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ABSTRACT

In this paper, we present a physics-informed approach to tailor the lift profile of an unsteady airfoil through the execution of an appropriate maneuver. In previous research, a low-order aerodynamic model based on the unsteady thin airfoil theory was developed for predicting the flowfield and loads on airfoils undergoing arbitrary motions. The theory was phenomenologically augmented using the concept of leading edge suction parameter (LESP) to incorporate the capability to predict intermittent leading edge vortex (LEV) shedding. The criticality of LESP was used to predict the onset and termination of LEV shedding and thus model the effect of LEVs on the flowfield and loads for a prescribed motion. In the current work, an inverse aerodynamic formulation is developed based on this framework for tackling the inverse problem: to obtain the motion kinematics required for generating a prescribed lift profile for an airfoil operating in the dynamic-stall regime. The LEV-modeling capability of the aerodynamic model enables the motion-design algorithm to take into account the effect of complex phenomena, such as dynamic stall and LEV shedding, which are not taken into account in previous research approaches. Several case studies are presented to demonstrate various scenarios such as lift tracking using pitching and heaving motions, lift cancellation during unsteady motion, and the generation of a given lift profile using two equivalent motions. The kinematic profiles generated by the inverse formulation are also simulated using a high-fidelity unsteady computational fluid dynamics solver to validate the predictions.

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I. INTRODUCTION

Unsteady aerodynamic flows are relevant in many modern engineering applications such as flapping wing micro-air vehicles, urban air mobility, formation flight and energy harvesting mechanisms, to name a few. Unsteady flow phenomena, such as leading edge vortex (LEV) shedding and dynamic stall, cause highly unsteady load fluctuations, and stability, control, noise, and vibration issues in these applications.1–9 It is therefore desirable to tailor the loads suitable to the application under consideration to minimize the detrimental effects. In this paper, we present a physics-informed low-order approach to tailor the lift of an unsteady airfoil operating in the dynamic stall regime.

The recent advancement in experimental and computational facilities and the improved understanding of the flow physics of unsteady wings have facilitated several approaches to control flow phenomena and modulate the loads on them. A variety of flow-control strategies have been explored for dynamic-stall control including pointwise flow control using electric or fluidic actuators5–10 and leading-edge suction and blowing1–11 to alter the LEV shedding process and thus its effect on the transient loads. Dynamically deforming airfoils have also been suggested as an approach to control dynamic stall.14 Another popular strategy is the use of deployable actuators such as trailing edge flaps,16–18 leading-edge minitabs,19 and vortex generators.19 Nature-inspired concepts, such as leading-edge tubercles20 and bio-inspired alula,21,22 have also been explored for altering the dynamics of the LEV and the process of dynamic stall.

Several recent research efforts have considered motion design as an approach for flow and load control on unsteady airfoils. Rival et al.17 conducted a detailed study of the effect of various kinematics on the LEV formation on unsteady airfoils and suggested the possibility of regulating the LEV characteristics by carefully tuning the airfoil
motion. Recently, Wei et al. demonstrated the generation of gusts with carefully controlled character using an airfoil actuated in heave and pitch obtained using an analytical model. Xu and Lagor presented lift-equivalent motion design for an unsteady airfoil using effective angle of attack. Sedky used a similar approach for tailoring the lift of a wing encountering a transverse gust. Angulo and Babinsky showed that gust mitigation on a wing experiencing transverse gusts can be achieved by using a suitable pitch-angle profile. McGowan et al. demonstrated the use of kinematics to modulate the lift of airfoils operating in the dynamic stall regime. Elfering and Granlund presented an approach to achieve lift modulations of an airfoil by the use of flap motion designed using an analytical approach based on Theodorsen theory.

Most analytical approaches for motion design rely on simplified aerodynamic models based on restrictive assumptions such as small motion amplitudes, attached flow, planar wake, and no LEV formation. LEV formation and dynamics is known to have a significant impact on the transient lift of an airfoil, and hence, it is important to take into account the role of LEVs in motion design. Further, multiple empirical parameters are often used in low-order models, which increases the reliance of the models on experimental or computational data.

In previous research, Ramesh et al. developed a low-order method to predict the flowfield and loads of unsteady airfoils with intermittent LEV shedding using the leading-edge suction parameter (LESP). The method, called the LESP-modulated discrete vortex method (LDVM), is built upon a large-angle unsteady thin airfoil theory applicable for airfoils undergoing arbitrary large-amplitude motions. This theory was phenomenologically augmented by an LEV-modelling capability formulated based on the LESP. The LESP is a measure of the leading-edge suction that can be supported by an airfoil before flow separation and shear-layer rollup occur at the leading edge, resulting in LEV formation. The flowfield is modeled using discrete vortices shed from the airfoil, which gives the capability to model the roll up of the vortical wake, thus extending the application of the method to non-planar wakes. The LDVM uses a single empirical parameter, the threshold value of LESP for an airfoil at a given Reynolds number, determined from experimental or computational data to predict the flow phenomena such as intermittent LEV shedding and wake rollup as well as the resulting loads of the airfoil for arbitrary unsteady motions. In a recent research effort, the authors presented an inverse-aerodynamic formulation based on LDVM to achieve a desired LEV shedding pattern from an airfoil. In the current work, we present an extension of this approach to design motion kinematics for an airfoil to achieve a desired lift variation, taking into account the role of LEV formation and dynamic stall. The theory behind the LDVM method is outlined, and the use of LESP to predict LEV shedding from unsteady airfoils is described in Sec. II. The inverse-aerodynamic approach is formulated in Sec. III. Several case studies are presented in Sec. V to demonstrate the application of the inverse formulation to various scenarios. The predicted motion kinematics are simulated using unsteady Reynolds-averaged Navier–Stokes (RANS) computational fluid dynamics (CFD), and the results are presented to validate the low-order results from the inverse-aerodynamic formulation. Finally, the concluding remarks are presented in Sec. VI.

II. THEORETICAL BACKGROUND

The LDVM uses a large-angle unsteady thin airfoil theory to model the airfoil, while the flowfield is represented using discrete vortices released from the airfoil edges. Figure 1 shows an airfoil of chord c undergoing arbitrary pitching and heaving motion. At time $t = 0$, the airfoil leading edge is at the origin of an inertial coordinate system OXZ. For $t > 0$, the airfoil translates to the left with a speed $U$. The pitch angle and the heave position of the airfoil are denoted by $\theta$ and $h$, respectively, and the corresponding velocities are denoted by $\dot{\theta}$ and $\dot{h}$. A body-fixed coordinate system, Bxz, is defined such that the origin is at the airfoil leading edge and the x axis aligns with the chord. The airfoil pitches about the pivot point $x_p$. The airfoil is represented using a bound-vorticity distribution along its camberline, given by

$$\gamma(\nu, t) = 2U_{\text{ref}} \left[ A_0(t) \frac{1 + \cos \nu}{\sin \nu} + \sum_{n=1}^{\infty} A_n(t) \sin(n\nu) \right],$$

where $U_{\text{ref}}$ is an arbitrary reference velocity. An appropriate choice of $U_{\text{ref}}$ for the current unsteady airfoil problem is the forward speed of the airfoil. Hence, $U_{\text{ref}}$ is set equal to $U$ in this work for nondimensionalization of various quantities. The coefficients $A_n(t)$ can be obtained by imposing the zero-normal-flow boundary condition,

$$A_0(t) = -\frac{1}{\pi} \int_0^\infty \frac{W(\nu, t)}{U_{\text{ref}}} d\nu,$$

and

$$A_n(t) = \frac{2}{\pi} \int_0^\infty \frac{W(\nu, t)}{U_{\text{ref}}} \cos(n\nu) d\nu, \quad n \geq 1.$$
\[
W(x, t) = \frac{\partial n}{\partial x}(U \cos \theta + \dot{h} \sin \theta + u_{\text{ind}}(x)) - U \sin \theta - \dot{\theta}(x - x_p) + \dot{h} \cos \theta - w_{\text{ind}}(x),
\]

(4)

where \( \frac{\partial n}{\partial x} \) is the slope of the camberline, and the terms \( u_{\text{ind}}(x) \) and \( w_{\text{ind}}(x) \) denote the components of velocity induced on the camberline in the \( x \) and \( z \) directions, respectively, by the discrete vortices. The flowfield consists of LEVs and trailing-edge vortices (TEV) released from the airfoil. A TEV of strength \( \Gamma_{\text{TEV}}^j \) is released at every time step \( j \) to satisfy Kelvin condition,

\[
\Gamma_B^j + \Gamma_{\text{TEV}}^j = \Gamma_B^{j-1},
\]

(5)

where \( \Gamma_B = \int_0^L \gamma(\nu) \, d\nu = \pi c U_{\text{ref}} [A_0(t) + A_1(t)/2] \) is the airfoil bound circulation. Ramesh et al. \(^{32} \) observed that the initiation of LEV shedding occurs when the chordwise suction force at the airfoil leading edge exceeds a threshold that can be supported by the airfoil. The leading-edge suction parameter (LESP, denoted henceforth by \( \zeta \)) was defined as a non-dimensional measure of this force. \( \zeta \) is closely related to the leading coefficient \( A_0 \) and can be obtained as

\[
\zeta(t) = -\frac{1}{\pi} \int_0^L W(x, t) \, dx,
\]

(6)

where \( U_{\text{ref}}(t) \) is the magnitude of the kinematic velocity at the airfoil midchord, as derived by Narsipur et al. \(^{32} \). In LDVM, discrete LEVs are released during the time steps when \( \zeta \) would exceed a critical value \( \zeta_{\text{crit}} \) without releasing an LEV. The strengths of the latest LEV and the TEV are calculated simultaneously at those time steps so as to satisfy the Kelvin condition (5) and to maintain \( \zeta \) at \( \zeta_{\text{crit}} \). The critical value of LESP can be predetermined for an airfoil-Reynolds number combination using CFD or experimental results for a representative motion and then be used in the LDVM simulations for any arbitrary motions of that airfoil at the particular Reynolds number.

The chord-normal force and chordwise suction force per unit span of the airfoil are, respectively, given by

\[
F_N = \rho c U_{\text{ref}} \left[ (U \cos \theta + \dot{h} \sin \theta) \left(A_0(t) + \frac{1}{2} A_1(t)\right) + \rho c^2 U_{\text{ref}} \frac{3}{4} A_0(t) + \frac{1}{4} A_1(t) + \frac{1}{8} A_2(t) \right] + \rho \int_0^L u_{\text{ind}}(x) \gamma(x, t) \, dx + \rho c \Gamma_{\text{lev}}
\]

(7)

and

\[
F_S = \rho c U_{\text{ref}}^2 A_0^2,
\]

(8)

where \( \Gamma_{\text{lev}} \) is the rate of vorticity shedding from the leading edge and \( \rho \) is the fluid density. The lift and drag coefficients can then be obtained as

\[
C_L = C_N \cos \theta + C_S \sin \theta,
\]

(9)

\[
C_D = C_N \sin \theta - C_S \cos \theta,
\]

(10)

where \( C_N \) and \( C_S \) are the normal-force coefficient and the suction-force coefficient, respectively, obtained by non-dimensionalizing the corresponding forces using the quantity \( \frac{1}{2} \rho U_{\text{ref}}^2 c \).

The LDVM method was validated in previous research against computational and experimental results for a variety of scenarios involving different geometries, Reynolds numbers, and motion kinematics. \(^{32,34-38} \)

More details on the LDVM method and the LESP concept can be found in these studies.

III. METHODOLOGY: THE INVERSE-AERODYNAMIC FORMULATION FOR LIFT REGULATION

The LDVM is implemented using a time-stepping scheme as shown in Fig. 2. The input is the prescribed motion kinematics of the airfoil. At each time step, the downwash \( W^0 \), due to the kinematics

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**FIG. 2.** Flowchart for the LDVM algorithm.
and the existing discrete vertices, is computed. A new TEV is now shed so as to satisfy the Kelvin condition (5). An intermediate value of $L$, denoted by $L/C_3$, is then calculated based on this solution of the flowfield. If this intermediate value of $L$ exceeds $L_{crit}$, a new LEV is released, and the strengths of the latest LEV and TEV are now determined simultaneously to maintain $L$ at $L_{crit}$ and to satisfy (5). With the latest vortex strengths known, the flowfield solution is completed, and the algorithm proceeds to calculate the Fourier coefficients and then to obtain the loads. The unsteady loads and flowfield are the final outputs. The discrete vortices are convected to their future positions, and the simulation advances to the next time step.

The flow chart for the current inverse-aerodynamic formulation (henceforth referred to as the IAF) algorithm is shown in Fig. 3, where the differences from the LDVM algorithm are highlighted using a different color. The desired lift profile, $C_l_{desired}$, is the input. The outputs are the kinematics required to achieve the lift profile, along with other loads and the flowfield. At each time step $j$, the algorithm iteratively solves the nonlinear equation

$$C_l(h_j, \theta_j) - C_l_{desired} = 0$$

(11)

to obtain the pitch and/or heave position of the airfoil at that time step that will generate the desired value of lift. The IAF algorithm uses initial guess values of the positions $h_j$ and $\theta_j$, which are set equal to the respective values from the previous time step. The corresponding velocities are then approximated using a first order backward difference scheme. Using the guessed kinematics, the flowfield solution and the loads are then calculated using the regular LDVM procedure. The guess values are updated if Eq. (11) is not satisfied. This procedure is repeated until the equation is solved within a specified tolerance. In this work, MATLAB’s `fsolve` equation is used to solve Eq. (11). The simulation proceeds to the next time step once the kinematics are obtained. It can be noted that the effect of LEV strength and evolution on the lift is considered by the IAF algorithm for generating the motion. Based on the desired lift profile, the algorithm generates a kinematic profile that may result in LEV shedding, and the effect of the previously shed leading-edge vorticity on the lift is taken into account while determining the kinematics for a time step.

Clearly, given the one-equation-two-variable scenario in Eq. (11), there may be multiple combinations of pitch and heave profiles that can be used to achieve a desired lift variation. In this work, we assume that one degree of freedom is known, so that a unique solution can be obtained for the other degree of freedom.

IV. CFD METHOD

CFD calculations were performed using North Carolina State University’s REACTMB-INS code, which solves the time-dependent incompressible Navier–Stokes equations using a finite-volume method. The governing equations are written in arbitrary Lagrangian/Eulerian (ALE) form, which enables the motion of a body-fitted computational mesh in accord with prescribed rate laws. Spatial discretization of the inviscid fluxes uses a low-diffusion flux-splitting method valid in the incompressible limit. This method is extended to higher-order spatial accuracy using piecewise parabolic method interpolations of the primitive variables $[p, u, v, w]$ and transported variable for the $S$–$A$ model, $\nu$. Viscous terms are discretized using second-order central differences. A dual time-stepping method is used to integrate the equations in time. An artificial compressibility technique, discretized in a fully implicit fashion and solved approximately using ILU decomposition, is used to advance the solution in pseudo-time. Typically, eight sub-iterations per physical time step were needed to reduce the residual errors two orders of magnitude. The Spalart–Allmaras model, as implemented by Edwards and Chandra, is used for turbulence closure. The CFD method has been validated against experimental wind and water tunnel data. In addition to experimental validation, recent research by Sudharsan et al. showed that force, moment, and flow data for multiple unsteady motions from the current CFD method compared well with validated large eddy simulation (LES) results.
A two-dimensional body-fitted O-grid for the NACA 0012 airfoil containing 140,400 cells was generated for this work and is shown in Fig. 4. Inlet and outlet boundary conditions were defined for the forward (left) and rearward (right) half of the O-grid, respectively, with the wall boundary condition being defined for the airfoil surface. The wall $y^+$ for the grid generated in the current study was less than 5. A grid sensitivity study, carried out to ensure grid convergence, showed that decreasing the wall $y^+$ to a tenth of that used in the final grids does not alter the results noticeably. The $y$-spacing at the wall for the final grid of the NACA 0012 airfoil was $4.3 \times 10^{-3} c$. All motions were simulated with a dimensionless time, $t^* \left(= \frac{U_{ref} \tau}{c} \right)$ of $1.7 \times 10^{-3}$, which translates to a physical time step ($t$) of $5 \times 10^{-5}$ s.

V. RESULTS

Case studies to demonstrate the capability of the IAF in computing the motion kinematics required to achieve the desired $C_l$ profiles for unsteady airfoils is presented in this section. For all cases, the NACA 0012 airfoil, pivoted at the leading edge and operating in a chord-based freestream Reynolds number ($\frac{u_{ref}^2}{\mu}$, where $\mu$ is the dynamic viscosity of the fluid) of 30,000, is used as the baseline geometry and operating conditions. An $X_{out}$ value of 0.17, obtained in previous research, is used for the above-mentioned airfoil-Re combination in the IAF.

A. Obtaining a desired lift profile using a pure pitch motion

In the current case study, the IAF is used to design a pitching motion to produce a commanded $C_l$ profile. Figure 5 plots the target $C_l$ profile (green curve) and the resulting pitch motion generated by the IAF (gray curve). In order to verify the accuracy of the method, the IAF-generated motion is numerically simulated using CFD and the resulting actualized $C_l$ prediction (black curve) is compared with the target $C_l$. Observations indicate that for $t^* > 1$, the actualized $C_l$ is lower than the target $C_l$, with the maximum difference of $\sim 14\%$ occurring at $t^* = 3.5$, which corresponds to peak $C_l$. However, barring the fluctuations in lift the range of $5 < t^* < 6$, the trends in the actualized $C_l$ compare well with the target $C_l$.

To better understand the reason behind the discrepancies in the IAF and computationally predicted lift, flow data (vorticity contours from CFD and LEV/TEV positions from the IAF) are co-plotted and presented in Fig. 6 at the time stamps (a)–(h) plotted in Fig. 5. The first indication of divergence occurs at $t^* = 2.5$ where computations show the upper-surface shear layer starting to separate from the airfoil’s trailing edge. Given that the IAF does not account for unsteady trailing-edge separation, the resulting motion generated is based on the assumption of fully attached flow at the trailing edge. Therefore, the IAF-generated motion will always result in a lower $C_l$ (when compared to the target $C_l$) during parts of the motion that evidence unsteady trailing-edge separation, thereby explaining the difference between target and actualized $C_l$ up to $t^* = 3.5$. However, once LEV initiation is triggered and the vortex gains strength, the LEV dynamics start to dominate ($t^* > 3.5$) and influence the aerodynamics, thereby explaining the decrease in the difference between target and actualized $C_l$. 

FIG. 4. Computational grids used in the CFD simulations: (a) computational domain and (b) grids around the symmetric NACA 0012 airfoil.

FIG. 5. Case A: commanded $C_l$ and the resulting pitch motion and $C_l$ variation.
between \( t^* \) of 4 and 5.5. Further observation of the flow data shows the vortex core in CFD convecting slower than what is predicted by the IAF, which could contribute to the increase in \( C_l \) in the range of 5 \( < t^* < 6 \), with the fluctuations most likely due to the nascent upper-surface vortex that is present aft of the main LEV at \( t^* = 4.5 \), which then produces a counter-rotating vortex and convects as a pair downstream, as observed at \( t^* = 6.5 \) in Fig. 6(f). Finally, the large drop in actualized \( C_l \) occurring at \( t^* = 6 \) can be attributed to the LEV pinching off and the delay in the re-attachment of the upper-surface shear layer. Said dynamics are not accounted for by the IAF, which instead assumes fully attached flow at the trailing edge, and therefore a higher \( C_l \) as soon as the LEV convects off of the airfoil surface. In conclusion, while the IAF-generated motion correctly accounts for the LEV-influenced aerodynamics, the lack of an unsteady trailing-edge separation model leads to the observed discrepancies between the target and the actualized lift.

B. Obtaining a desired lift profile using a pure heave motion

The target \( C_l \) used in case A (Sec. VA) is supplied to the IAF to generate a pure-heave motion to achieve the commanded \( C_l \) in the current case study. Similar to Fig. 5, Fig. 7 plots the target \( C_l \), IAF-generated heave motion, and actualized \( C_l \) from CFD. While the general trend in actualized \( C_l \) is similar to that observed for the pitch case (Sec. VA), the divergence of the actualized \( C_l \) from the target for \( t^* < 3.5 \) is lower for the heave case. Vortex flow data from Fig. 8 help justify the above-mentioned observation by showing that the magnitude of the shear-layer separation from the airfoil trailing edge is lower for the heave case as compared to the pitch case. The above-mentioned observation is quantified by studying the upper-surface skin friction data at \( t^* = 2.5 \) (not presented here), which revealed that separation had progressed up to 80% of the chord for the heave case as compared to 75% in pitch, thereby explaining the larger loss in lift in the latter case. Beyond \( t^* = 3.5 \), LEV dynamics start to dominate, leading to closer modeling of \( C_l \) to the target. Additionally, in line with the behavior observed for case A, the IAF simulation results in the vortex being...
more afterward as compared to CFD predictions. However, on comparing the flows at \( t^* = 5.5 \) for the pitch [Fig. 6(f)] and heave [Fig. 8(f)] cases, we observe the vortex core to be more afterward in the latter case, which in turn results in an earlier drop in \( C_l \) (at \( t^* = 5.5 \)). Once again, the delay in shear layer reattachment leads to a lower \( C_l \) prediction in CFD as compared to the target.

C. Lift cancellation during an unsteady maneuver

The current case study demonstrates how the IAF can be used to design motion kinematics that can be superimposed on a given baseline motion with a non-zero \( C_l \) profile to produce a zero lift profile. The baseline motion considered here is a pitchup-return-motion, as shown in Fig. 9 (purple curve). The resulting \( C_l \) for the baseline motion from computations is plotted in Fig. 9(a) (black curve). Additionally, the flow images for the motion have been plotted in the left column of Fig. 10 at time instances corresponding to markers (a)–(d) in Fig. 9(a).

The IAF is used to design a heaving motion which, when simultaneously performed along with the baseline pitch motion, will cancel out \( C_l \) due to pitch and maintain zero-lift for the entirety of the motion. The resulting heave motion generated by the IAF is plotted in Fig. 9(b) (orange curve). When the combined pitch-heave motion was simulated using the numerical method, the resulting \( C_h \) plotted in Fig. 9(b), showed that the IAF was successfully able to generate the motion kinematics required to cancel out the baseline lift. The snapshots in the right column of Fig. 10, which show the flowfield evolution.
for the combined pitch-heave motion at time stamps corresponding to markers (e)–(h) from Fig. 9(b), demonstrate that as the airfoil pitches up, the simultaneous heave-up motion induces a positive heaving velocity, which in turn results in an increased flow velocity at the airfoil’s leading edge. This allows the airfoil to maintain a higher suction at the leading edge, thereby facilitating flow attachment and eliminating the formation of the LEV. Additionally, the heave-up results in increased pressure on the airfoil’s upper surface, which counters and cancels out the pressure on the lower surface, resulting in net zero lift. The slight fluctuations in the actualized $C_l$ for the pitch-heave motion can be attributed to the inability of the IAF to account for unsteady trailing-edge boundary layer separation.

D. Pitch-heave equivalence

In this case study, we seek to use the IAF to design a pure-heave motion that produces the same lift variation as that of a pure-pitch motion. The baseline pure-pitch motion, shown in Fig. 11(a), is the same as the pure-pitch motion used in case C ($0°−45°−0°$ pitchup–hold–return motion about the leading edge). The corresponding flow images at four time instances, marked as (a)–(d) in Fig. 11(a), have been presented in the left column of Fig. 12.

The target $C_l$ for this case study is the CFD-generated $C_l$ for the pitch motion, shown in Fig. 11(a). Figure 11(b) plots the IAF-generated equivalent heave motion to produce the target $C_l$. Also, shown in Fig. 11(b) is the actualized $C_l$ generated by the CFD simulation of the IAF-generated heave motion. While the comparison between the actualized and target $C_l$ is quantitatively accurate until $t^* = 1$, differences start to emerge in the succeeding portion of the motion. On comparing the flow data between the pitch and heave motions at equivalent time instances in Fig. 12, we observe that while the flow dynamics are qualitatively similar up to $t^* = 1$, slight differences start to emerge at $t^* = 3$ and more noticeable differences are

![Fig. 10. Case C: flow images for the pure-pitch motion (left column) and the combined pitch-heave motion (right column). (a)–(h) correspond to the respective time instants marked in Fig. 9.](image)

![Fig. 11. Case D: (a) baseline pure-pitch motion and the associated $C_l$ variation. (b) Commanded $C_l$, the resulting heave motion, and the $C_l$ variation from the pure-heave motion.](image)
Additionally, the shear layer closer to the leading edge is observed to be more concentrated in the pitch motion as opposed to a more spaced out structure in heave. It is, however, interesting to note that, despite the striking differences in the flow dynamics, there is a quantitative match in $C_l$ behavior between the pitch and heave motions for $4 < t' < 5$. By the end of the motion ($t' > 5$), both pitch and heave motions exhibit qualitatively similar $C_l$ trends, with the heave motion producing lower lift, which can be ascribed to the difference in flow behavior [Figs. 12(d) and 12(h)].

VI. CONCLUSIONS

In this paper, we presented an inverse-aerodynamic formulation for generating desired lift profiles on unsteady airfoils operating in the dynamic stall regime. The inverse methodology is built upon LDVM, a large-angle-thin-airfoil-theory-based low-order method that can predict the LEV dynamics and associated loads on unsteady airfoils executing arbitrary prescribed pitch and/or heave motions. In the current work, the LDVM is modified to inverse design a pitch or heave motion that would result in a prescribed lift profile, while taking into account the effect of LEV formation and evolution. Several case studies were presented to demonstrate various capabilities of the inverse method. The generated motions were also simulated using high-fidelity RANS CFD to verify the accuracy of the inverse approach. Overall, the predictions of the loads and the flowfield are in good agreement with the CFD results.

In all the cases presented here, the CFD simulations show the presence of trailing-edge boundary-layer separation, which is not currently modeled in the inverse formulation. It is expected that the prediction accuracy can be improved by augmenting the model with this aspect of the flow-physics in future research. The inverse formulation is applicable for cambered airfoils even though a symmetric airfoil is considered for all the case studies in this work. It can also be easily extended to deforming foils to generate optimal kinematics for undulating fish-like geometries, or for exploring gust alleviation using optimal foil kinematics by incorporating a gust model into the aerodynamic theory. It is to be noted that a smooth motion profile is not always guaranteed, even though the kinematics generated by the inverse formulation for all the cases considered in this study were smooth (devoid of any large spikes in the positions, velocities, and accelerations). However, this issue may be tackled by posing the problem as an optimization problem with constraints on the kinematics. The same strategy can also be employed to generate motions with bounds on the allowable aggressiveness of the resulting kinematics.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Arun Vishnu Suresh Babu: Conceptualization (equal); Writing – original draft (equal); Writing – review & editing (equal). Shreyas...
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