Analytical derivation and numerical simulation of the ablation rate of a spherical target

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This paper presents a quasi-steady-state analytical model of the plasma conduction region of a spherical target, and the model is used to analyze the factors that influence the mass ablation rate during laser ablation. Unlike in the case of planar geometry, the mass ablation rate changes as the distance to the ablation front increases. For the plasma in the heat-conduction region of the spherical target under certain conditions, the new analytical model provides relevant parameters such as the density, pressure, and sound velocity, and its results align with those from one-dimensional hydrodynamic simulations. The model and results presented here are valuable resources for investigating mass ablation rates in laser fusion processes.

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I. INTRODUCTION

In direct-drive and fast-ignition laser fusion solutions, the laser irradiation of spherical targets is common.1–16 When a laser irradiates a spherical target, plasma is generated outside the target, in a region that is usually divided into a heat-conduction region and a corona region based on the plasma properties, and those properties in these two regions profoundly affect the success or failure of the ignition scheme. The heat-conduction region has been studied extensively. For example, for planar plasma, De Groot et al.11 proposed an analytical model that explicitly included the time evolution of the heat-conduction region, and Li and Zhang12 and Zhang et al.13 formulated a one-dimensional (1D) model of plasma in the heat-conduction region of a planar target. Also, Christopherson et al.14 proposed a semi-analytical model of hot-spot ignition for a compressible shell and studied the physical characteristics of combustion propagation in inertial-confinement plasma by analysis and simulation. For planar targets, the parameters of the heat-conduction region have been the subjects of much experimental15 and analytical11 research. Studies have shown how laser intensity and incident angle affect laser ablation.15,19 However, despite extensive research on laser ablation from different perspectives, analytical research is lacking on the factors that influence the ablation pressure and mass ablation rate in spherical systems. These parameters are important for guiding research into laser targeting, so knowing which factors affect them in spherical systems is essential.
This study takes the 1D hydrodynamic equations in spherical coordinates and solves them via similarity variables to obtain the required analytical solutions. In Sec. II, the rules for how the density, pressure, mass ablation rate, and other related physical quantities vary in the heat-conduction region are obtained via the plasma properties in the heat-conduction and corona regions, and the modeling differences between spherical and planar targets are elucidated. In Sec. III, the analytical results are compared with those from numerical simulations, and the mass ablation rate of planar and spherical targets is compared. Finally, the results are summarized in Sec. IV.

II. MODELING DETAILS

In laser fusion, it is well known that plasma is generated when a laser irradiates a solid target. The plasma generation region is usually divided into three regions, i.e., a space-compression region, a heat-conduction region, and a corona region, and the interface between the latter two regions is the critical density surface. Plasma formation in the heat-conduction region after laser irradiation is divided into three steps for a spherical target. First, the laser acts on the critical density surface of the plasma but cannot penetrate it, so energy enters the heat-conduction region via electron heat conduction, and thermal energy is the primary source for plasma generation. Furthermore, the plasma that results from ablation of the solid target flows outward from the ablation front, and the mass of ablated material per unit time is known as the mass ablation rate $dm/dt$. Finally, when driven by a long laser pulse (nanosecond scale), the heat-conduction region reaches a steady state after a certain time (approximately 2 ns), i.e., the fluid behavior in the heat-conduction region reaches a quasi-steady state. In the heat-conduction region, the electron heat conduction increases the region’s temperature, the plasma expansion decreases the temperature and density of the region, and the mass ablation increases the density and decreases the region’s temperature. All these factors make the plasma in the heat-conduction region reach a quasi-steady state.

A. Model building

We begin by considering the heat-conduction region, which is in equilibrium because of heat conduction, expansion, ablation, and other factors, and so is quasi-steady. Because the laser energy is wholly absorbed in the corona region, in the heat-conduction region, we have $P \approx P_e$ and $q \approx q_e$, where $P_e$ is the electron pressure and $q_e$ is the electron heat flow. At the same time, assuming the laser irradiation is uniform, the fluid conforms to the steady-state assumption in the heat-conduction region. The above considerations lead to the following 1D single-fluid hydrodynamic equations in spherical coordinates:

$$\frac{\partial p}{\partial t} + \frac{\partial p v_r}{\partial r} + \frac{2 \rho v_r}{r} = 0, \quad (1)$$

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} = -\frac{1}{\rho} \frac{\partial P}{\partial r}, \quad (2)$$

$$\frac{3}{2} \frac{\partial c_s^2}{\partial t} + \frac{3}{2} \rho v_r \frac{\partial c_s^2}{\partial r} + \rho c_s^2 \frac{\partial v_r}{\partial r} + \frac{2 \rho c_s^2 v_r}{r} = -\frac{\partial q}{\partial r} + \frac{2 q}{r} + \frac{dI}{dr}, \quad (3)$$

Equation (1) is the equation for conservation of mass, Eq. (2) is the equation for conservation of momentum, and Eq. (3) is the equation for conservation of energy, where $\rho$ is the mass density, $v$ is the fluid velocity, $c = (P/\rho)^{1/2}$ is the isothermal sound velocity, $P$ is the pressure, $q$ is the heat flow, and $I(x)$ is the spatially correlated laser energy flux.

The laser energy is assumed to be deposited near the sonic surface so that the inverse bremsstrahlung absorption can be neglected in the region of $r > r_s$, where $r_s$ is the critical density surface. The ponderomotive force is neglected because the radiation pressure due to the laser light is small compared to the plasma pressure for cases of interest.20 Radiation from the plasma and the inertial force due to the ablative acceleration of the target have also been neglected.21 The effect of these terms on laser fusion target shooting is usually small for the parameters studied herein. It is assumed that the temperatures of electrons and ions are equal, giving $c^2 = (Z + 1)T/(AM_p)$, where $Z$ is the ionic charge state, $A$ is the atomic weight, and $M_p$ is the proton mass. The heat flow $q$ in Eq. (3) is the minimum of the classical heat flow23 and the finite heat flow:25

$$q = \min \left[ K_0 T_e^{3/2} \frac{\partial T_e}{\partial r} \left| T_e, T_i, \frac{m_e}{m_i} \right|^{1/2} \frac{\partial T_e}{\partial x} \right], \quad (4)$$

where the heat flux coefficient is $K_0 = 1.8 \times 10^{29} / Z \ln \Lambda \text{ cm}^{-1} \text{ s}^{-1} \text{ keV}^{-3/2}$, with $\ln \Lambda$ the Coulomb logarithm, $m_i$ the electron mass, and $f$ the flux limit.

B. Model for heat-conduction region

Below, we develop a self-similar solution for the heat-conduction region. We take the similarity variable to be $s = (r - r_s)/\Delta_1(t)$, where $\Delta_1(t) = (r_t - r_s)$ is the width of the heat-conduction region, with $r_s < r < r_t$ and $0 < s < 1$. Guided by numerical simulations,26 the dependent variables are written as

$$\rho(r, t) = \rho_i(t) R(s), \quad (5)$$

$$v_i(r, t) = c_i(t) V(s), \quad (6)$$

$$c_i^2(r, t) = c_i^2 E(s), \quad (7)$$

where $R(s), V(s),$ and $E(s)$ are the scaling factors regarding the sonic surface, $\rho_i(t)$ and $c_i(t)$ are the density and sound velocity at the sonic surface, and the boundary conditions are $E(0) = V(0) = 1/R(0) = 0$ and $E(1) = V(1) = R(1) = 1$.

First, we deal with Eq. (1). Substituting Eqs. (5) and (6) into Eq. (1) gives
where \( \rho \frac{\partial R}{\partial t} + \rho V \frac{\partial \rho}{\partial t} + \rho \frac{C_s V}{r} \frac{\partial \rho V}{\partial t} + \frac{3}{r} \rho C_s V = 0 \),

where \( R \) is a function of \( s \) and \( t \) is a function of \( t \). Thus, \( \partial R/\partial t \) is transformed to give

\[
\frac{\partial R}{\partial t} = \frac{dR}{ds} \frac{ds}{dt}.
\]

where

\[
\frac{ds}{dt} = -\frac{\Delta_0'(t)(r - r_0)}{\Delta_0^2(t)} = -s \frac{c_s'(t) + c_s(t)}{\Delta_0(t)}.
\]

We also assume that

\[
\frac{\partial \rho_s}{\partial t} = \frac{a \rho_s}{t},
\]

\[
\frac{\partial c_s}{\partial t} = \frac{b c_s}{t},
\]

where \( a \) and \( b \) are constants, and then we have

\[
\frac{\partial R}{\partial t} = -s (b + 1) \frac{dR}{ds}.
\]

Because \( s = (r - r_0)/\Delta_0(t) \), we have \( ds/\partial t = 1/\Delta_0(t) \), and by using Eq. (13), Eq. (1) becomes

\[
aR - s(b + 1) \frac{dR}{ds} = -c_s(t) \frac{d(RV)}{ds} = \frac{10 RV}{3 s}.
\]

A similar approach yields the transformed momentum and energy conservation equations, and the complete set of transformed conservation equations is as follows:

\[
aR - s(b + 1) \frac{dR}{ds} = -c_s(t) \frac{d(RV)}{ds} = \frac{10 RV}{3 s},
\]

\[
(b + a)VR - s(b + 1) \frac{d(RV)}{ds} = -c_s(t) \frac{d(RE + RV^2)}{ds} = \frac{10 RV^2}{3 s},
\]

\[
\frac{3}{2} \left[ 2bE - s(b + 1) \frac{dE}{ds} + \frac{3}{2} c_s(t) \frac{dE}{ds} + \frac{c_s(t) E}{\Delta_0} \frac{dE}{ds} + \frac{10 E V}{3 s} \right]
\]

\[
= \frac{c_s(t) K_0 T_s^{9/2}}{\Delta_0} \frac{dE}{ds} + \frac{c_s(t) K_0 T_s^{9/2} E_s^{5/2}}{\Delta_0} \frac{dE}{ds}.
\]

Next, these equations must be solved, but Eqs. (15)–(17) constitute a very complex set of nonlinear equations that is very difficult to solve. Instead, simulation results suggest \( E \approx s^{1/2} \), \( V \approx s^{3/2} \), and \( R \approx s^{-11/10} \) as approximate solutions, so the approximate solutions for the conduction region are

\[
T \approx T_s (r - r_0)/\Delta_0, \quad v \approx c_s (r - r_0)/\Delta_0, \quad \rho \approx \rho_s (r - r_0)/\Delta_0^{-11/10}.
\]

Equations (18)–(20) are the solution of the self-similar equation, and the form of the solution is similar to that of the flat plate target. By comparison, it is found that the index of \( (r - r_0)/\Delta_0 \) in the spherical target is different from that in the planar target, which will affect the properties of the heat-conduction region plasma.

### C. Model for corona region

Next, we incorporate the corona-region model and use the solutions of the above equations to solve the width and laser energy equations for the heat-conduction region. As the model for the corona region, we use the approximate model of an isothermal plasma expanding in a vacuum, i.e.,

\[
\rho(r, t) = \rho_s \exp \left[ (r - r_0)/c_s t \right],
\]

\[
v(r, t) = c_s + (r - r_0)/t.
\]

Moreover, the validity and applicability of this assumption have been verified.\(^{11}\) The mass flow \( \rho_s c_s \) from the heat-conduction region and the energy flow from the laser absorption maintain the isothermal expansion of the plasma. Next, Eq. (3) is integrated to give the heat flow density in the corona region as

\[
q(r) = -a(r)I_0 + \frac{13}{5} \ln \left[ (r - r_0)/\Delta_0(t) \right] \rho(r) c_s^3.
\]

where \( a(r) = 1 - \exp[-(r - r_0)/c_s t] \) is the laser absorption coefficient of the plasma, with \( t(r) \) being the inverse bremsstrahlung optical depth of the plasma. In the model, the corona region is isothermal. In Eq. (3), the first and second terms can be ignored compared to the third. Moreover, the heat flux outside the plasma sonic surface is zero, so the second term on the right-hand side of Eq. (3) can be ignored. Because the corona region is isothermal, the velocity therein is closely related to the temperature in the microscopic state, and \( v_s \) can also be approximated as a constant \( c_s \) in the corona region. Thus, the heat flux on the sonic surface is

\[
q_r = -a r_0 I_0 + \frac{13}{5} \ln s r c_s^3.
\]

where \( s = c_s/r_0 \) and when \( q_r = 0 \), this model becomes a steady-state model.

### D. Calculation of width of heat-conduction region

The key features of this model are that the conduction region is subsonic and evolves with time and space. For laser-heated plasma, the width of the heat-conduction region is estimated as \( \Delta_0 = c_s t \) following the literature.\(^{13}\) However, this estimate assumes that the plasma expands as an isothermal plasma. However, the subsonic region in laser-heated plasma must be smaller than this because it is certainly not isothermal. In this model, the sonic surface is the boundary between the heat-conduction and corona regions, and the width of the heat-conduction region is obtained by solving for the heat flow [Eq. (23)] in the corona region at the sonic surface and the heat flow in the heat-conduction region. By using Eq. (18), the heat flow at the sonic surface in spherical coordinates is found to be

\[
q_r = -\frac{1}{2} r_0 T_s^{9/2}/\Delta_0.
\]

and substituting the heat flow in the corona region [Eq. (23)] into the above equation leads to the width \( \Delta_0 \) of the heat-conduction region as
\[ \Delta \approx \frac{1}{2} K_0 T_0^{7/2} \left( \frac{9}{11 - 5 \ln s \rho^2} \right). \]  

(26)

Another equation for the width of the heat conduction region can be obtained by using the spatial derivative of the speed of sound at the sonic surface. The approximate solution for the spatial derivative of sound velocity on one side of the heat-conduction region is [Eq. (19)]

\[ \frac{\partial v}{\partial r} (r \leq n) \approx 0.6 \frac{c_s}{\Delta_r}. \]  

(27)

The spatial derivative of the sound velocity on one side of the corona region is [Eq. 22]

\[ \frac{\partial v}{\partial x} (x \leq n) \approx 1/t. \]  

(28)

The above two equations for the spatial derivative of the velocity are combined to obtain another equation for the width of the heat-conduction region, i.e.,

\[ \Delta_r \approx 0.6c_s t. \]  

(29)

\section*{E. Plasma characteristics at sonic surface}

As it is known, the energy per unit area in the corona region is \(4 \rho_c c_s^2\). The energy per unit area in the heat-conduction region is obtained by calculating the energy density from Eqs. (18)–(20) and then integrating over the heat-conduction region. The energy per unit area in the heat-conduction region is then added to that in the corona region to obtain

\[ I_0 \approx 5.5 \rho_c c_s^2, \]  

(30)

and substituting Eq. (30) into Eq. (26) gives

\[ \Delta_r = \frac{5.5}{11 - 5 \ln s} \left( \frac{K_0 T_0^{7/2}/I_0} \right). \]  

(31)

Calculated as the partial derivative of the velocity, Eq. (29) is coupled with Eq. (31) obtained from the heat flow at the sonic surface to give

\[ c_s t \approx \frac{9}{11 - 5 \ln s} \left( \frac{K_0 T_0^{7/2}/I_0} \right). \]  

(32)

The modeling herein assumes that the laser energy is deposited in the corona region, i.e., \(x_0 \approx 1\), and the plasma characteristics at the sonic surface are deduced as follows. In this model, it is assumed that the ion and electron temperatures are equal, from which we have

\[ c_s = 0.91 \times 10^4 \left( \frac{9}{11 - 5 \ln s} \right)^{5/6} \left( \frac{I_0 Z \ln \Lambda}{A} \right)^{1/6} \left( \frac{Z + 1}{A} \right)^{7/12} \text{cm/s}. \]  

(35)

In the above equations, the units of \(I_0\) are W/cm², \(A\) is in micrometers, time is in nanoseconds, and the range of \(r\) (cm) is \([0, \Delta_r]\). Equations (34) and (35) show that for given laser intensity and pulse time, the greater the distance from the sonic surface to the ablation front, the greater the sound velocity at the sonic surface. From this, the ablation rate and ablation pressure can be derived as

\[ \dot{m} \approx 2.18 \times 10^{-4} \left( \frac{11 - 5 \ln s}{9} \right)^{5/3} \left( \frac{I_0^{2/5}}{Z \ln \Lambda} \right)^{1/5} \left( \frac{A}{Z + 1} \right)^{7/6} \text{g/cm}^2\text{s}. \]  

(36)

\[ P_a \approx 4.4 \times 10^{-11} \left( \frac{11 - 5 \ln s}{9} \right)^{5/6} \left( \frac{I_0^{6/5}}{Z \ln \Lambda} \right)^{1/6} \times \left( \frac{A}{Z + 1} \right)^{7/12} \text{mbar}, \]  

(37)

where \(s = \frac{r - r_0}{A0}\) is the similarity variable, with \(\Lambda_r(t) = (r - r_0)\) the wide of the heat-conduction region. Compared with the formulas of the planar target, the formulas of the spherical target [(34)–(37)] have an extra term about \(\ln s\). As a result of this, the properties of the plasma change. Due to the different indices of the relevant terms in the formulas, the density of the plasma in the conduction region of the spherical target decreases faster than that of the planar target while the velocity changes little. The mass ablation rate decreases with the increase in the similarity variable \(s\) due to different changes in density and velocity. There is no similar change in the mass ablation rate in the planar target due to the balance of density and velocity.

Through Eq. (36) and the relationship between \(s\) and \(r\), we can get the following conclusion. Unlike the planar target model, the mass ablation rate is related to the distance from \(r\) to \(r_0\), with the former decreasing as the latter increases. As \(r\) converges to \(r_0\), with \(s = 1, \frac{11 - 5 \ln s}{9}\) tends to a constant, which means when the distance between the sonic surface and the ablation front is increased to a certain extent, the plasma outside a spherical target can be treated approximately as that outside a planar target.

\section*{III. VERIFICATION AND DISCUSSION}

To verify the accuracy of the analytical results, we compare them with those from hydrodynamic simulations using the ideal-gas equation of state and a single-fluid, single-temperature hydrodynamic specification based on the Euler equation.\(^\text{26}\) The governing equations are as follows:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0, \]  

(38)

\[ \frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho uu) + \nabla p = 0, \]  

(39)

\[ \frac{\partial pE}{\partial t} + \nabla \cdot [(\rho E + p)u] = \nabla \cdot \kappa_n \nabla T + S_L, \]  

(40)

where \(\rho, u, E, T, p\) represent density, velocity, total energy, temperature, and pressure, respectively, and \(S_L\) is the laser energy deposition term for inverse bremsstrahlung absorption. The electron thermal...
conduction is modeled using the Spitzer–Härm model, and $k_e$ is the electron heat exchange coefficient. The size of the flux limiter is 0.1, and the flux limiter for each wavelength is the same in this paper. The governing equations are solved using the finite-volume method, with a nonuniform grid used to minimize the computational effort. The ablation material is an infinitely thick CH target with an initial density of 1.0 g/cm$^3$. The above equation is solved in spherical coordinates.

The same conditions were set for the computational model and the simulation analysis, i.e., a driving laser power density of $I = 3 \times 10^{14}$ W/cm$^2$, a wavelength of $\lambda = 0.53 \mu$m, and a pulse width of $t = 2$ ns. Figure 2 shows the spatial distribution of density and temperature of the laser-heated plasma in the simulation. The outwardly ejected plasma ($x \geq x_s$, where $x_s$ is the position of the ablation front) is divided into two regions: (i) the heat-conduction region, through which the laser energy absorbed at the critical density surface is transmitted to the ablation front, and (ii) the corona region, where the laser energy is deposited. The sonic surface ($x_c$) in Fig. 2 is the boundary surface between the two regions. In previous models, the critical density surface and the sonic surface were considered to coincide. However, the results herein show a difference between the sonic surface and the critical density surface and that the location of the former is determined by thermal conduction. Figure 2 shows the spatial variation of temperature and density in the heat-conduction and corona regions, and it is clear that the corona region is isothermal.

Crucially, Eqs. (18)–(20) predict that the Mach number ($M = v/c$) has a consistent relationship with the normalized temperature, and in Fig. 3, comparison of the computed normalized temperature [solid line, calculated from Eq. (18)] with the temperature (hollow circles) and Mach number (solid circles) obtained from the simulation reveals that this is indeed the case (the range of the simulation is set to the width of the conduction zone). This holds for different wavelengths ($\lambda = 0.35, 0.53,$ and 1.059 $\mu$m). As shown in Fig. 3(c), a certain discrepancy occurs at larger wavelengths. The sonic surface is the boundary of the heat-conduction region, and the wavelength influences the temperature and velocity at the corona region. The longer the wavelength, the larger the temperature ($T_c = 3.84 \times 10^{11} A^2/\lambda^3 (\text{K})$ and velocity ($v_c = 7.2 \times 10^{14} A^{1/2} \lambda^{-1/2} (\text{km/s})$) at the interface between the corona region and the heat-conduction region, and then the temperature and velocity in the heat-conduction region are affected through electron heat conduction. As the wavelength increases, the velocity is more evident than the temperature increase. Velocity will affect the Mach number of the heat-conduction region, and temperature will affect the normalized temperature of the heat-conduction region. At larger wavelengths, the differences between the two will be more pronounced, so visible discrepancies appear in Fig. 3(c).

Under the same conditions, Fig. 4 compares the analytical and simulation results for the plasma density [Fig. 4(a)], pressure [Fig. 4(b)], and ablation rate [Fig. 4(c)] in the heat-conduction region of the spherical target.

Figure 4 shows that the results of the analytical model constructed herein are very close to those of the numerical simulation, with obviously consistent trends, and the slight deviations between them have a reasonable explanation. When the laser is incident from the right side, laser compression occurs on the left side of the heat-conduction region, resulting in a higher density. Elsewhere in the heat-conduction region, the density decreases slightly near the sonic surface [e.g., as in Fig. 4(a)].

In Fig. 4(b), the analytical pressure results are consistent with the simulation ones, with the pressure decreasing gradually from the ablation front to the sonic surface. There are slight discrepancies between the analytical and simulation pressure results, particularly near the ablation front, and the main reason for these is that as the distance to the ablation front decreases, the influence of the simulation procedure by boundary conditions increases.

The ablation rate is the critical parameter. The analytical and simulation results are compared [Fig. 4(c)], which follow the same trend with little difference. However, unlike the mass ablation rate for the flat target, that for the spherical target depends on the distance between $r$ and $r_m$ as can be seen, the mass ablation rate decreases with distance increase. Figure 4(c) also shows that the mass ablation rate decreases less rapidly as the similarity variable $s$ increases. This result is consistent with the actual situation in the geometrical model. In spherical coordinates, the solution of the problem can be approximated as solving for a flat-plate target in planar coordinates when the plasma expands to a certain degree. This result is also consistent with the analytical solution [Eq. (36)].

Figure 5 shows the relationship between mass ablation rate and laser intensity to validate the analytical results further. Equation (36) predicts that the mass ablation rate can be increased by increasing the laser intensity for a given laser pulse time and distance from the sonic surface to the ablation front, and Fig. 5 shows this effect of the laser intensity on the mass ablation rate.

The analytical results presented in this study exhibit a crucial characteristic. In contrast to the planar target model, the spherical target model displays a significant correlation between the mass ablation rate and the distance separating $r$ and $r_m$. To effectively observe this characteristic, the mass ablation rates of the spherical and planar target models are compared in Fig. 6, and the relative rate of change of the mass ablation rate in the heat-conduction region and at the sonic
FIG. 3. Spatial distributions of normalized temperature (hollow circles) and Mach number (solid circles) obtained from simulation agree well with the results of the proposed model [solid line, from Eq. (18), $s = (r - r_s)/\Delta_0(t)$]. Results are shown for three wavelengths $\lambda$ = (a) 0.351 $\mu$m, (b) 0.53 $\mu$m, and (c) 1.059 $\mu$m, and the peak laser energy is $I = 3 \times 10^{14}$ W/cm$^2$ with $t = 2$ ns.

FIG. 4. Plots of (a) density, (b) pressure, and (c) mass ablation rate in heat-conduction region [solid line, from Eq. (18), $s = (r - r_s)/\Delta_0(t)$]. The ablation front is used as the coordinate origin, and the peak laser energy is $I = 3 \times 10^{14}$ W/cm$^2$ with $t = 2$ ns.
surface is plotted. Notably, the curve demonstrates that the mass ablation rate remains constant concerning $s$ in the case of the planar target; in this scenario, the mass ablation rate remains consistent throughout the heat-conduction region. Conversely, the mass ablation rate diminishes in the spherical target model as $s$ increases. In the spherical target, a larger arc in the $r$-normal direction, resulting from the geometrical properties, leads to a larger area for the region and a smaller ablation rate of mass per unit area. This physical phenomenon does not occur with the planar target because the area remains unchanged as the distance from $r$ to $r_s$ increases.

Equations (34), (36), and (37) provide analytical solutions for the density, pressure, and mass ablation rate. These results are specifically associated with the laser intensity, interaction time, and the similarity variable $s$, which are the main focus of this study. Consequently, an analysis was conducted to investigate and fit the factors influencing each parameter to the numerical simulation outcomes. This research has revealed the relationship between the distance between $r$ and $r_\text{a}$ and the mass ablation rate in spherical coordinates. Unlike the planar target model, the mass ablation rate decreases gradually as the distance between the two increases.

**IV. CONCLUSIONS**

This paper has presented an extension and improvement of the planar model of laser-heated absorbing plasma. In order to address the challenges associated with solving the mass ablation rate with a biconical ignition scheme, a self-consistent model of the spatial structure of a laser-heated spherical-target plasma was formulated in spherical coordinates. The results obtained from theoretical modeling and numerical simulations were consistent. In contrast to the planar target model, the mass ablation rate in the spherical target is closely related to the distance between $r$ and $r_\text{a}$. When this distance is sufficiently large, the properties of the planar target model are exhibited [see Fig. 4(c)]. It is important to note that previous studies failed to propose an analytical method for studying the plasma parameters in the heat-conduction region of a spherical target.

Equation (36) shows that the laser intensity and the similarity variable $s$ impact the mass ablation rate. According to the relationship between $s$ and $r$, the mass ablation rate decreases as the distance between $r$ and the ablation front $r_\text{a}$ increases, while increasing the laser intensity enhances the mass ablation rate. Equations (34), (35), and (37) present the plasma characterization parameters of the heat-conduction region of a spherical target, and their adjustability facilitates real-time analysis of this region. However, this paper did not explore the influence of angle on the outcomes with a spherical target. In a 1D spherical coordinate system. In future work, to approach the actual situation more closely, we will establish and solve a 2D spherical-target model, investigating the impact of angle on the physical quantities to better align with the real-world situation of a spherical target. Compared to numerical simulations of fluid dynamics, the analytical solution method proposed herein offers a theoretical approach and mechanism for analyzing the action process. The self-consistent model developed herein provides clear physical representations and stable results in the heat-conduction region, which is advantageous for analyzing the state and influence of various parameters in the spherical target. Additionally, it serves as a reference point for future research on spherical targets.

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**AUTHOR DECLARATIONS**

**Conflict of Interest**

The authors have no conflicts to disclose.

**Author Contributions**

Yan-Zhao Han: Conceptualization (equal); Data curation (equal); Validation (equal); Visualization (equal); Writing – original draft (equal).
Yun-Xing Liu: Data curation (equal); Methodology (equal); Supervision (equal). Ying-Jun Li: Conceptualization (equal); Supervision (equal); Writing – review & editing (equal).

DATA AVAILABILITY
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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