications, depending on their specific situations, e.g., temperature, strain rate, and composition.

In order to satisfy the suggestion by the reviewer, Table 1 is enclosed, which shows the HISS parameters for one typical temperature and strain rate from those reported for different temperatures and strain rates by Chia (1994) and Wang (2001). In the discusser's opinion, "correct" parameters are those that have been determined from reliable test data and have satisfied the above validation criteria.

Some of the research reported by Desai and coworkers has considered creep and disturbance (softening or degradation). Parameters for creep and disturbance are also subject to the conditions stated above, e.g., use of reliable tests data and satisfaction of the validation criteria.

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