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ABSTRACT
In the present study, the evolutionary multi-objective optimization is employed to design the optimum amplitude and wavelength of sinusoidal leading-edge tubercles for the NACA0012 (National Advisory Committee for Aeronautics) airfoil to improve its aerodynamic performance at Reynolds number of 5.0 × 10^4 than that of the NACA0012 smooth leading-edge or baseline airfoil. Here, the optimum tubercle is found to have an amplitude of 11.71% and a wavelength of 25% of the baseline airfoil chord, respectively. Through a combination of in-house water tunnel experiments and numerical simulations, it is additionally established that the optimized tubercle airfoil exhibits superior lift and reduced drag characteristics compared to the baseline airfoil, particularly in the post-stall high angle of attack regime. Furthermore, it is noticed that the optimized tubercle design enhances the gap between large separation regions or stall cells along the tubercle airfoil span during the post-stall regime. Consequently, a more pronounced attached flow regime is developed between the consecutive stall cells, contributing to the tubercle airfoil’s improved aerodynamic characteristics compared to the baseline airfoil. Our investigations also revealed that the formation and arrangement of the stall cells on the tubercle airfoil span are associated with a biased wake mechanism similar to the one observed in the wake of side-by-side arranged circular cylinders.

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I. INTRODUCTION
Humpback whales are known for their impressive swimming abilities, reaching cruising speeds of more than 7.4 km/h. They use tail flukes and flippers to propel, steer, balance, and perform tight maneuvers through the waterbody, unlike other cetaceans.1 Fish and Battle2 studied the intricate morphology of humpback whales’ long flippers, shown in Fig. 1, and investigated the functionality and role of rounded protuberances or tubercles on the flipper’s leading-edge. They found that the cross section of the flipper is similar to the shape of a symmetric thick airfoil like NACA0021 with an average thickness-to-chord (t/c) ratio of 21%. In addition, they observed that the airfoil chord length typically varies along the span (b) of the flipper. Although smooth leading-edge or baseline airfoil aerodynamics is well established in the literature, the tubercle leading-edge effects on airfoil aerodynamics are sparsely documented. The experimental1–11 and numerical investigations12–19 on different static leading-edge tubercle airfoils have revealed that the tubercles retain attached flow conditions on the airfoil surface even at high angles of attack (α), thereby improving the aerodynamic lift performance and delaying the stall angle compared to the baseline airfoil. This discovery holds potential implications in designing the Micro Air Vehicles (MAVs) and the Under Water Vehicles (UWVs), where the improved aerodynamic lift is highly essential, especially for operating at low chord-based Reynolds number (Re ranges from 10^4 to 10^7) regime.

The leading-edge tubercle airfoil is typically constructed by introducing the sinusoidal variations on the local chord length (c) at the leading-edge side of the baseline airfoil. Note that the tubercles can be designed at an isolated location1,20,21 or in the form of a continuous pattern22 along the leading-edge of the airfoil. In general, these tubercle airfoils’ are named with respect to the amplitude (A) and wavelength (λ) of the sinusoid profile followed in constructing the tubercle shape, which are generally expressed as percentages of the baseline airfoil’s...
chord length \((c)\), as shown in Fig. 2(a). For instance, a NACA0021 airfoil with sinusoidal leading-edge tubercles having an amplitude \((A)\) of 5% and a wavelength \((\lambda)\) of 50% of the baseline NACA0021 airfoil chord length is denoted as NACA0021 A5250 tubercle airfoil. The same nomenclature style for the tubercle airfoil is adopted in the present study.

So far in the literature, the tubercles airfoil aerodynamics are investigated by taking the baseline airfoil shape as NACA0020, \(^{3,4,5,12,14,15,18,23,24}\) NACA0021, \(^{16,15,19,25,29}\) and NACA634-021 \(^{19,17,22,30-34}\) with \(A\) ranging from 1.3% to 12% of \(c\) and \(\lambda\) between 11% and 50% of \(c\). Notice that the shape of the baseline NACA0020, NACA0021, and NACA634-021 airfoils is more or less similar, as shown in Fig. 2(b), where the \(t/c\) \(\approx 21\%\). Therefore, it is intuitive that their aerodynamic time-averaged lift \((C_l)\) and drag \((C_d)\) coefficient characteristics at various \(Re\) will be more or less the same. For verification purposes, the readers may compare the \(C_l\) features, the maximum time-averaged lift coefficient \((C_{l\text{max}})\), and the stall angle of attack of NACA0021 and NACA634-021 at \(Re = 1.2 \times 10^6\) and \(1.83 \times 10^5\), respectively, in the references.\(^{10,34}\) Furthermore, since the aerodynamic performance of the baseline airfoil NACA0020, NACA0021, and NACA634-021 are alike, the tubercle’s effects on these airfoils will presumably be equivalent. Specifically, it is observed that irrespective of \(A\) and \(\lambda\), the tubercle airfoils show decreased \(C_l\) and increased \(C_d\) in the pre-stall angle of attack compared to the baseline airfoil.\(^{5,12,25,27}\) On the other hand, in the post-stall regime for smaller \(A\) and \(\lambda\) \((A \leq 6\%\) and \(\lambda \leq 21\%\) of \(c)\), higher \(C_l\) and lower \(C_d\) are produced than the corresponding baseline airfoil.\(^{6,15,22,35}\) In addition, it is reported that for a small \(A/\lambda\) ratio between 0.05 and 0.075, the \(C_l\) characteristics of tubercle airfoil follow the trend of baseline airfoil with decreased peak \(C_l\) or \(C_{l\text{max}}\) and stall angle.\(^{5,12,22}\)

The above literature review implies that tubercle airfoils are aerodynamically advantageous over the baseline airfoil if the \(A/\lambda\) ratio is larger than 0.075. However, a question may be posed: is this observation valid for different sets of \(A\) and \(\lambda\) resulting in the same \(A/\lambda\) ratio? Wei et al.\(^{36}\) examined various tubercle airfoils that maintained an identical \(A/\lambda\) ratio. They noted that despite the fixed \(A/\lambda\) ratio, the tubercle airfoils with distinctive \(A\) and \(\lambda\) values exhibited differing aerodynamic performance characteristics. Hansen et al.\(^{22}\) and Mattos et al.\(^{17,22}\) have reported similar observations. Chen et al.\(^{7}\) investigated the effects of \(A\) by keeping the \(\lambda\) \((25\%\) of \(c)\) constant and varying \(A\) to 5%, 10%, and 15% of \(c\). The analysis was performed on the NACA0012 airfoil at \(Re = 1.23 \times 10^5\). It was found that the tubercle airfoil with \(A = 10\%\) of \(c\) shows better \(C_l\) characteristics than the other two tubercle airfoils (i.e., \(A = 5\%\) and 15% of \(c\)). They remarked that the tubercle of dimension \(A = 10\%\) and \(\lambda = 25\%\) of \(c\) constructed on the NACA0012 airfoil is the optimum tubercle airfoil configurations for \(Re = 1.23 \times 10^5\). However, the study conducted by Chen et al.\(^{7}\) lacked a comprehensive exploration of a large set of \(A\) and \(\lambda\) values. In a subsequent investigation, Taheri\(^{1}\) employed a neural network trained on existing literature data of \(C_l\) and \(C_d\) characteristics from diverse tubercle airfoils. The
neural network was combined with Genetic Algorithm (GA) optimization to identify the most favorable tubercle configuration to enhance the aerodynamic performance characteristics compared to the baseline NACA0012 airfoil, specifically at $Re = 1.2 \times 10^5$. The results of Taheri\textsuperscript{37} suggested that $A = 3\%$ and $\lambda = 19\%$ of $c$ is the optimum tubercle airfoil configuration to achieve the maximum aerodynamic performance benefits over the baseline NACA0012 airfoil at $Re = 1.2 \times 10^5$. It is worth noting that $A$ and $\lambda$ values on the optimal tubercle airfoil configurations are significantly different in the studies of Chen et al.\textsuperscript{36} and Taheri\textsuperscript{37} even though the considered $Re$ is nearly the same in both the studies. The discrepancy could be attributed to potential inadequacies in the objective function of the optimization algorithm of Taheri.\textsuperscript{37} This motivated us to conduct the current investigation to methodically assess a more suitable objective function(s) and optimum tubercle configuration for tubercle airfoil’s improved aerodynamic performance by considering representative baseline airfoil as NACA0012 and $Re = 5.0 \times 10^4$. Here, an evolutionary multi-objective optimization (EMO) is implemented for minimizing two objectives corresponding to $C_l$ and $C_d$, ensuring that the $C_l$ and $C_d$ of the tubercle airfoil surpass those of the baseline airfoil within the range of $0^\circ \leq \alpha \leq 20^\circ$. It is worth mentioning here that the proposed optimization algorithm can be suitably extended to other airfoil profiles and $Re$ ranges by adopting a more extensive training dataset.

Furthermore, it has been observed that the enhanced aerodynamic performance of most tubercle airfoils at high $\alpha$ or in the post-stall regime is associated with counter-rotating vortex pairs (CVPs).\textsuperscript{4,13,25,30} Specifically, in the post-stall regime, the baseline airfoil experiences extensive flow separation across the airfoil span. In contrast, the tubercle airfoils exhibit intermediate flow separation along the airfoil span due to the presence of CVPs, leading to a more sustained $C_l$ than the baseline airfoil in the post-stall regime. These CVPs originating from the tubercles energize the boundary layer flow,\textsuperscript{4,6,19} enabling it to counteract the adverse pressure gradient on the airfoil surface that causes flow separation at high $\alpha$. However, the CVPs cannot ensure a completely attached flow regime throughout the airfoil span. Several researchers have additionally reported the intermittent appearance of large separation zones or stall cells\textsuperscript{12,25,30,38} on the tubercle airfoil. Interestingly, it has been found that the development of these CVPs and stall cells depends on parameters like $A$, $\lambda$, and $\alpha$.\textsuperscript{25,30} Nevertheless, previous studies have failed to elucidate the underlying flow physics that led to the formation of stall cells.

In this study, the optimal design of the tubercle configuration is achieved by minimizing two objectives. These objectives are defined as $f_1$ and $f_2$ [as defined in Eq. (1)]. This evolutionary multi-objective optimization (EMO) approach evaluates the lift and drag performance separately, as outlined in the following equation:

Minimize \( \{ f_1, f_2 \} \)

\[
\begin{align*}
    f_1 &= \frac{1}{C_{l_{opt}}}, \\
    f_2 &= \frac{1}{C_{d_{opt}}}, \\
    C_{l_{opt}} &= \frac{1}{n} \sum_{i=1}^{n} (C_{l_i} - C_{l_{baseline}}), \\
    C_{d_{opt}} &= \frac{1}{n} \sum_{i=1}^{n} (C_{d_i} - C_{d_{baseline}}), \\
    C_{l_i} &= F(A/c, \lambda/c), \\
    C_{d_i} &= F(A/c, \lambda/c), \\
    \text{Constraints} \quad 0^\circ \leq \alpha \leq 20^\circ, \\
    0.03 \leq A/c \leq 0.15, \\
    0.03 \leq \lambda/c \leq 0.25.
\end{align*}
\]
In the above Eq. (1), the time-averaged lift and drag coefficients for the tubercle airfoil are represented as $C_{l_{\text{tubercle}}}$ and $C_{d_{\text{tubercle}}}$, while $C_{l_{\text{Baseline}}}$ and $C_{d_{\text{Baseline}}}$ corresponds to the baseline airfoil’s time-averaged lift and drag coefficients. The $\alpha$ varies from $0^\circ$ to $20^\circ$ in steps of $1^\circ$, therefore, the total number of intermittent steps $n = 21$. From Eq. (1), it is required to establish a methodology that can generate the data of $C_{l_{\text{tubercle}}}$ and $C_{d_{\text{tubercle}}}$ of the tubercle airfoil at $Re = 5 \times 10^4$ for $\alpha$ in the range from $0^\circ$ to $20^\circ$. The potential approaches include conducting wind/water tunnel experiments or implementing numerical simulations. Nevertheless, both methods are expensive and time-consuming. Furthermore, the experimental or numerical data of $C_{l_{\text{tubercle}}}$ and $C_{d_{\text{tubercle}}}$ are sparsely available in the literature, which is insufficient for the search algorithm to determine the optimum solution. Thus, for populating the data
A. Data generation using CFNN

The cascade forward neural network (CFNN)\(^{37,47}\) model is a type of artificial neural network (ANN), a potent tool that emulates the human brain’s learning process. On the successful training of ANN with known inputs and outputs of any discrete information, the ANN can predict the unknown outcomes for the given inputs. ANN models can understand complex and non-linear relations between the input (x) and output (y) information. This understanding is achieved through artificial neurons and synapses. ANNs consist of multiple layers of neurons, including input, hidden, and output layers, interconnected by synapses. These synapses are assigned weights \(w(0 < w < 1)\), which multiply the input signal [see Eq. (2)], where \(kn\) represents the total number of neurons] to modulate signal strength between different layers of neurons. Each neuron incorporates bias \(s(0 < s < 1)\) value to introduce a shift in the signal [see Eq. (2)]. Furthermore, an activation function (usually a sigmoid function\(^48\) or hyperbolic tangent function\(^48\)) is applied at each neuron to introduce non-linearity in the system and determine the neuron’s output. The present CFNN includes all these features of ANN. However, the CFNN architecture is only the flow of information from the input layers to hidden layers and from hidden layers to the output layers.

\[
\hat{y}_j = f \left( \sum_{i=1}^{kn} (w_i \times x_i + s_i) \right),
\]

\[
\text{mse} = \frac{1}{td} \sum_{j=1}^{td} (y_j - \hat{y}_j)^2.
\]

The Levenberg–Marquardt back-propagation\(^{37,49}\) method is chosen as the CFNN training algorithm in the present study. This algorithm iteratively (epochs) adjusts the \(w\) and \(s\) [Eq. (2)] and calculates the output \(\hat{y}_j\) corresponding to the input \(x_j\). Simultaneously, it calculates the mean squared error \(\text{mse}\), see Eq. (3), where \(td\) is the training dataset] between the calculated or predicted output \(\hat{y}_j\) and actual output \(y_j\), to check the convergence of the mse. After achieving the mse within the convergence limit of \(O(10^{-3})\), completing the CFNN training, the algorithm stops the iterations.

The CFNN tool, available in the Deep Learning Toolbox\(^{50}\) in MATLAB, is customized for the present study to predict the outputs of \(C_l\) and \(C_d\) of a tubercle airfoil by feeding nine inputs corresponding to the airfoil geometry and flow conditions. The inputs are the thickness-to-chord ratio \(t/c\) of the airfoil, the location of the maximum thickness \(x_{\text{max}}/c\), the ratio of span \(b/c\) and chord \(c\) of the airfoil [also known as Aspect Ratio (AR)], the angle of attack \(\alpha\), the ratio of the airfoil’s chord length at the tip and root of the wing \(c_{\text{tip}}/c_{\text{root}}\), the swept back angle of the wing \(\Lambda\), the ratio of tubercle wavelength to chord length \(\lambda/c\) of the airfoil, the ratio of tubercle amplitude to the chord \((h/c)\) of the airfoil, and the free stream Reynolds number \(Re\). This structure of inputs and outputs is adopted from Taheri,\(^{37}\) which consists of finite-span wing parameters \((AR, c_{\text{tip}}/c_{\text{root}}, \Lambda)\). However, the present study investigates only for the infinite-span wing where the airfoil has \(AR = \infty, c_{\text{tip}}/c_{\text{root}} = 1, \text{and} \Lambda = 0\).

B. Solving the multi-objective optimization problem using CFNN and GA

In the present multi-objective optimization problem, the well-known evolutionary optimization algorithm and genetic algorithm\(^*\) are used in conjunction with CFNN, as shown in Fig. 3. Genetic algorithms (GA) are computational approaches derived from natural evolution. They are employed in the solution of complex optimization and search issues.\(^{33}\) GA begins with a group (population) of potential problem solutions (guess and random solutions). A collection of chromosomes (or solutions) represents the population, each encoding a unique attribute or characteristic. GA aims to increase the quality of these solutions through successive iterations (generations), emulating natural selection. The objective of the optimization serves as a metric.
to calculate the fitness value (probability of being the optimum solution) for every solution, and then the GA rank and sorts the set of solutions based on their fitness value. The Pareto chart further helps to rank the solutions, in the case of multiple solutions with least fitness values. For a minimization problem, the solutions with higher fitness values are prone to be rejected. Here, genetic operators like crossover and mutation combine and change the current generation solutions to build new ones. The solutions with the most negligible fitness value in the previous generation (in the case of minimization problem) will likely appear in the next generation population without participating in crossover and mutation. This process is known as elitism. Furthermore, the GA looks to converge toward better solutions over time through repeated cycles of assessment, selection, and genetic operations, ideally finding an optimal or near-optimal solution to the problem at hand.

This study uses the GA tool available in MATLAB Global Optimization Toolbox\textsuperscript{52} which is built based on the Goldberg and David algorithm.\textsuperscript{51} The GA searches for the optimal $A$ and $k$ for a
The potential solutions \((A/c\) and \(\lambda/c\)) with the least positive fitness values \((f_1, f_2)\) are segregated, ranked, and sorted. Crossover and mutation are performed to generate the new population set with the present population. The population in the new generation consists of the traits of potential solutions in the earlier generation. Therefore, this new population may have optimum or higher workable solutions than the last generation. The best-fitted solution is passed down to successive generations, and the process repeats until the solution converges to a tolerance of \(10^{-6}\). The solution is considered converged if the new generation’s population has the same traits as earlier generations.

In the present study, after 353 generations, convergence is achieved, and the best solution obtained in each generation is stored. The present study utilized the available workstation in the Bio-Inspired Aero and Hydrodynamics (BIAH) research lab at IIT Kharagpur with a configuration of 32 cores Intel Xeon(R) Gold 5218 processor and 256 GB of memory. The Pareto-optimal front of the present multi-objective optimization problem is shown in Fig. 6. This optimal amplitude and wavelength are found to be \(A/c = 11.71\%\) and \(\lambda/c = 25\%\). Thus, the optimum tubercle configuration of the NACA0012 airfoil is NACA0012 A11.71\%25 for operating at \(Re = 5.0 \times 10^6\). Additional numerical and experimental investigations are done on this optimum tubercle configuration and baseline airfoil to determine the associated flow physics responsible for improved aerodynamic performance. In Secs. II C and II D, discussions on the experimental and numerical methodology are presented. For the sake of conciseness in the text, the NACA0012 baseline and NACA0012 A11.71\%25 tubercle airfoils are called "baseline" and "tubercle" airfoils, respectively, in the following sections.

C. Experimental methodology

The present study performs the aerodynamic force measurements on the baseline and tubercle airfoils using an in-house low-speed recirculating free-surface water tunnel, as shown in Fig. 7. The test section’s flow velocity \((V_{inf})\) is set to \(0.2\) m s\(^{-1}\). The turbulence intensity \((TI)\) at this flow speed is measured to be \(2\%\). Additional details and calibration of the present water tunnel can be found in Pinapatruni et al.,\(^{56}\) which are not repeated here for brevity. The water tunnel test section has linear and angular actuation mechanisms coupled with programmable
MDrive 23 plus stepper motors for precise airfoil position and pitch control. In Fig. 8, Computer Numerical Control (CNC) manufactured baseline and tubercle airfoils are shown with the mean chord of $c = 0.1$ m and the span of $b = 0.4$ m. The airfoil is coupled to an ATI mini 40 six-axis force-torque sensor to measure the acting aerodynamic forces and moments on it. This ATI mini 40 force-torque sensor has a resolution of $\pm 0.005$ N in the $X$ (parallel to the chord of the airfoil) and $Y$ (normal to the chord of the airfoil) directions and $\pm 0.01$ N in the $Z$ (along the span of the airfoil) direction. The sensing range in the $X$ and $Y$ directions is $\pm 20$ N, and the sensing range in the $Z$ direction is $\pm 60$ N. The torque sensing range in each direction is $\pm 1$ Nm. The maximum drift in the force reading along three axes is found to be $\pm 0.01$ N, which corresponds to a non-dimensional force coefficient of $\pm 0.0125$. However, no significant drift is noticed in the torque readings. The sampling frequency for both force and torque is set as 1 kHz. In order to ensure the flow regime around the airfoil is two-dimensional, the formation of the wing-tip vortices is restricted by the use of the end plates. More details on the actuation mechanism can be found in Sinha et al., which are not repeated here for brevity.

A FASTCAM Mini U50 high-speed camera is used to position the airfoil precisely at the desired angle of attack in the water tunnel test section (see Fig. 9). The camera’s field of view is set parallel to the test section’s horizontal plane to view the cross section of the airfoil, and the camera’s horizontal centerline is aligned parallel to the flow direction. Note that the camera’s horizontal centerline is taken as a reference, and the airfoil’s chord is set parallel to the flow direction (i.e., $\alpha = 0^\circ$). The $\alpha$ of the airfoil gradually increased to $20^\circ$, where the time for the airfoil 1° movement is set to 10.125 s. Here, the airfoil’s motion was precisely kept slow, so the wake flow regime resembles a quasi-steady flow. Additionally, this method eases the continuous data acquisition of $C_l$ and $C_d$ with $\alpha$ variation. It is worth mentioning that the actual forces acquired from the force sensor are aerodynamic forces applied to the airfoil.
parallel ($F_x$) and perpendicular ($F_y$) to the airfoil’s chord [refer to Eqs. (4) and (5)], respectively, where $\rho_\infty$ is free stream flow density), which are then resolved in parallel and perpendicular to free stream flow direction to obtain the lift [$C_l$ in Eq. (6)] and drag [$C_d$ in Eq. (7)] coefficients, respectively. Note that the $C_x$ and $C_y$ are the time-averaged axial and normal aerodynamic force coefficients, respectively.

$$C_x = \frac{2F_x}{\rho_\infty V_\infty^2 bc},$$

(4) $$C_y = \frac{2F_y}{\rho_\infty V_\infty^2 bc},$$

(5) $$C_l = C_y \cos (\alpha) - C_x \sin (\alpha),$$

(6) $$C_d = C_x \cos (\alpha) + C_y \sin (\alpha),$$

(7)

### D. Numerical methodology

The incompressible uniform fluid flow simulation over the baseline and tubercle airfoils is performed using the ANSYS Fluent v202 commercial CFD solver. The solver uses the finite volume methodology (FVM) to solve the continuity [Eq. (8)] and momentum conservation [Eq. (9)] equations. The fluid flow parameters within the control volume ($cv$) enclosed by the control surface ($cs$) are density, velocity, pressure, and shear stress represented by $\rho$, $V$, $p$, and $\tau$, respectively. Note that term $\mathbf{n}$ represents the unit normal vector of the control surface and $t$ represents time. Additionally, the Transition SST\textsuperscript{58,59} (shear stress transport) model is used to model the turbulence in the flow field. In this model, the turbulence parameters, kinetic energy ($k$), specific rate of dissipation ($\omega$), intermittency ($\gamma$), and the momentum thickness Reynolds number ($Re_\tau$) are computed by using an additional transport equation. The general form of a scalar transport equation is given in Eq. (10). The term $\varnothing$ represents the turbulence parameter, and the terms $\nabla$ and $S_\varnothing$ are the diffusion coefficient and the source of the turbulence parameter. The accuracy and potential of the solver and turbulence model in simulating the flow field over the airfoil at low $Re$ are well documented in the literature.\textsuperscript{58,59}

$$\frac{d}{dt} \int_{cv} \rho \, dv + \int_{cs} \rho V \cdot \mathbf{n} \, dS = 0,$$

(8) $$\frac{d}{dt} \int_{cv} \rho V dV + \int_{cs} \rho \left( V_i \frac{\partial V_i}{\partial x_j} - p_i + \tau_{ij} \right) \cdot \mathbf{n} dS = \int_{cs} \left( \tau_{ij} \frac{\partial V_j}{\partial x_i} - p \mathbf{n} \right) \cdot \mathbf{n} dS,$$

(9) $$\frac{d}{dt} \int_{cv} \rho \varnothing dV + \int_{cs} \rho \varnothing \left( V \cdot \mathbf{n} + \mathbf{n} \cdot \nabla \varnothing \right) dS + \int_{cv} S_\varnothing dv.$$  

(10)

The coupled scheme\textsuperscript{58,59} is used for pressure–velocity coupling in the above governing equations [Eqs. (8) and (9)], and the gradients are evaluated using the least squares cell-based method. The second-order and third-order MUSCL (Monotone Upstream-Centered Schemes for Conservation Laws) schemes are used for spatial discretization of pressure and momentum. A second-order upwind scheme is used to discretize all of the turbulent parameters. The convergence criteria are set to $10^{-4}$, $10^{-5}$, and $10^{-6}$ for continuity, momentum, and turbulence parameters. The present simulations are time-dependent, and the temporal derivates are solved using a second-order implicit scheme.

### 1. Computational domain

Figure 10(a) shows the computational domain considered for the numerical simulation of the three-dimensional flow around the tubercle airfoil. The tubercle airfoil is modeled with $c = 0.1$ m and $b = 1.25c$. Here, $c$ is the mean-chord length of the tubercle airfoil, which is the same as the baseline airfoil chord. The outer radius of the computational domain is set to $30c$, which is twice the size of the domain used by Counsil and Boulama\textsuperscript{58} to avoid the boundary influence on the

![Diagram](https://example.com/diagram.png)

**FIG. 10.** (a) Computational domain around the NACA0012 A11.725 airfoil and (b) poly-hexacore meshing.
results. In addition, the airfoil surface is configured to have a no-slip wall boundary condition \((V_x = V_y = V_z = 0 \text{ m/s})\). The inlet and outlet boundary conditions are set to velocity inlet \((V_x = 0.5; V_y = 0; \text{ and } V_z = 0 \text{ m/s})\) and pressure outlet \((p_{gauge} = 0 \text{ Pa})\). On the lateral face, the periodic boundary conditions are applied. The solution initialization is performed by setting \(V_x = 0.5 \text{ m/s}, V_y = V_z = 0 \text{ m/s}, \text{ and } p = p_{gauge} = 0 \text{ Pa}\) at all mesh points.

2. Mesh generation and validation

Figure 10(b) shows the poly-hexagons (polyhedral cells are between poly-prism cells and hexahedral cells) mesh used to discretize the computation domain around the NACA0012 tubercle airfoil. On the airfoil’s surface, the height of the first inflation layer (mesh) is set to \(1 \times 10^{-6} \text{ m}\) to satisfy the \(y^+ = 1\) (non-dimensional first cell height) criteria for adequately capturing the boundary layer characteristics.\(^{18,58,60}\) The time step size is set to 0.001 \text{s} after performing a time-independent study. The corresponding Courant number based on the minimum cell size is 0.588. The numerical solution is said to be converged at each time step when the residuals from continuity, momentum, and turbulence equations are less than \(10^{-3}, 10^{-5}, \text{ and } 10^{-6}\). This is achieved by performing up to 50 sub-iterations in each time step.

The mesh independence test is performed on tubercle airfoil at \(a = 4^\circ\) and \(Re = 5.0 \times 10^4\) with a Turbulence Intensity (TI) of 2\%. As shown in Table 1, the mesh size of 37.43 \(\times 10^5\) with a minimum cell size of \(8 \times 10^{-4} \text{ m}\) on the airfoil surface produces a numerical error of below 2.5\% with reasonable computational resource requirement. Hence, it has been followed in the present simulations. Now, to validate the adopted Ansys Fluent CFD solver, the numerical simulations are performed on NACA63\(x\)-021 A12\(25\) airfoil using the above mesh resolution and compared \(C_l\) characteristics at \(Re = 5.0 \times 10^4\) with the results of the Zhang et al.,\(^{33}\) as shown in Fig. 11.

The \(C_l\) of the NACA63\(x\)-021 tubercle airfoil obtained in the present solver agrees with the experimental results of Zhang et al.,\(^{33}\) This further confirms that the adopted Ansys Fluent CFD solver can accurately simulate fluid flow over tubercle airfoil. It is worth to mention the leading-edge tubercle curvature in this study has been modeled by following the non-linear shear transformation methodology proposed by Wei et al.,\(^{33}\) which ensures the continuity of curvature at the tubercles while preserving the geometric shape of the baseline airfoil.

III. RESULTS AND DISCUSSION

Section II B derived the optimized tubercle configuration, i.e., A11.7\(25\) for the NACA0012 baseline airfoil at \(Re = 5.0 \times 10^4\) using CFNN and GA. This optimized tubercle airfoil has enhanced \(C_l\) and \(C_d\) characteristics compared to the baseline airfoil. To further verify the accuracy of the prediction of the CFNN and GA, the CFD simulations are performed at \(Re = 5.0 \times 10^4\) on both baseline and tubercle airfoils. These CFD results will enable us to determine the underlying flow physics responsible for the tubercle airfoil’s enhanced aerodynamic performance. The results from the CFD simulations are further qualitatively compared with those of the in-house water tunnel experiments. Note that the water tunnel experiments on the baseline and tubercle airfoil are conducted at a reduced \(Re = 2.0 \times 10^4\) due to hardware limitations.

A. Aerodynamics characteristics

In Fig. 12(a), the \(C_l\) vs \(a\) characteristics of the baseline and tubercle airfoils are plotted at \(Re = 2.0 \times 10^4\) and \(Re = 5.0 \times 10^4\), obtained from in-house water tunnel experiments and numerical simulation, respectively. At \(Re = 5.0 \times 10^4\) and \(0^\circ < a < 2^\circ\), the \(C_l\) of the tubercle airfoil is slightly higher than that of the baseline airfoil. However, when \(a > 2^\circ\), the \(C_l\) of the tubercle airfoil is found to be smaller compared to the baseline airfoil, and the difference is more pronounced within the range of \(6^\circ < a < 10^\circ\). Notice that, on the baseline airfoil, the \(C_l\) reaches the maximum, \(C_{l_{max}} = 0.74\) at \(a = 10^\circ\), and stalls beyond this \(a\). The \(C_l\) of the baseline airfoil stays relatively constant at \(C_l \approx 0.56\) in the post-stall range of \(10^\circ < a < 20^\circ\). Conversely, the \(C_l\) of the tubercle airfoil did not stall at \(a = 10^\circ\), unlike the baseline airfoil, and it shows an increasing trend for \(a\) up to \(20^\circ\). Furthermore, it can be observed in Fig. 12(a) that the \(C_l\) vs \(a\) of the baseline and tubercle airfoils obtained from in-house water tunnel experiments at \(Re = 2.0 \times 10^4\) shows a similar trend but with a lesser magnitude than that of at \(Re = 5.0 \times 10^4\) which are obtained from CFD simulations. This additionally confirms the accuracy of the present CFD simulation results,

\[
\Delta C_l = \frac{C_{l_{tubercle}} - C_{l_{baseline}}}{C_{l_{baseline}}} \times 100, \quad (11)
\]

\[
\Delta C_d = \frac{C_{d_{tubercle}} - C_{d_{baseline}}}{C_{d_{baseline}}} \times 100. \quad (12)
\]
Furthermore, the percentage difference of $C_l$ between the tubercle and baseline airfoil ($\Delta C_l$) is calculated using Eq. (11) and compared in Fig. 13(a). At $Re = 5.0 \times 10^4$ and in the pre-stall regime, $\Delta C_l$ is negative, indicating the tubercle airfoil is underperforming in terms of lift characteristics compared to the baseline airfoil. The lowest $\Delta C_l$ is $-23.72\%$, observed at $\alpha = 10^\circ$. On the other hand, in the post-stall regime, the tubercle airfoil has higher $C_l$ than the baseline airfoil, reaching a maximum $\Delta C_l$ of $38.82\%$ at $\alpha = 16^\circ$. Notice that due to the Reynolds number effect, the trend of $\Delta C_l$ at $Re = 2.0 \times 10^4$ is not the same as that of $Re = 5.0 \times 10^4$, as shown in Fig. 13(a). The slope of the $\Delta C_l$ curve at $Re = 2.0 \times 10^4$ increases almost linearly from $\alpha = 5^\circ$ to $20^\circ$ and reaches the maximum $\Delta C_l$ of $33.23\%$ at $\alpha = 20^\circ$.

The $C_d$ vs $\alpha$ characteristics of the baseline and tubercle airfoils are plotted in Fig. 12(b). At $Re = 5.0 \times 10^4$ and $0^\circ \leq \alpha \leq 6^\circ$, the $C_d$ of the tubercle and baseline airfoil are more or less equal. On the other hand, the $C_d$ of the baseline airfoil is less than the tubercle airfoil from $6^\circ < \alpha \leq 10^\circ$. In the post-stall regime, i.e., $\alpha > 10^\circ$, the $C_d$ of the baseline airfoil is considerably more than the tubercle airfoil. Additionally, we have compared the $C_d$ vs $\alpha$ characteristics of the baseline and tubercle airfoils obtained from our in-house water tunnel experiments at $Re = 2.0 \times 10^4$ with that of the CFD simulation results at $Re = 5.0 \times 10^4$. They show good agreement in the trends, but the $C_d$ magnitude is comparatively more at $Re = 2.0 \times 10^4$ than at $Re = 5.0 \times 10^4$.
Furthermore, the percentage difference of \( C_d \) between the baseline and tubercle airfoil (\( \Delta C_d \)) is calculated using Eq. (12) and plotted in Fig. 13(b). At \( Re = 5.0 \times 10^4 \), in the pre-stall regime, the \( C_d \) of the tubercle airfoil is typically higher than the baseline airfoil, reaching a maximum \( \Delta C_d \) of 79.11% at \( x = 10^\circ \). However, the \( C_d \) of the tubercle airfoil is less than the baseline airfoil in the post-stall regime. In particular, the tubercle airfoil maintained a minimum of 10.42% lesser \( C_d \) than the baseline airfoil in the post-stall regime for \( x \) between 12° and 20°. Notice that due to the Reynolds number effect, the trend of \( \Delta C_d \) at \( Re = 2.0 \times 10^4 \) is not the same as that of \( Re = 5.0 \times 10^4 \), as shown in Fig. 13(b). Except at low \( x \) (\( x < 4^\circ \)), the \( C_d \) of the tubercle airfoil is less than the baseline airfoil at \( Re = 2.0 \times 10^4 \), and the maximum difference is 39.10% observed at \( x = 6^\circ \).

### B. Time-averaged flow properties

In order to thoroughly explain the \( C_l \) vs \( x \) and \( C_d \) vs \( x \) characteristics discussed above, the time-averaged flow features of the tubercle and baseline airfoils are investigated at \( x = 0^\circ \), 6°, 8°, 12°, and 16° at \( Re = 5.0 \times 10^4 \) and plotted in Figs. 15 and 16, respectively. The time-averaged (i) pressure coefficient (\( C_p \)), (ii) \( z \)-vorticity (\( \omega_z \)), and (iii) iso-surfaces of \( x \)-velocity (\( V_x = 0 \)) contours on the suction surface of tubercle airfoil are shown Fig. 15. Note that the time period selected for deriving the time-averaged flow properties is obtained from the dominant frequency observed in the fast Fourier transform (FFT) of transient \( C_l \) data on the tubercle airfoil at each angle of attack. For example, the averaging time period at \( x = 10^\circ \) is 0.125 s, as shown in Fig. 14. At \( x = 0^\circ \), the separation bubbles are formed along the trough regions [see Fig. 15(iii-a)]. Interestingly, these separation bubbles are periodic along the span of the tubercle airfoil, and the flow remains attached between the two separation bubbles or behind the crest regions. Conversely, no separation bubble is noticed on the suction surface of the baseline airfoil at \( x = 0^\circ \), as can be seen in the \( z \)-vorticity plot in Fig. 16(ii-a). However, at \( x = 0^\circ \), near the trailing edge of the baseline airfoil, a wake regime is developed. At \( x = 6^\circ \), the pressure coefficient (\( C_p \)) on the trough section of the tubercle airfoil is comparatively smaller than the \( x = 0^\circ \) case, as shown in the contours in Fig. 15(i-b). The smaller \( C_p \) induces a high adverse pressure gradient, leading to an enhanced flow separation regime, as shown in Fig. 15(iii-b). Notice that the flow separation regimes at \( x = 6^\circ \) remain periodic along the span of the tubercles, similar to that of \( x = 0^\circ \) case. Later, when \( x \) is raised to 8°, the separated shear layer, emerging from the trough regions, oscillates in the spanwise direction [see Fig. 15(iii-c)]. This transmission of trough region flow regime from periodic separation bubbles type at \( x = 6^\circ \) to spanwise shear layers oscillations type at \( x = 8^\circ \) results in the change in the slope of \( C_l \) vs \( x \) characteristics curve of the tubercle airfoil at \( x = 6^\circ \) for \( Re = 5.0 \times 10^4 \) as shown in Fig. 12(a). On the contrary, the flow separation regime on the suction side of the airfoil is augmented for the baseline airfoil when \( x \) is changed from 0° to 6° then to 8° without any alteration of the bulk flow features [see in Figs. 16(ii-b) and 16(ii-c)]. As a result, the baseline airfoil’s \( C_l \) vs \( x \) characteristics curve continues to rise for \( x \) up to 8° without any slope change, as shown in Fig. 12(a). Note that both tubercle and baseline airfoil behave as the streamline objects at \( x < 6^\circ \). Additionally, the separation bubbles on the tubercle airfoil contribute insignificantly along the drag direction. In consequence, the \( C_d \) vs \( x \) of both baseline and tubercle airfoils attain near zero value for \( x < 6^\circ \). However, the transition of the surface flow pattern to a shear layer oscillations type [Fig. 15(iii-c)] at \( x = 8^\circ \) increases the \( C_d \) on the tubercle airfoil compared to the \( C_d \) on the baseline airfoil [see Fig. 12(b)] in the range \( 6^\circ < x \leq 10^\circ \).

When \( x > 8^\circ \), large separation zones, also known as stall cells,\(^{38} \) emerge from the trough regions of the tubercle airfoil, as shown in Figs. 15(iii-d) and 15(iii-e). Notice that the stall cells are initiated at the trough region and propagated along the span of the tubercle airfoil [see Fig. 15(iii-d)], and their initiation location is not unique along the airfoil span. At \( x = 12^\circ \), stall cell appeared at the mid-span of the airfoil, and at \( x = 16^\circ \), it appeared at both ends of the airfoil. Furthermore, the flow is mostly attached to the suction surface of the
FIG. 15. Contours of the time-averaged (i) pressure coefficient ($C_p$), (ii) z-vorticity ($\omega_z$), and (iii) iso-surfaces of $V_z = 0$ on the airfoil’s suction surface.
tubercle airfoil, except the surface occupied by the stall cell and the surface near the trough regions. On the other hand, complete flow separation on the suction surface of the baseline airfoil is observed at \( \alpha > 10^\circ \), causing the airfoil to stall and bringing the reduction in the \( C_l \) value and increase in \( C_d \) value, as shown in Figs. 16(ii-d) and 16(ii-e) and 12(a) and 12(b). The attached flow on the suction surface of the tubercle airfoil helps to maintain the high \( C_l \) and less \( C_d \) values compared to the baseline airfoil at \( \alpha > 10^\circ \) [see Figs. 12(a) and 12(b)].

The stall cells with bi-periodic, quad-periodic, and more \(^{16,20,42} \) repetition patterns have been previously reported on different tubercle airfoils with various amplitudes, wavelengths, and Reynolds numbers. However, on the presently optimized tubercle airfoil, the sexta periodic

FIG. 16. Contours of time-averaged (i) pressure coefficient and (ii) \( z \)-vorticity (\( \omega_z \)) of the baseline airfoil at various angles of attacks.
pattern [i.e., (repeats after every six tubercles)] of the stall cell is observed along the airfoil span [see Figs. 15(iii-d) and 15(iii-e)]. Note that in our CFD simulations, the tubercle airfoil’s transverse faces are set to periodic boundary conditions in the computation domain, which allows for duplication and translation of the flow field variables by a geometric shift equal to the span of the airfoil, i.e., $1.5c$ taken for a simulation case. For example, the time-averaged streamline plots on the tubercle airfoil at $\alpha = 12^\circ$, as shown in Fig. 17(b), clearly highlight the sexta periodic pattern of stall cells. Several authors\textsuperscript{13,16,18,19,30} described the salient features of the stall cells on the tubercle airfoil. In the present study, the stall cell flow features proposed by Cai et al.\textsuperscript{16} are adopted and marked in Fig. 17. In the type 1 regime, the flow is attached to the adjacent crests containing the stall cell, which has transverse velocities in opposite directions. The type 2 regime is associated with the converging flow between two successive stall cells. In the type 3 regime, the attached flow on the neighboring crests of the stall cell has the transverse velocity component in the same direction. Although these salient features of the stall cell are known, its formation...
mechanism and flow regime patterning on the tubercle airfoil surface have not been thoroughly investigated. In Sec. III C, the flow physics responsible for the stall cell’s temporal evolution is systematically analyzed using the transient flow features.

C. Transient flow properties

To investigate the stall cell flow characteristics over the tubercle airfoil, two sectional planes—one parallel to the incoming flow as shown in Fig. 18(a) and another one perpendicular to the chord line at a distance of 0.1c from the origin [refer to Fig. 18(d)]—are considered. In addition, the sectional plane’s top and front views are shown in Figs. 18(c) and 18(f). Here, $\alpha = 12^\circ$ is taken as a representative case for discussion. For the definition of origin, refer to Fig. 2(a).

The $y$-vorticity ($\omega_y$) and $y$-velocity ($V_y$) on these sectional planes are calculated and plotted in Figs. 19 and 22, respectively. Note that the tubercle airfoil observes significant $\omega_y$ on its span where the transverse velocity component ($V_z$) merges with the freestream flow ($V_x$) on the tubercle airfoil, as observed in Fig. 17(a). Hence, understanding the evolution of the $\omega_y$ on the tubercle airfoil will provide insights into the stall cell formation.

The $y$-vorticity contours displayed in Figs. 19(i-a)–19(i-e) bear a resemblance to side-by-side flow-over triangles, which is an intriguing observation. To understand this flow physics, we have compared the present flow characteristics with that observed in the case of side-by-side circular cylinders. The flow regime of the side-by-side circular cylinders depends on the gap between the cylinders, as shown in Figs. 20(a)–20(d). These flow types can be classified as follows: (a) the wake behind each of the circular cylinders placed side by side is

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**FIG. 18.** (a)–(c) show the side, isometric, and top views of the section plane, made parallel to the freestream at $\alpha = 12^\circ$, on the tubercle airfoil, respectively. Similarly, (d), (e), and (f) show the side, isometric, and front views of the tubercle airfoil with a sectional plane perpendicular to the chord at 0.1c from the origin [for the definition of origin refer to Fig. 2(a)].
FIG. 19. (i) The y-vorticity ($\omega_y$) contours on the sectional plane defined in Fig. 18(c), and (ii) shows the streamlines on the tubercle airfoil at different time instances.

(i-a) Shear layer oscillations

(i-b) Antiphase vortex shedding

(i-c) Strong clockwise vorticity

(i-d) Strong anti-clockwise vorticity

(i-e) Fully developed separation zone

(ii-a) In-phase vortex shedding

(ii-b) Antiphase vortex shedding

(ii-c) Spiral with clockwise vorticity

(ii-d) Spiral with anticlockwise vorticity

(ii-e)
uninfluenced by the neighboring cylinder, (b) the wake region of one cylinder is biased toward the wake region(s) of the neighboring cylinder(s), (c) in-phase, and (d) anti-phase coupled vortex shedding behind the cylinders. For additional information on these wake structures, readers are advised to refer to the study of Sumner et al. 61

Figure 19(i-a) and 19(ii-a) show an in-phase vortex shedding pattern on the flow over the tubercle airfoil at time \( t = 0.19 \) s, where the vortices from adjacent tubercles shed synchronously. In this instance, the vortices \( C_1 \) and \( C_2 \) on the tubercle airfoil turn into the gap (or the trough region) between the tubercles, as shown in Fig. 19(ii-a). Vortices turning into the gap between tubercles contrast the typical vortex rotation observed in the flow regime of a side-by-side circular cylinder, where the vortices turn into the cylinder’s wake away from the gap region, as illustrated in Fig. 20. It is important to note that in-phase vortex shedding occurs in all trough regimes of tubercle airfoils at different time instants.

At a later time, \( t = 0.51 \) s, the anti-phase vortex shedding (asynchronous vortex shedding) between the tubercles is observed, as shown in Figs. 19(i-b) and 19(ii-b). In contrast to in-phase vortex shedding Fig. 19(ii-a), anti-phase vortex shedding appears at a specific tubercle trough region [see Fig. 19(i-b)]. The \( C_p \) contour with \( x \)-velocity \( (V_x = 0 \text{ m/s}) \) iso-surface on the tubercle airfoil surface shown in Fig. 21(a) revealed that the anti-phase vortex shedding is associated with a large separated flow region, which propagates in the spanwise direction on the airfoil surface. It is important to note that the flow separation is severe at the trough regime \( T_3 \), as shown in Fig. 21(a). However, at the same time, the flow separation at the \( T_1, T_2, T_4 \), and \( T_5 \) trough regimes is suppressed. This suppressed flow separation is because the strong \( \omega_y \) vorticity from the tubercles is turning into the trough regimes \( T_1, T_2, T_4 \), and \( T_5 \), exchanging the momentum in the boundary layer and keeping the flow attached to the airfoil surface as shown in Fig. 21(b). However, at the trough \( T_3 \) regime, the turning of \( \omega_y \) vorticity from the tubercles is not significant. As a result, the \( T_3 \) trough regime has witnessed a large separated flow region, unlike other \( T_1, T_2, T_4 \), and \( T_5 \) troughs.

Note that the progression of this large separated flow is restricted toward the leading-edge of the tubercle airfoil due to the presence of counter-rotating vortex pairs (CVPs), which are discussed in the subsequent section. However, the separation can progress toward the trailing-edge and spanwise directions of the airfoil as there is no sufficient strength of vorticity (or momentum exchange) on the airfoil surface [see Fig. 21(b)] to overcome the adverse pressure gradient associated with the large separated flow. As a result, this large separated flow is further expanded in both the chordwise (toward trailing-edge) and spanwise direction by pushing the boundaries of large separated flow with strong clockwise [see in Fig. 19(i-c)] and anti-clockwise [see in Fig. 19(i-d)] vortices at \( t = 0.58 \) s and \( t = 0.65 \) s, respectively. Finally, at \( t = 0.90 \) s, a complete stall cell appears with a
large recirculation zone [see Figs. 19(i-e) and 19(ii-e)]. Furthermore, the appearance of this stall cell [Fig. 19(i-e)] resembles the biased wake similar to the one behind the side-side circular cylinders, as shown in Fig. 20(b).

Figure 22 shows the contours of the $y$-velocity ($V_y$) at different time instances on the plane perpendicular to the chord line, as shown in Fig. 18(f). The $V_y$ contours also revealed the presence of the biased wake on the tubercle airfoil. At $t = 0.45$ s, the wake behind all the
tubercles is straight and identical. From \( t = 0.58 \) s, the diversion of the wake is noticed. Finally, the fully developed flow with biased wake is observed at \( t = 0.90 \) s [Fig. 22(c)].

From these similarities between the flow over a tubercle airfoil with stall cells and the flow over side-by-side arranged cylinders, it can be concluded that the stall cell is indeed a form of biased wake formed due to the side-by-side arranged tubercles on the leading-edge of the airfoil.

D. Effects of counter-rotating vortex pairs on the stall cells

It has been reported that the critical factor that governs the in-phase, anti-phase vortex shedding, and biased wake pattern behind the side-by-side arranged cylinders is linked to the gap between the cylinders. Note that the gap between two successive tubercles varies along the crest to trough because of the curvature of the tubercles. Here, to further understand the tubercle gap effects on the stall cell characteristics, the contours of \( x \)-vorticity (\( \omega_x \)) are plotted on three different planes, one at the origin and the other two on both sides of the origin, at a distance of \( 0.1c \) (refer Fig. 23). For the definition of origin, refer to Fig. 2(a). This \( x \)-vorticity on these planes shows counter-rotating vortex pairs or CVPs. The CVPs at \( x/c = -0.1 \) and 0.0 show that the flow is mostly attached to the surface of the tubercle. However, at \( x/c = 0.1 \), the flow separation and reattachment are noticed on two adjacent tubercles marked by \( T_1 \) and \( T_2 \) (refer to Fig. 23). A separation bubble is formed between the separation and reattachment region, as shown in Fig. 23. By correlating these separation bubbles and the stall cell in Fig. 15(iii-d), it is understood that the CVPs clip and hold the stall cell and do not allow it to propagate toward the leading-edge of the tubercle airfoil. Hence, the stronger the CVPs are, the lesser the flow separation will be, leading to suppression of the stall cell development on the airfoil surface.

IV. CONCLUSIONS

The genetic algorithm coupled with the cascade forward neural network is developed to predict the optimum sinusoidal tubercle wavelength and amplitude suitable for the NACA0012 baseline airfoil at the Reynolds number of \( 5.0 \times 10^4 \). Here, a novel fitness function is adopted that ensures both the \( C_l \) and \( C_d \) characteristics of the tubercle airfoil are better than the baseline airfoil for the \( 0^\circ \leq \alpha \leq 20^\circ \). The search algorithm has estimated the optimum tubercle configuration must have an amplitude and wavelength of 11.71\% and 25\% of the baseline airfoil chord, respectively. Furthermore, numerical simulations and water tunnel experiments are conducted on this optimum tubercle airfoil to validate its aerodynamic advantages over the baseline airfoil. It is noticed that the tubercle airfoil at the high angle of attacks or post-stall regime yields significantly larger lift and reduced drag compared to the baseline airfoil. The flow features comparison of both the airfoils has revealed that the flow on the baseline airfoil span is completely separated at the high angle of attacks, leading to a substantial reduction in the lift and increment in drag. On the other hand, the flow regime of the tubercle airfoil at the high angle of attack is associated with the intermediate flow reattachment induced from counter-rotating vortex pairs, which helps it achieve enhanced aerodynamic characteristics over the baseline airfoil. In addition, intermediate large separation regions or stall cells are observed on the tubercle airfoil span, which hinders the aerodynamic performance of the tubercle airfoil, specifically at the high angle of attack. The formation of the stall...
cell is linked to the development of a biased wake pattern due to the side-by-side arrangement of the tubercles on the airfoil span.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Gangadhar V. R. Pinapatruni: Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Resources (equal); Supervision (equal); Project administration (equal); Resources (equal); Supervision (equal); Validation (equal); Writing (equal); Review & editing (equal).

Sunil Manohar Dash: Conceptualization (equal); Formal analysis (equal); Methodology (equal); Writing (equal); Investigation (equal); Methodology (equal); Resources (equal); Supervision (equal); Writing – review & editing (equal).

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DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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