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ABSTRACT

Flashover is a major limiting factor for the transmission and miniaturization of high-power microwave (HPM) devices. We conducted a study to investigate the developmental process of surface flashover on HPM dielectric windows through particle-in-cell-Monte Carlo collision simulations. A one-dimensional spatial distribution and three-dimensional velocity distribution model is established, encompassing the entire process of surface flashover, which includes electrode field emission, single-surface multipactor, outgassing, and gas breakdown. The nonuniform mesh generation method is employed to enhance the simulation accuracy. The growth rates of electron and ion densities increase as gas pressure rises. Additionally, the discharge transitions gradually from multipactor to gas ionization dominance. Notably, a space-charge-limited (SCL)-like sheath occasionally forms during an rf cycle near the surface under intermediate background pressure (~0.05 Torr). The SCL-like sheath cannot exist stably. Instead, it periodically disappears and appears as the rf electric field changes. The underlying physics are explained by the variations of the rf electric field, which lead to the variations in the surface charge density, thereby affecting the normal electric field. The normal electric field interacts with the spatial distribution of charged particles, ultimately leading to the formation of the SCL-like sheath. This work may facilitate a comprehensive understanding of the developmental processes of surface flashover.

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I. INTRODUCTION

Over the past few decades, high-power microwave (HPM) technology has rapidly advanced due to its promising applications in space technology, military, and industrial applications. Nevertheless, rf multipactor and flashover persist in HPM sources and equipment such as relativistic backward-wave oscillators (RBWO), traveling wave tubes (TWT), and relativistic magnetrons.1 Despite advancements in HPM technology, flashover along the dielectric surface of HPM devices continues to pose a significant obstacle to further development. Understanding the mechanism of the rf multipactor and breakdown on the dielectric surface under the influence of HPM is essential for the development of HPM technology. Surface flashover is the culmination of complex physical processes, and several theoretical models have been proposed to explain this phenomenon, such as the secondary electron emission avalanche (SEEA) theory and the electron triggered polarization relaxation (ETPR) theory.2 Presently, the SEEA model remains the predominant theory for surface flashover in vacuum.

Over recent decades, great progress has been made in both theoretical and experimental research on the mechanism of flashover along the dielectric surface of HPM devices.1,3–26 In the meantime, the particle-in-cell-Monte Carlo collision (PIC-MCC) simulation is widely used in the numerical research of HPM breakdown. Kim and Verboncoeur1,25 investigated the time-dependent physics of the single-surface breakdown in HPM systems using a one-dimensional electrostatic PIC-MCC simulation. Their findings reveal that the number of electrons oscillates twice the rf frequency over time. Wang and Cai constructed a model involving outgassing and investigated the impact
of various outgassing rates on the breakdown through simulation.\textsuperscript{6,7} Zhang \textit{et al.} established a PIC simulation model covering the complete process of surface flashover. Their study also explored the effects of using angled dielectric insulators and graded permittivity in enhancing the breakdown threshold.\textsuperscript{8–11} Wang \textit{et al.}\textsuperscript{12} found through PIC simulations that multipactor on dielectric window induced by circularly polarized microwave could be mitigated with a proper normal gyro-magnetic field. Additionally, Chang \textit{et al.} conducted comprehensive theoretical and experimental research on the suppression of multipactor using periodic grooves.\textsuperscript{13–15} Dong \textit{et al.}\textsuperscript{16} and Zuo \textit{et al.}\textsuperscript{17} both developed the one-dimensional spatial distribution and three-dimensional velocity distribution (1D3V) PIC-MCC model and corresponding codes. The effects of different gas pressures and outgassing rates on discharge were investigated by them, respectively. Wang \textit{et al.}\textsuperscript{18} proposed an energy-balance model of multipactor based on its similarity to collisional gas discharge. Theoretical predictions of multipactor in envelope microwaves and under an extra magnetic field align closely with PIC simulation results. Furthermore, Wen \textit{et al.}\textsuperscript{19} observed higher harmonic generation of the normal electric field near the dielectric surface and formulated a corresponding theory to elucidate the mechanism.

However, previous research has rarely focused on the plasma sheath during the surface flashover, leaving certain details to be discussed. The final surface flashover is finished with plasma formation, accompanied by plasma-sheath interaction with the dielectric surface. Hence, the investigation of sheath dynamics may provide insights into a more comprehensive understanding of the developmental processes involved in surface flashover. In our previous study of DC surface flashover, Sun \textit{et al.}\textsuperscript{20} proposed a physical mechanism for the development of surface flashover, grounded in observed alterations in sheath structures derived from simulations and theoretical derivations. This underscores the importance of paying attention to sheath dynamics in the research of surface flashover.

This paper presents a 1D3V model of surface flashover, encompassing the discharge process from the initial stage of the multipactor to the gas ionization based on the PIC-MCC algorithm. The time-varying trend of the plasma sheath observed in our simulation is depicted and analyzed in detail. This paper is organized as follows: in Sec. II, the established 1D3V model is described in detail. The simulation results are presented and analyzed in Sec. III. Section IV outlines the conclusions of the current study and future plan for upgrades to the simulation model.

II. MODEL SETUP

The model of surface flashover, which taking into account the secondary electron emission (SEE) of the dielectric under low background pressure, is shown in Fig. 1. In our simulation, the electromagnetic (EM) wave propagates along the \(-y\) axis, while the rf electric field is polarized along the \(x\)-direction. The 1D3V electrostatic PIC-MCC code is developed based on this model, enabling simulation of the surface discharge process.

To begin with, field emission (FE) resulting from field distortion serves as the main source of seed electrons. The electric field near the junction, where electrodes, dielectric, and vacuum meet, aka triple junction, is distorted, leading to FE. The Fowler–Nordheim formula is used to calculate the equivalent emission current density;\textsuperscript{21,22}

\[
J_F = \frac{e^2}{8\pi \hbar^2 \varphi_m} E_{FE}^2 \exp\left(\frac{8\pi \sqrt{2m_e \varphi_m^3}}{3 \hbar E_{FE}}\right),
\]

where \(J_F\) denotes the current density, \(\varphi_m\) is the work function of the dielectric, \(E_{FE}\) represents the electric field at the location of field emission, \(h\) is Planck’s constant, \(m_e\) refers to the rest mass of electrons, and \(e\) is the elementary charge. Each macroparticle contains \(N\) electrons, and the emission number of macroparticles per time step can be calculated,

\[
\text{num} = \frac{J_F \cdot S}{e \cdot N \cdot \Delta t},
\]

where \(\text{num}\) is the number of macroparticles, \(S\) stands for the surface area of FE, and \(\Delta t\) is the time step. The initial velocity of seed electrons obeys three-dimensional Maxwell’s velocity distribution,

\[
f(v) = \frac{m}{2\pi kT} \frac{1}{4\pi v^2} \cdot e^{-\frac{v^2}{2kT}},
\]

where \(f(v)\) is the velocity distribution function, \(m\) represents the rest mass of electrons, \(k\) is the Boltzmann constant, and \(T\) signifies the temperature of the whole system. The initial direction of seed electrons follows the cosine distribution, with its probability density function as follows:

\[
f(\theta) = \cos(\theta)/2, \quad \frac{\pi}{2} < \theta < \frac{\pi}{2},
\]

where \(\theta\) is the angle between the initial velocity of the seed electron and the direction perpendicular to the dielectric surface. Secondary electrons are emitted from the dielectric surface when electrons, accelerated by the electric field, collide with the surface. The secondary
electron yield (SEY) coefficient, denoted as $\delta$, is the function of impact energy $E_i$ and angle $\beta$ of the incident electrons. Vaughan’s empirical formula is adopted to calculate the SEY:

$$
\delta(E_i, \beta) = \delta_{\text{max}} \left( 1 + k_1 \frac{\beta^2}{2\pi} \right) \times f(w, k),
$$

where $\delta_{\text{max}}$ is the maximum value of $\delta$, and $E_{\text{max}}$ is the corresponding energy of $\delta_{\text{max}}$. They are parameters related to the type of material. $E_{\text{min}}$ denotes the minimum energy of SEE. Secondary electrons will not be generated when $E_i$ is below $E_{\text{min}}$, typically set as 12.5 eV. The parameter $k_1$ quantifies the surface smoothness of the material, ranging from 0 for rough surfaces to 2 for polished surfaces. Normally, the value of $k_1$ for a typical dull surface is set to 1. While the calculated SEY may contain a decimal part, the simulation requires an integer number of electrons. To address this, we record the integer part and the fractional part of $\delta$ as $\delta_i$ and $\delta_f$, respectively. We then round $\delta_f$ to a random number $R$, which falls within the range of 0–1,

$$
\delta = \begin{cases} 
\delta_i + 1, & \delta_f > R, \\
\delta_i, & \delta_f \leq R.
\end{cases}
$$

FIG. 2. The initial energy distribution of secondary electrons.

The initial direction of secondary electrons also follows the cosine distribution, as illustrated in Eq. (4). Von Seggern has fitted an energy distribution function of secondary electron, which is suitable for most insulating materials,

$$
g(E_0) = \sum_{n=1}^{2} a_n e^{-b_n E_0}.
$$

With $a_1 = 0.476, a_2 = -0.476, b_1 = 0.4$, and $b_2 = 0.5$, $E_0$ is the initial energy of secondary electrons. The energy distribution curve obtained from Eq. (9) is displayed in Fig. 2. It reveals the initial energy of most secondary electrons is concentrated within the range of 0–4 eV. In order to improve the simulation speed, we set the initial energy of secondary electrons to 2.9 eV in this paper, which is the expected value of the energy distribution function.

The movement of charged particles in the electromagnetic field is determined by the Newton–Lorentz equation,

$$
dr \frac{dt} = \vec{v},
$$

$$
m \frac{d\vec{v}}{dt} = q(E_{\text{rf}} + E_y + \vec{v} \times \vec{B}),
$$

where $r$ and $v$ are the position and velocity ofcharges, respectively. $E_{\text{rf}}$ is the rf electric field, $B$ represents the rf magnetic field, and $q$ signifies the charge quantity of the charged particle. The rf electric field $E_{\text{rf}}$ is set as $E_{\text{rf}} = E_{\text{rf}} \cos(2\pi f t + \phi)$, where $E_{\text{rf}}$ is the amplitude of the electric field, $f$ indicates the frequency of the EM wave, and $\phi$ is the initial phase of the rf electric field. $E_y$ is the static electric field related to charges on the surface and charged particles in space. In this paper, the effect of rf magnetic field on the process of discharge is not considered. The central difference formats of Eqs. (10) and (11) are given as follows:

$$
r_{x+1}^- = r_x^- + v_n^- \Delta t,
$$

$$
m \frac{v_n^- - v_{n+0.5}}{\Delta t} = q(E_{\text{rf}} + E_y^-).$$

This particle integration scheme, called leap-frog, is used in our PIC-MCC code. Prior to determining the electron motion, it is necessary to calculate the spatial electric field distribution.

Surface charge and the charged particles in the simulation area jointly generate the normal space charge electric field $E_{\text{dc}}$. The normal electric field $E_{\text{dc}}$ and the electric potential $\varphi_E$ are determined by solving the one-dimensional Poisson equation,

$$
\frac{\partial^2 \varphi_E}{\partial y^2} = -\frac{\rho}{\varepsilon_0},
$$

$$
E_{\text{dc}} = -\frac{\partial \varphi_E}{\partial y}.
$$

where $\varepsilon_0$ is the vacuum permittivity and $\rho$ signifies the charge density. The surface area of the dielectric, measuring 50 cm$,^2$, is employed in calculating the charge density $\rho$. The space in the y-direction is divided into multiple grids, with the normal electric field $E_{\text{dc}}$ located at the center of each grid. $\varphi_E$ and $\rho$ are located at the grid nodes.

The charge density $\rho$ of each node is calculated based on the position information of charged particles in the y-direction, which is acquired by solving Eqs. (12) and (13). Upon determining the charge density, the space charge field $E_{\text{dc}}$ can be computed by solving the one-dimensional Poisson equation. In our simulation, the left boundary of the Poisson equation is set at the dielectric surface ($y = 0$). The right boundary is located at $y = L$, assumed to be dielectric with a secondary electron yield of zero. The boundary conditions for the Poisson equation are expressed as

$$
E_{\text{dc}} \left( \frac{1}{2} \right) = \frac{\varphi_E(0) - \varphi_E(1)}{Ay} = \frac{1}{2\varepsilon_0} [\sigma + \rho(0)Ay],
$$

where $\varphi_E(0)$ and $\varphi_E(1)$ are the electric potentials at the left and right boundaries, respectively.
\[ \varphi_e(L) = 0, \quad (17) \]

where \( \Delta y \) represents the size of the grid and \( \sigma \) refers to the charge density on the dielectric surface. The positive or negative sign of \( \sigma \) indicates whether the surface of the dielectric accumulates positive or negative charges. Positive charges accumulate on the surface when the SEY \( \delta \) exceeds unity, while negative charges can be found on the surface when SEY is less than 1. Upon obtaining the SEY \( \delta \) through calculation, there will be \((\delta - 1)\) positive charges accumulated on the dielectric surface. In the one-dimensional case, dividing the charge quantity on the dielectric surface by the surface area yields the surface charge density \( \sigma \). \( \varphi_e(0) \) represents the electric potential at the first grid point, with similar expressions in the equations of this paper having analogous interpretations. As we mentioned earlier, \( E_{dc} \) is located at the center of every grid, while \( \varphi_e \) and \( \rho \) are located at the grid nodes. Therefore, the left boundary condition used in solving Poisson’s equation should be \( E_{dc}(1/2) \) rather than \( E_{dc}(0) \).

Additionally, it should be noted that \( E_{dc}(0) = \sigma/2\epsilon_0 \) is commonly employed to estimate the normal electric field generated by positive surface charges, according to the model proposed by Anderson and Brainard. However, this formula defaults the relative permittivity of the dielectric to 1, which deviates from the actual situation. Zhang et al. replaced this formula with a more rigorous one in the model he developed,

\[ E_{dc}(0) = \frac{\sigma}{(1 + \epsilon_0)\epsilon_0}, \quad (18) \]

where \( \epsilon_0 \) denotes the relative permittivity of dielectric. In our simulation, we find that incorporating the actual value of dielectric permittivity has little effect on the results. Therefore, we decide to continue applying the approximate expression from Anderson’s model in this paper.

At the end of each time step, \( E_{dc} \) is updated by solving Eqs. (14) and (15). Normally, the discretized form of the one-dimensional Poisson equation is expressed as follows:

\[ \varphi_e^{i+1} - \frac{1}{D_y} \varphi_e^i + \frac{1}{D_y} \varphi_e^{i+1} = \frac{\rho^{i+1}}{2\epsilon_0} \Delta y^2, \quad (19) \]

where \( \Delta y \) represents the size of the grid. \( i \) ranges from 1 to \((n - 2)\), where \( n \) is the total number of grids. Equation (19) is applicable when all grid sizes are uniform. We utilize a nonuniform grid division method in our simulation to reduce the number of grids while maintaining accuracy and enhancing computational efficiency. The size of each grid is determined through exponential fitting following the establishment of minimum and maximum sizes of grids, along with the total number of grids. The size of the grid exponentially increases with the distance from the surface, as depicted in Fig. 3. The grid density distribution function is expressed as

\[ \Delta y(i) = a \cdot \exp(b \cdot i), \quad (20) \]

where \( \Delta y(i) \) represents the size of the \( i \)th grid and \( a \) and \( b \) are the fitting parameters determined from the simulation parameters. This approach involves employing fine grid partitioning in regions proximal to the dielectric surface and coarse grid partitioning in distant regions. The reason why exponential fitting is chosen is that in this case, the grid size grows more gradually near the surface. Finer grid division near the surface yields more accurate results for the Poisson equation, as the majority of charged particles are concentrated near the surface. After employing the nonuniform grid division method, the discretized form of the one-dimensional Poisson equation is revised as

\[ \frac{1}{D_y (\Delta y_i + \Delta y_{i+1})} \varphi_e^i - \frac{1}{D_y (\Delta y_i + \Delta y_{i+1})} \varphi_e^{i+1} + \frac{1}{D_y (\Delta y_i + \Delta y_{i+1})} \varphi_e^{i+2} = -\frac{\rho^{i+1}}{2\epsilon_0}, \quad (21) \]

where \( i \) still ranges from 1 to \((n - 2)\), \( n \) is the number of grids, and \( \Delta y_i \) is the size of the \( i \)th grid. Nonuniform grid division allows for more accurate results of the Poisson equation compared to the uniform grid. Simultaneously, employing larger grid sizes in remote regions from the surface can enhance simulation efficiency without compromising accuracy, given the low concentration of charged particles in these areas.

Except for SEE, electrons colliding with the dielectric surface of the dielectric also induce the desorption of absorbed gas. The composition of desorbed gas is highly complex. To simplify this process and improve the simulation efficiency, we substitute argon (Ar) for the actual desorbed gas in this study. In the conventional MCC algorithm, all electrons in the space must be traversed in each time step, necessitating extensive computational resources. To speed up the simulation, we employ the “null collision” algorithm. The maximum frequency of collision is defined as

\[ \nu_{\text{max}} = \max \left[ n_g(x) \sigma_T(E) v \right] = \max \left[ n_g(x) \max [\sigma_T(E) v] \right], \quad (22) \]

where \( n_g(x) \) is the gas density and \( E \) and \( v \) denote the energy and velocity of the electron, respectively. \( \sigma_T \) is the total cross section for a given electron, which is related to its energy. \( At \) represents the unit time step. The total collision probability, independent of particle energy and position, is written as

\[ P_t = 1 - \exp(-\nu_{\text{max}} At). \quad (23) \]

\( P_t \) is the proportion of electrons colliding with gas molecules related to the total number of electrons in each time step. The number of electrons hitting the surface can be easily counted, after which electrons can be randomly selected to collide with gas molecules based on this count. For each selected electron, the type of collision it participates in is determined randomly.
The types of collisions specifically considered in this model include the elastic collision, excitation collision, and ionization collision between electrons and the Ar molecules. Other collision types are ignored in this paper owing to their minimal collision cross sections. In the case of elastic collision, no new particles are produced, but the electron’s velocity changes and the scattering angle \( \chi \) is determined as

\[
\cos \chi = \frac{2 + E_e - 2(1 + E_e)^R}{E_e},
\]

(24)

where \( E_e \) represents the energy of the electron before the collision and \( R \) is a uniformly distributed random number ranging from 0 to 1. The electron speed after the elastic collision is written as

\[
v' = \sqrt{\frac{1 - 2m_e m_i (1 - \cos \chi)}{(m_e + m_i)^2}},
\]

(25)

where \( m_e \) and \( m_i \) are the rest mass of the electron and the argon ion, respectively. The speed of the electron after the elastic collision can be calculated according to Eqs. (24) and (25).

For the excitation collision, no new particles are generated, and the electron energy after the collision is written as

\[
E' = E_e - E_{exc},
\]

where \( E_{exc} \) denotes the excitation energy. The velocity of the electron becomes

\[
v' = \sqrt{2E' / m_e}.
\]

Subsequently, the final speed of the electron after excitation collision is determined using Eqs. (24) and (25), that is, \( v' \) and \( E' \) are used as the \( v \) and \( E \) in Eqs. (24) and (25).

For the ionization collision, a new electron and an Ar ion are created, while the original electron is scattered. Initially, the energy of the incident electron becomes

\[
E'' = E_e - E_{ion},
\]

and then, the remaining energy of the incident electron is randomly divided between itself and the newly created electron. Therefore, the energy and velocity of the incident electron are presented as

\[
E'' = R \times E',
\]

(26)

where \( R \) is a random number uniformly distributed between 0 and 1. At last, Eqs. (24) and (25) are utilized to calculate the final speed of the electron after ionization collision. The initial direction of both the newly generated electron and ion is determined randomly.

According to the above-mentioned model, the algorithm flow is illustrated in Fig. 4.

The simulation parameters in this paper are set as follows. The amplitude of rf electric field \( E_{rf} = 5 \text{ MV/m} \) and its frequency \( f = 10 \text{ GHz} \). The initial phase of the rf electric field is set as \( \theta = 0 \). The surface area of FE is set as \( S = 5 \text{ \mu m} \). In this paper, we choose polytetrafluoroethylene (PTFE) as the dielectric material where its \( \delta_{r0} = 3.0 \) and the corresponding energy \( E_{ion} = 300 \text{ eV} \). The smoothness factor \( k_s \) is set as 1. The initial energy of electrons emitted from the dielectric surface is 2.9 eV. The simulation region extends from \( y = 0 \) to \( y = L = 160 \mu m \), as the discharge area is concentrated close to the dielectric surface. The dielectric is located at \( y = 0 \). The surface area of the dielectric is 50 \text{ cm}^2. Each macroparticle (electron or ion) represents \( N = 5 \times 10^6 \) real particles. The size of the grid \( \Delta y \) varies between 0.5 and 2 \text{ \mu m} \), and the expression of the grid density distribution function is set as

\[ J(y) = 0.4954 \cdot \exp(0.0095 \cdot y), \]

and the time step is set as \( 1 \times 10^{-4} \text{ ns} \). At the beginning of each time step (\( t_{\text{emission}} = 0.5 \text{ ns} \)), five macroparticles (electron or ion) are emitted into the simulation space, corresponding to an initial seed current density of

\[ J = 8.011 \times 10^4 \text{A/m}^2. \]

III. SIMULATION RESULTS AND DISCUSSION

This section begins with a benchmark test of our model in the case of vacuum. Subsequently, simulation results under varying background gas pressures are compared, and the factors influencing these changes are analyzed. In addition, we observe the occasional formation of a space-charge-limited (SCL)-like sheath during an rf cycle near the
surface under intermediate background pressure (≈0.05 Torr) and provide a detailed explanation of the underlying physical mechanisms.

A. Saturated multipactor

In the case of vacuum, only the process of multipactor occurs, devoid of ionization, resulting in the absence of ions in space. In the beginning, electrons are emitted from the dielectric surface with a relatively low initial energy (2.9 eV), subsequently accelerated by the rf electric field. Some electrons, after being accelerated, will collide with the dielectric surface at high speeds, leading to the emission of secondary electrons into the vacuum. Given the SEY exceeds 1 due to the high energy of these incident electrons, more secondary electrons are emitted than the incident electrons, leading to the deposition of positive charges on the surface. These positive charges, in turn, attract more electrons to impact the surface, thereby generating more secondary electrons. This process is known as the secondary electron emission avalanche (SEEA). As the multipactor develops, the number of electrons in the vacuum swiftly increases from 0 to around $10^{10}$ and the multipactor eventually covers the surface. In the absence of gas ionization in vacuum, the multipactor saturates and stabilizes over time. At this stage, the number of electrons in space ceases its increase and begins to oscillate over time at twice the frequency of the rf electric field, following the saturation of the multipactor, as shown in Fig. 5(a).

During the saturation stage of the multipactor, both the mean energy and average SEY of electrons oscillate at twice the frequency of the rf electric field. We average the energy and SEY for each rf period, and the results are shown in Figs. 5(c) and 5(d). Figure 5(c) illustrates that the mean energy of electrons in vacuum initially undergoes rapid increase, followed by a gradual decline, with the maximum value of mean energy reaching 500 eV. The average energy of electrons for each period remains stable ($\approx 120$ eV) after the multipactor reaches saturation. As discussed earlier, electrons strike the dielectric surface with high energy during the initial stage of the multipactor, resulting in the average SEY higher than 1. As a consequence, positive charges gradually accumulate on the surface. These positive charges deposited on the surface generate a normal electric field $E_{dc}$, attracting electrons in vacuum to strike the surface. The presence of $E_{dc}$ also changes the energy of electrons.

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$E_{dc}$ further increases the energy of electrons in vacuum when few positive charges accumulate on the surface. However, as the

\[ \text{FIG. 5.} \quad \text{(a) The number of electrons; (b) surface charge density; (c) the average energy; and (d) average SEY for each rf period as a function of time in vacuum.} \]
multipactor progresses, the accumulation of positive charges on the surface continues to increase and strengthens $E_{dc}$, consequently shortening the transit time of electrons and diminishing their energy.

According to Vaughan’s empirical formula, the SEY is related to the energy $E_i$ and angle $\beta$ at which electrons strike the surface. Initially, electrons exhibit high mean energy, resulting in the average SEY much greater than 1. With decreasing impinging energy, SEY decreases accordingly and eventually stabilizes around unity, as shown in Fig. 5(d), so the number of electrons remains steady. The SEY curve employed in this paper is shown in Fig. 6. This curve intersects with $E_1 = 55$ eV and $E_2 = 1.9$ keV, indicating that SEY exceeds 1 when the impinging energy falls between $E_1$ and $E_2$. It is observed that the average energy of electrons at the saturation stage of the multipactor ($\approx 120$ eV) surpasses $E_1 = 55$ eV, while the corresponding SEY approaches 1. This phenomenon arises because the mean energy is influenced by all electrons in vacuum, including those farther away from the surface, which are accelerated for a longer duration in the rf electric field due to the shielding effect of the space charge field $E_{dc}$. Ions will be absorbed by the boundary when they approach the surface, thereby influencing the amount of positive charge on the surface. However, the quantity of ions absorbed by the surface is significantly smaller than that of electrons hitting it, as the positive charges accumulated on the surface repel these ions, resulting in negligible influence on surface charge density. Hence, we ignore the effect of ions on surface charge density in this simulation.

Ionization collisions between electrons and gas molecules generate positive ions and new electrons, increasing both the quantity of electrons in space. Figure 7 illustrates the numbers of electrons and ions as a function of time at low pressure (0.05 Torr) and average SEY. Electrons closer to the surface experience more frequent collisions with the dielectric surface, but they are accelerated for a shorter time and have lower impinging energy, causing a lower SEY. Therefore, the average SEY at the saturation stage of the multipactor remains close to 1 despite the mean energy of electrons being higher than $E_1$.

B. Simulation results at different gas pressures

Apart from generating secondary electrons, collisions between electrons and the dielectric surface also release absorbed gas, leading to a gradual increase in pressure near the surface. With rising gas pressure, electrons moving in the electric field may collide with gas molecules, resulting in gas ionization. The gas desorption rate $\gamma$, representing the number of gas molecules released when an electron collides with the dielectric surface, is commonly used in simulations to characterize the rate of gas release. Nevertheless, the gas desorption rate $\gamma$ typically remains well below 1, indicating a slow increase in gas pressure. Consequently, in this paper, we maintain a fixed background gas pressure to expedite simulations and investigate the impact of varying gas pressures on discharge by adjusting the background gas pressure.

Gas ionization leads to the appearance of positive ions near the dielectric, which will have impacts on the space charge field $E_{dc}$. Ions may collide with gas molecules, and collisions between ions may also occur. However, these processes are omitted in this simulation due to their small cross sections. Only collisions between electrons and gas (elastic collision, excitation collision, and ionization collision) are considered. Ions will be absorbed by the boundary when they approach the surface, thereby influencing the amount of positive charge on the surface. However, the quantity of ions absorbed by the surface is significantly lower than that of electrons hitting it, as the positive charges accumulated on the surface repel the ions, resulting in negligible influence on surface charge density. Hence, we ignore the effect of ions on surface charge density in this simulation.

Ionization collisions between electrons and gas molecules generate positive ions and new electrons, increasing both the quantity of electrons in space. Figure 7 illustrates the numbers of electrons and ions as a function of time at low pressure (0.05 Torr) and

![FIG. 6. Secondary electron yield coefficient curve of PTFE.](image)

![FIG. 7. The numbers of electrons and ions as a function of time at (a) 0.05 Torr and (b) 0.5 Torr.](image)
relatively higher gas pressure (0.5 Torr). In contrast to the vacuum condition shown in Fig. 5(a), the number of electrons increases over time while oscillating at twice the rf frequency when the gas pressure exceeds 0. Furthermore, with the development of surface flashover, the number of ions also gradually grows, being comparable to that of electrons. The number of electrons and ions exhibit quasi-linear growth when the gas pressure is not high, as depicted in Fig. 7(a). As the gas pressure increases, the number of particles goes up gradually faster. As depicted in Fig. 7(b), at high pressure, both the number of electrons and ions demonstrate exponential growth rates significantly exceeding those observed at low pressure.

Equations (22) and (23) in Sec. II indicate that the rate of ionization collision correlates directly with the density of gas and electrons, as well as the energy of electrons. Since the mean energy of electrons changes little between 100 and 1000 eV, the ionization rate is primarily determined by the density of gas and electrons. Given the consistent spatial range used in this simulation, the density of electrons is proportional to the number of electrons. At the initial stage, the number of electrons at 0.5 Torr is similar to that at 0.05 Torr, but the density of gas molecules at 0.5 Torr is notably greater. Consequently, the probability of ionization collision at 0.5 Torr is higher, causing a more rapid electron density growth compared to 0.05 Torr, thereby amplifying the ionization collision rate at high pressure (0.5 Torr) substantially over that at lower pressure (0.05 Torr). Ionization collision is an important source of electrons alongside the process of SEE, with the probability of ionization collision at 0.5 Torr being much greater than that at 0.05 Torr. Therefore, the number of charged particles experiences swifter growth at 0.5 Torr compared to that at 0.05 Torr. Figure 7 illustrates a drastic increase in the number of electrons at 0.5 Torr, soaring by approximately 3980%, whereas at 0.05 Torr, the increase is a mere 18%. Thus, the rate of ionization collision at 0.05 Torr increases little, resulting in a quasi-linear increase in the number of particles.

Figure 8 displays the spatial distribution of charged particles under vacuum, low-pressure (0.05 Torr), and high-pressure (0.5 Torr). In vacuum, the multipactor serves as the only source of electrons, resulting in their concentration near the surface, with the peak of electron density observed very close to the surface, as shown in Fig. 8(a). At 0.05 Torr, the electron density near the surface is significantly lower than that in vacuum. The peak of electron density is observed farther from the surface compared to that of electron density, with the electron density slightly farther away from the surface being higher than that in vacuum. This phenomenon arises from ionization collisions, which generate electrons and ions at a greater distance from the surface, consequently increasing the density of both particles in those regions. At 0.5 Torr, the impact of ionization collisions on both electron and ion density becomes more pronounced due to its increased frequency. Both the density of electrons and ions away from the surface notably rise compared to vacuum and low pressure, as illustrated in Fig. 8(c). The variations in the spatial distribution of electrons and ions also indicate that, as gas pressure increases, the influence of ionization collision on discharge progressively strengthens. Ionization collision could emerge as the predominant factor in surface flashover under sufficiently high gas pressure.

Figure 9 depicts the spatial distribution of normal electric field $E_{dc}$ at different gas pressures. Above the multipactor layer, the normal electric field decreases rapidly to close to 0 in the case of vacuum. This implies that the electric field generated by charges accumulated on the dielectric surface is effectively shielded by the multipactor electrons. As a result, the majority of electrons are confined near the surface, as depicted in Fig. 8(a). Subsequently, with the appearance of the plasma, i.e., as the gas pressure increases, this shielding effect is disrupted. Consequently, the spatial distribution of the normal electric field changes, causing some multipactor electrons to no longer be confined around the surface but escape to distant regions, participating in ionization collisions with gas molecules. As shown in Fig. 9, with gas pressure increasing, the disruption of the shielding effect becomes increasingly pronounced. This phenomenon also explains why surface flashover gradually transitions from being dominated by multipactor electrons to surface flashover.

C. Evolution of plasma sheath structure near the dielectric surface during flashover

With the increase in gas pressure, plasma gradually forms near the surface of the dielectric. Plasma is generally an electrically quasi-neutral substance composed of electrons and ions. Since electrons move faster in the electric field than ions, a non-electrically neutral region known as the plasma sheath is formed between the plasma and the solid surface, including dielectric surfaces and electrode surfaces. In this simulation, the electrodes are not considered; thus, the electrode sheath is not included in the present work. The sheath is the transition region from plasma to the solid surface, shielding the space charge field and influencing the interaction of charged particles with the dielectric surface. The concept of sheath was initially described by Langmuir and Tonks in 1929. A Debye sheath usually forms where the plasma meets a solid surface due to the higher mobility of electrons compared
to ions, resulting in the surface being negatively charged relative to the plasma. It is the most common type of sheath, and for a conventional Debye sheath, the interaction between the charged particles and the solid boundary is disregarded. However, some electrons and ions may collide with the surface when the plasma is close to it. Incident electrons and ions can induce secondary electron emission on the surface through various mechanisms. The model of plasma sheath considering electrons emitted by the solid boundary was first proposed by Hobbs, but it is only valid for weak surface emission scenarios. When the ratio of the emission to primary electron fluxes exceeds 1, a potential well forms between the plasma and surface, transforming the sheath into other forms. The sheath in this case is called the space-charge limited (SCL) sheath. In addition to secondary electron emission on the solid boundary, the thermal electron emission and photo-electron emission also contribute to the formation of the SCL sheath.

During the process of surface flashover, the development of the multipactor induces gas desorption, creating a low-pressure environment near the surface. Subsequently, plasma gradually forms in the surrounding space, coexisting with the multipactor. Chang et al. conducted a theoretical analysis of the space charging shielding in this case and found that a potential distribution analogous to the SCL sheath formed near the surface. A similar phenomenon is observed in our simulation. As depicted in Fig. 10, a potential well appears near the surface, with the potential reversing at the point approximately 2 μm away from the surface, at a pressure of 0.05 Torr. The electric potential is derived by solving the Poisson equation, wherein the spatial distribution of charged particles plays a crucial role in determining the electric potential according to Eq. (14). As demonstrated in Fig. 9(b), the electron density surpasses the ion density near the surface, decreasing as the height y increases. The ion density exhibits an increase near the surface, followed by a decrease with y. The peak of ion density is observed farther from the surface than that of electron density, resulting in an initial increase and subsequent decrease in the net charge density. This distribution of charged particles contributes to the formation of potential well shown in Fig. 10.

The time-varying trends of the electric potential near the surface are illustrated in Figs. 11(a)–11(b), corresponding to different phases ϕ of the rf electric field, with a gas pressure of 0.05 Torr. During the early stages of discharge, the potential distribution similar to the SCL sheath is not observed due to the lower ion density compared to the electron density at this time. As the discharge develops, ionization collision leads to an increase in ion density and the gradual accumulation of ions above the surface. The ions move more slowly than electrons, with the space charge field $E_{dc}$ exerting differently on electrons and ions. Consequently, electrons tend to gather very close to the surface, while ions accumulate above them. This spatial distribution of charged particles leads to the formation of the potential well. As shown in Fig. 11(a), a potential distribution analogous to the SCL sheath gradually forms near the surface as the ion density increases.

It should be emphasized that the potential distribution observed in our simulation, though similar to the SCL sheath, does not conform precisely to this model. First, achieving the flux balance at the solid boundary, that is, the electron flux equal to the ion flux, is a fundamental requirement for solving the sheath potential. Thus, ion flux cannot be ignored in the theoretical model. However, in our simulation, plasma coexists with the multipactor, causing the surface to become positively charged during the development of the multipactor, which repels ions, resulting in a very small ion flux ($0-\times 10^{-12}$ m$^{-2}$s$^{-1}$), while the electron flux ranges from around $10^{25}$ to $10^{26}$ m$^{-2}$s$^{-1}$. Additionally, as depicted in Fig. 10(a), the plasma potential aligns closely with the potential of the surface. Therefore, it is difficult for ions to cross the potential well and collide with the surface, which is quite different from the theoretical model of the SCL sheath.

Furthermore, the SCL-like sheath in this simulation does not persist stably, and it periodically disappears and appears as the rf electric
field undergoes periodic variations, as shown in Fig. 11. The presence of SCL-like sheath near the surface corresponds to specific phase of the rf electric field, where $\varphi$ equals $n\pi$ ($n = 0, 1, 2, \ldots$). The potential distribution undergoes variation with changes in the phase $\varphi$. Specifically, when the phase $\varphi$ equals $\pi/2 + n\pi$ ($n = 0, 1, 2, \ldots$), the SCL-like sheath completely vanishes.

Since the potential distribution near the surface changes periodically, we choose an rf cycle at $t = 9$ ns for detailed investigation. Our findings indicate that, besides the number of electrons, the SEY also oscillates periodically at twice the rf frequency, which is evident because the SEY directly affects the number of electrons in the simulation area. Figure 12(a) displays the variation in average SEY with the phase $\varphi$ of the rf electric field.

According to Eqs. (5)–(7), SEY is the function of the impact energy of electrons. The initial energy of secondary electrons is low (2.9 eV), and most of them have a short transit time. Therefore, most multipactor electrons experience only a very small fraction of the rf field phase, making the change in the rf phase $\varphi$ during their transit time negligible. In other words, the energy of multipactor electrons gained from the rf electric field depends on the phase $\varphi$ at which they are emitted. As previously mentioned, the rf electric field $E_{rf}$ is defined as $E_{rf} = E_{rf0} \cos \varphi$ with the initial phase set to 0. Consequently, the magnitude of $E_{rf}$ reaches its maximum when $\varphi$ equals $n\pi$ ($n = 0, 1, 2, \ldots$), and it has the minimum value when $\varphi$ equals $\pi/2 + n\pi$ ($n = 0, 1, 2, \ldots$). Thus, the average SEY reaches its peak when $\varphi$ equals $n\pi$ ($n = 0, 1, 2, \ldots$), gradually declining as $\varphi$ changes from $n\pi$ to $\pi/2 + n\pi$ ($n = 0, 1, 2, \ldots$), with the minimum value attained at $\varphi = \pi/2 + n\pi$ ($n = 0, 1, 2, \ldots$), as displayed in Fig. 12(a). As the energy gained by electrons from $E_{rf}$ depends only on its magnitude, regardless of polarity, the oscillation frequency of SEY and the number of electrons is twice that of the rf field $E_{rf}$. The periodic variation of SEY induces a corresponding periodic change in the number of electrons, as depicted in Fig. 12(b). The average SEY is greater than unity when $\varphi$ equals $n\pi$ ($n = 0, 1, 2, \ldots$), causing the number of electrons to increase as $\varphi$ changes from $n\pi$ to $\pi/4 + n\pi$ ($n = 0, 1, 2, \ldots$). Subsequently, the average SEY drops below 1 when $\varphi$ equals $\pi/4 + n\pi$ ($n = 0, 1, 2, \ldots$), resulting in a
decrease in the number of electrons. The above-mentioned process repeats cyclically, yielding a periodic variation in the number of electrons.

As previously discussed, the process of SEE on the dielectric surface causes the accumulation of positive charges. In the absence of ionization collisions, the accumulated charge on the surface should theoretically match the total charge of electrons in the simulation area, according to the principle of charge conservation. Figure 12(c) illustrates the variation of surface charge density relative to the phase $\varphi$ at a pressure of 0.05 Torr. It is observed that the variation trend of surface charge density still aligns with that of the number of electrons.

The electrons in space include both multipactor electrons and plasma electrons. Plasma electrons are generated due to ionization collisions, while the process of SEE is mainly caused and sustained by multipactor electrons. At a gas pressure of 0.05 Torr, where the ionization rate remains modest, the discharge process is still dominated by the multipactor. This indicates that in this situation, the variation trend of surface charge density should still be consistent with that of the number of electrons, as illustrated in Figs. 12(b) and 12(c).

The spatial distribution of charged particles determines the potential distribution. Figure 13(a) displays the spatial distribution of electrons and ions at various phases within the same rf period, corresponding to the phases $\varphi$ of 0, $\pi/4$, and $\pi/2$, respectively. It can be observed from Fig. 13(a) that the spatial distribution of ions remains almost unchanged at different phases, contrasting with the variation observed in that of electrons. Due to the relatively slow movement speed of ions and the low ionization rate under low gas pressure, the growth rate of ion quantity is not rapid. Therefore, the spatial distribution of ions does not undergo significant changes with time, as depicted in Fig. 13(b). This underscores that the primary factor causing the variation in potential distribution is the dynamic spatial distribution of electrons.

The accumulation of positive charge on the surface exerts an attractive force on electrons, influencing their spatial distribution. Surface charge density serves as a boundary condition in solving Poisson’s equation, influencing the normal electric field $E_N$ and thus affecting the spatial distribution of electrons and ions, with a primary effect on the spatial distribution of electrons. The surface charge density is relatively high when the phase $\varphi$ equals $n\pi$ ($n = 0, 1, 2, \ldots$), causing electrons to concentrate near the surface, as shown in Fig. 13(a). This spatial distribution of charged particles contributes to the formation of the SCL-like sheath, evident from Fig. 11. As we mentioned earlier, the spatial distribution of charged particles interacts with the potential distribution. Secondary electrons emitted from the dielectric surface, with a low initial energy (<3 eV), are unable to cross the potential well, causing a fraction of the newly generated electrons to be returned to the surface by the potential well. Meanwhile, under the influence of the SCL-like sheath, a portion of the plasma electrons near the surface is driven away from the surface. Consequently, as $\varphi$ changes from $n\pi$ to $n\pi/2 + n\pi$ ($n = 0, 1, 2, \ldots$), electrons in the...
The attractive force on electrons is enhanced, leading to a higher peak density. When the gas pressure exceeds 0, both the number of electrons and the surface charge density reaching its maximum value, the attractive force on electrons is enhanced, leading to a higher peak electron density (∼2 × 10^20 m⁻³). Concurrently, as depicted in Fig. 13(a), electron density within the range of 5–20 μm from the surface also increases (∼10^18 m⁻³), indicating that electrons are becoming more dispersed under the influence of normal electric field $E_d$. By the time $\phi$ equals $\pi/2 + \pi$ (n = 0, 1, 2,...), electron density within the range of 5–30 μm from the surface has increased to 10^18 m⁻³, with the peak electron density decreasing to 6 × 10^19 m⁻³.

As the electrons gradually disperse, the SCL-like sheath near the surface diminishes. It completely disappears when the phase $\phi$ changes to $\pi/2 + \pi$ (n = 0, 1, 2,...), as demonstrated in Fig. 11(c). In this case, the majority of plasma electrons move toward the surface influenced by the space charge field $E_d$. Consequently, electrons gradually gather near the surface again, elevating electron density in this region while decreasing farther away. The spatial distribution of electrons becomes similar to that at $\phi = 0$ when $\phi$ transitions from $\pi/2 + \pi$ to $\pi + \pi$ (n = 0, 1, 2,...), and we can once again observe a SCL-like sheath near the surface at this point, as shown in Fig. 11(c). Later, the spatial distribution of electrons will undergo the aforementioned changes, inducing corresponding variations in electric distribution. This cyclic process results in the periodic formation and disappearance of the SCL-like sheath, as presented in Fig. 11. Figure 14 illustrates the spatial distribution of electrons within a range of 0–10 μm from the surface over time, confirming its periodic variation.

IV. CONCLUSION

In the present work, we investigate the dynamic evolution of surface flashover induced by high-power microwaves using PIC-MCC simulation. The nonuniform grid division method is employed in our model to improve the efficiency of calculation. The number of electrons oscillates in time at twice the frequency of the rf electric field after the multipactor reaches saturation, and it also keeps increasing when the gas pressure exceeds 0. Under low gas pressures, both the number of electrons and ions rise quasi-linearly. However, as the air pressure increases, the growth in the number of particles shifts exponentially, and the surface flashover will gradually transition from being dominated by multipactor to gas ionization. A SCL-like sheath near the surface is observed in the simulation under low gas pressures. It is not a stable presence but periodically appears and disappears with changes in the rf electric field. We attribute this phenomenon to variations in the electric field, which influence the energy acquired by electrons. This, in turn, results in corresponding variations in both the number of electrons and the surface charge density, ultimately causing changes in the normal electric field. This dynamic process affects the motion of electrons, giving rise to the periodic variations of the SCL-like sheath near the dielectric surface.

The current focus of our simulation research is on pure rf electric field. However, it should be noted that the external magnetic field will affect the properties of the sheath. Chodura found through research that in the presence of a background magnetic field, a magnetic presheath exists between the Debye sheath and the presheath. In the magnetic presheath, the velocity of ions gradually deviates toward the normal direction of the surface, leading to changes in the angle at which plasma incident on the surface. Moreover, the multipactor can be suppressed when the static magnetic field reaches a certain value. In the future, we plan to further investigate the influence of external magnetic fields on discharge and the sheath structure.

Based on our current research, we are developing an improved simulation model and corresponding code to investigate electrostatic discharge on spacecraft. In recent years, the electrostatic discharge on spacecraft, triggered by the strong electromagnetic environment, has become increasingly prominent due to the rapid development of the global aerospace industry. With the surface of the spacecraft already charged, the probability of surface flashover on spacecraft induced by HPM significantly increases, leading to a notable reduction in the discharge threshold and posing a serious threat to the operational safety of the spacecraft. However, this issue has not been studied in depth. Therefore, our future endeavors involve extending our 1D3V PIC-MCC simulation code into 2D3V, considering that the surface charge of the spacecraft may not be uniform. These upgrades to our model will enable us to conduct comprehensive research on the electrostatic discharge of spacecraft.

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APPENDIX: THE DERIVATION FOR THE LEFT BOUNDARY CONDITION OF POISSON EQUATION

Figure 15 displays the schematic of the left boundary in our model. According to the differential form of the Gauss’s law, we can obtain

\[
E_{dc}(1/2) - E_{dc}(0) = \frac{\rho(1/4)}{\varepsilon_0}.
\]

(A1)

In order to simplify the calculation, \(\rho(1/4)\) in Eq. (A1) is approximately replaced by \(\rho(0)\). Meanwhile, according to Anderson’s model, we can also obtain

\[
E_{dc}(0) = \frac{\sigma}{2\varepsilon_0}.
\]

(A2)

Substituting Eq. (A2) in Eq. (A1), we can obtain

\[
E_{dc}(1/2) = \frac{\varphi(0) - \varphi(1)}{\Delta y} = \frac{1}{2\varepsilon_0} [\sigma + \rho(0)\Delta y].
\]

(A3)

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