RESEARCH ARTICLE | JULY 14 2015

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AIP Advances 5, 077133 (2015)
https://doi.org/10.1063/1.4926974

AIP ADVANCES **5**, 077133 (2015)

Simultaneous evaluation of acoustic nonlinearity parameter and attenuation coefficients using the finite amplitude method

Shuzeng Zhang,[1] Hyunjo Jeong,[2,a] Sungjong Cho,[2] and Xiongbing Li[1]
[1]*School of Traffic and Transportation Engineering, Central South University, Changsha, Hunan, 410075, China*
[2]*Division of Mechanical and Automotive Engineering, Wonkwang University, Iksan, Jonbuk 570-749, Republic of Korea*

(Received 9 June 2015; accepted 6 July 2015; published online 14 July 2015)

A novel method to determine acoustic parameters involved in measuring the nonlinearity parameter of fluids or solids is proposed. The approach is based on the measurement of fundamental and second harmonic pressures with a calibrated receiver, and on a nonlinear least squares data-fitting to multi-Gaussian beam (MGB) equations which explicitly define the attenuation and diffraction effects in the quasilinear regime. Results obtained in water validate the proposed method. The choice of suitable source pressure is discussed with regard to the quasilinear approximation involved. The attenuation coefficients are also acquired in nonlinear regime and their relations are discussed. [http://dx.doi.org/10.1063/1.4926974]

Nonlinear acoustic methods have been widely used in recent decades as a nondestructive characterization tool for biological tissues and damaged solids.[1,2] The nonlinear effect is frequently determined by measuring the amplitude of the second harmonic signal generated when a pure sinusoidal wave of finite amplitude propagates through the material.[3] The measurement of nonlinearity parameter, β, receives significant attention and it is quantified in fluids as $\beta = 2\rho_0 c_0^2 p_2/kzp_1^2$, where ρ_0 is the density of the fluid, c_0 is the sound speed, k is the wave number, and p_1 and p_2 represent the plane-wave pressures of the fundamental and second harmonics at a propagation distance z.

The accurate measurement of acoustic nonlinear parameter β for fluids or solids generally requires making corrections for attenuation and diffraction effects due to material absorption and finite size radiation source, respectively.[4] These effects are well known for fundamental waves,[5,6] while those for second harmonic waves have not been well addressed and therefore not properly used in previous studies.[7–10] In this letter, we propose a novel technique to extract the nonlinearity parameter and the attenuation coefficients simultaneously based on a nonlinear least squares curve-fitting of measured pressure data to multi-Gaussian beam (MGB) equations which explicitly define the attenuation and diffraction corrections. The MGB model equations are developed from the quasilinear theory of the Westervelt equation that describes the combined effects of nonlinearity, diffraction and attenuation. The proposed approach is validated through experiments in water and shows that an accurate β value can be acquired together with other acoustic parameters including attenuation coefficients. The extracted attenuation coefficients of the fundamental and second harmonics are found to be both frequency- and pressure-dependent, which are much different from a linear behavior.

The experimental setup is shown in Fig. 1, where a through-transmission ultrasonic testing is conducted to measure the pressure amplitudes of fundamental and second harmonic waves in a water bath. Fig. 1 also shows a separate calibration experiment for the receiver in order to convert

[a]Corresponding author: hjjeong@wku.ac.kr

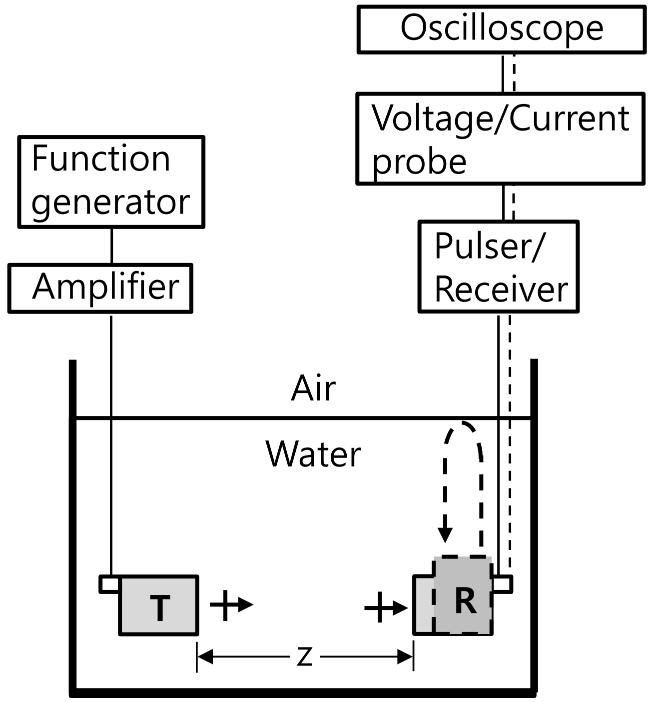
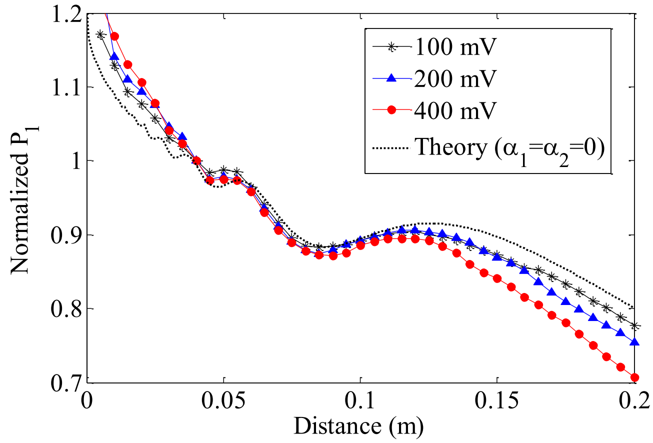


FIG. 1. Experimental setup for calibration measurement (dashed line circuit) and harmonic generation measurement (solid line circuit).

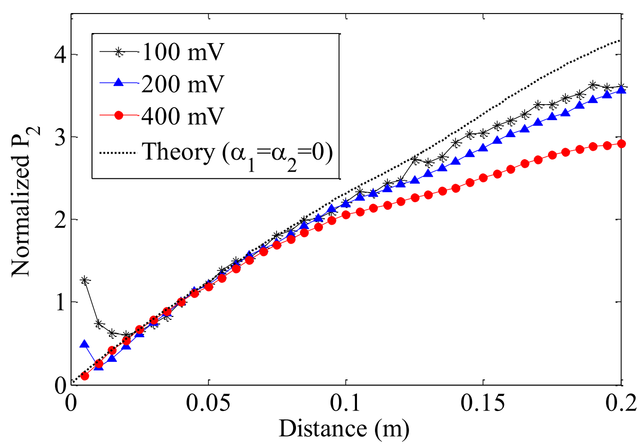
the measured current signals into the absolute sound pressures of fundamental and second harmonic waves. A broadband pulse-echo method is used, allowing a free surface reflection at the top water surface that satisfies reciprocity. A detailed calibration procedure is described in, Ref. 11 but the diffraction and attenuation corrections are made here for accurate determination of the receiver transfer function, $H(\omega)$. A Gaussian-enveloped toneburst of 30 cycles tuned to the fundamental frequency (3.5 MHz) with the excitation levels of 100 mV, 200 mV and 400 mV peak-to-peak is supplied by a function generator (33250A, Agilent Technologies, Inc., Santa Clara, CA), and then amplified by a linear amplifier (2100L, Electronics & Innovation, Ltd., Rochester, NY) to provide the high power monochromatic toneburst for the harmonic generation. The broadband transmitting and receiving transducers have the same size, 9.5 mm diameter and center frequency 5 MHz (V326-SU, Panametrics, Waltham, MA). The two transducers are fine set to align with each other for maximum output signal capture. The propagating distance z is increased from 0 to 200 mm in 5 mm steps by moving the receiver with a stepper motor. The receiver output is measured by a current probe (Tektronix CT-2, Tektronix, Inc., Wilsonville, OR) and digitized using a Waverunner oscilloscope (LT332, LeCroy, Chesnut Ridge, NY). From the quantitative experiments, the sound speed c_0 determined from the time-of-flight at several transmitter–receiver distances was 1482 m/s. The density of water was assumed to be 1000 kg/m^3.

Fig. 2 shows the variation of the fundamental and second harmonic pressures with propagation distance for three different input levels. The measured data at each input voltage was normalized by its respective data measured at z=40 mm. Also shown are the normalized pressures predicted by the multi-Gaussian beam (MGB) model of the quasilinear theory when the attenuation effects are neglected. The MGB model equations will be described later. It can be seen that the normalized pressures for different input voltages are not uniform along the propagation distance. These results indicate that in addition to the effects of attenuation and diffraction one needs to consider the effects of source pressure when determining the nonlinearity parameter β.

In general, when a finite amplitude wave radiates from a finite size source and propagates in a lossy medium, waveform distortion occurs and depends on the source pressure, attenuation, and diffraction effects. These three factors have to be quantified in order to obtain reliable measurements



(a)



(b)

FIG. 2. Measured pressures as a function of input voltage and comparisons with multi-Gaussian beam solutions of quasilinear theory with attenuations neglected: (a) Fundamental wave, and (b) Second harmonic wave.

of β. For this purpose, we assume weakly nonlinear, axisymmetric sound beams and employ the quasilinear theory of solutions. Then, the beam fields can be expressed in the form of plane waves modified by attenuation and diffraction corrections. To calculate the received pressure, the field is averaged over the active surface of the receiver to take into account its finite size. When a circular transducer of radius a radiates sound beam into fluids, the average pressures $\tilde{p}_1$ and $\tilde{p}_2$ received by a transducer of radius b at a coaxial distance z can be expressed as

$$\tilde{p}_1(z) = [p_0 \exp(ikz)]\,[M_1(\alpha_1, z)]\,[D_1(a, b, f, z)] \tag{1}$$

$$\tilde{p}_2(z) = \left[\frac{\beta k z}{2\rho_0 c_0^2} p_0^2 \exp(2ikz)\right] [M_2(\alpha_1, \alpha_2, z)]\,[D_2(a, b, 2f, z)] \tag{2}$$

where p_0 is the source pressure, and α_1 and α_2 are the attenuation coefficients at the fundamental and second harmonic frequencies, respectively, at the source pressure p_0. In these field equations, the first terms in the brackets represent the plane wave solutions, and the second and third terms are the attenuation and diffraction corrections, respectively.

The quasilinear system of equations resulting from the KZK equation or the Westervelt equation provide combined effects of attenuation and diffraction of the nonlinear acoustic fields, so that it is not easy to separate one effect from the other.[12] The MGB models[13] can be used to explicitly define these effects. Based on the quasilinear solutions of the Westervelt equation, the MGB model equations can be obtained in closed form and can be expressed as in Eqs. (1) and (2). The attenuation correction terms are given by[14]

$$M_1(\alpha_1, z) = \mathrm{e}^{-\alpha_1 z} \tag{3}$$

$$M_2(\alpha_1, \alpha_2, z) = \frac{e^{-2\alpha_1 z} - e^{-\alpha_2 z}}{(\alpha_2 - 2\alpha_1)\, z} \tag{4}$$

Considering the same transmitter and receiver radii ($a = b$) and using the concept of average pressure, the diffraction correction terms are given by

$$D_1(a,b,f,z) = \frac{1}{\pi b^2}\int_0^b \sum_{m=1}^{N} \frac{A_m}{1 + iB_m z/D_R} \exp\left(\frac{i\omega}{2}\frac{iB_m/c_0 D_R}{1 + iB_m z/D_R} r^2\right) 2\pi r dr \tag{5}$$

$$D_2(a,b,f,z) = \frac{1}{\pi b^2}\int_0^b \left\{\frac{1}{z}\int_0^z \sum_{m=1}^{N}\sum_{n=1}^{N} \frac{2A_m A_n}{(2 + B_a z) + (B_a - 2B_b z)z'} \times \exp\left(\frac{ikr^2 (B_a - 2B_b z')}{(2 + B_a z) + (B_a - 2B_b z)z'}\right) dz'\right\} 2\pi r dr \tag{6}$$

where $D_R = ka^2/2$ is the Rayleigh distance, $B_a = i(B_m + B_n)/D_R$, $B_b = B_m B_n/D_R^2$, and A_m and A_n are a set of complex-valued expansion coefficients.[15]

The β value can now be calculated by making necessary corrections for attenuation and diffraction to plane wave solutions. However, it is not easy to measure the attenuation coefficients since they depend on the source pressure[9] and interact nonlinearly. The diffraction corrections for known transmitter and receiver sizes can be calculated for a given fundamental frequency from Eqs. (5) and (6). If the pressure data can be obtained at large propagation distances, acoustic parameters p_0, α_1, α_2, and β can be extracted by fitting the measured data to Eqs. (1) and (2). The fitting process will fit the measured data to the pressure equation by adjusting these parameters to minimize the squared error according to the algorithm[16]

$$Min_z\left\|\tilde{p}_1\left(\{p_0, \alpha_1\}, z\right) - p_1^M\right\|^2 = Min_z \sum_i \left[\tilde{p}_1\left(\{p_0, \alpha_1\}, z_i\right) - p_{1,i}^M\right]^2 \tag{7}$$

$$Min_z\left\|\tilde{p}_2\left(\{p_0, \alpha_1, \alpha_2, \beta\}, z\right) - p_2^M\right\|^2 = Min_z \sum_i \left[\tilde{p}_2\left(\{p_0, \alpha_1, \alpha_2, \beta\}, z_i\right) - p_{2,i}^M\right]^2 \tag{8}$$

where $\tilde{p}_1$ and $\tilde{p}_2$ represent the objective pressure functions to be optimized for the fundamental frequency data and for the second harmonic data. Similarly, p_1^M and p_2^M represent the measured pressures at their respective frequencies. There are two steps to optimize the pressure amplitude and extract the acoustic parameters. Firstly, only the fundamental pressure is used to find values of p_0 and α_1. Secondly, with the known p_0 and α_1, the second harmonic pressure is used to extract α_2 and β. The proposed method will determine the nonlinearity parameter with high accuracy. Furthermore, it also provides the attenuation coefficients α_1 and α_2. Thus, separate measurements of attenuation coefficients are not necessary in determining the nonlinearity parameter β.

The nonlinear least squares data-fitting results are listed in Table I for three different input levels. The correlation coefficient of better than R=0.98 was obtained for all the curve-fit processes. Also shown are the literature values for some of these parameters. The literature values of α_1 and α_2 were obtained from attenuation experiments in linear regime and fitted to a quadratic function of frequency, $\alpha_f = 0.0253 f^2$ Np/m with f in MHz.[5] The extracted attenuation coefficients α_1 at 100 mV and 200 mV inputs agree with the literature value of 0.31 Np/m. The extracted attenuation coefficients α_2 at these inputs are close to but slightly smaller than or larger than $2\alpha_1$. These results indicate that the attenuation of the second harmonic increases as $\alpha_2 = 2\alpha_1$, (linearly with frequency), as opposed to $\alpha_2 = 4\alpha_1$, (quadratically with frequency), as would be expected from the linear thermoviscous absorption law. We further tested the effect of source pressure on the attenuation coefficients and the nonlinearity parameter β by increasing the input to 400 mV. Since the attenuation coefficients depend on the source pressure, the values of α_1 and α_2 and their relationship may change, and affect the β determination. Table I shows significant increases of α_1

TABLE I. Acoustic parameters extracted by the nonlinear least squares data-fitting process.

Input (mV)	p_0 (kPa)	α_1 (Np/m)	α_2 (Np/m)	β	m
100	22.3	0.36	0.61	3.51	0.82
200	46.5	0.38	0.82	3.62	1.13
400	110.5	0.78	2.38	3.75	1.57
Literature value	...	0.31	1.24	3.5	2

and α_2 at 400 mV compared to those at 100 mV or 200 mV input, and the relation of $\alpha_2 = 3\alpha_1$. The larger attenuation coefficients of both fundamental and second harmonics may be caused by the nonlinear losses from the fundamental and second harmonics to higher harmonics as the quasilinear approximation does not hold in higher input.[17] The input voltage of 400 mV or the source pressure of p_0 = 110.5 kPa does not seem to satisfy the quasilinear approximation used in his work since the extracted fundamental α_1 is much different from the linear α_1 at the same frequency. The change of α_1 and α_2 and their relationship seems to be very complicated in nonlinear regime. In nonlinear conditions, the attenuation of water shows both frequency- and pressure-dependent behavior as most soft tissues do.[18]

The frequency- and pressure-dependent attenuation is further investigated by assuming the zero attenuation at $f = 0$ MHz and fitting the attenuation coefficients α_f in the form[7]

$$\alpha_f = \alpha_0 f^m \tag{9}$$

The m values are given in Table I and the best-fit lines are shown in Fig. 3 with the literature quadratic curve in linear regime. The frequency dependence of attenuation is nearly linear at 100 mV and 200 mV inputs since m=0.82 and m=1.13, respectively. The frequency dependence at 400 mV input (m=1.57) is higher than those at lower inputs.

The extracted nonlinearity parameter β at 100 mV and 200 mV is very close to the literature value of 3.5[12] since these inputs satisfy the quasilinear approximation as observed in the extracted attenuation coefficients. Increasing input or source pressure increases the β value, and it differs by 7% at 400 mV input. This may be attributed to energy transfer from fundamental to higher harmonics,[17,19] or the source nonlinearity due to high input.[14] Based on the extracted attenuation coefficients and β values, the 100 mV and 200 mV inputs and their resulting source pressures seem to satisfy the quasilinear approximation used in this study. The acoustic parameters determination proposed in this work is based on the quasilinear theory of the nonlinear wave equation. Consequently, it is quite important to satisfy this condition experimentally by using a suitable input voltage (or source pressure).

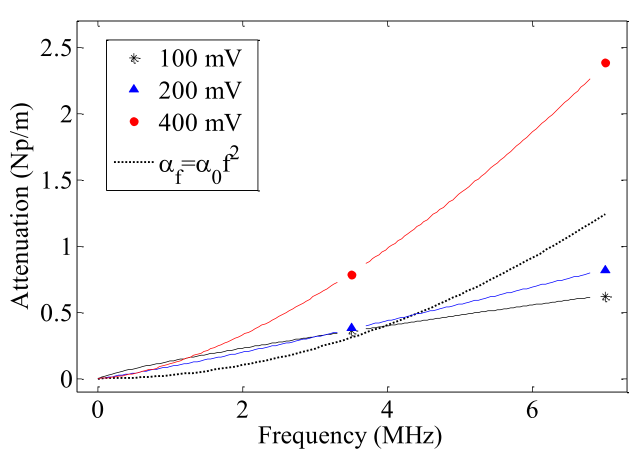


FIG. 3. The frequency- and pressure-dependent attenuation of water. Data points were fitted to the function $\alpha_f = \alpha_0 f^m$ Np/m with f in MHz. Dotted line shows the quadratic frequency dependence in linear condition for comparison.

A quantity called shock parameter is frequently used to provide a guidance for choosing the appropriate pressure amplitude. The shock parameter σ is defined as[9]

$$\sigma = \frac{2\pi p_0 \beta f z}{\rho c_0^3} \tag{10}$$

For water, β is about 3.5 and $\rho_0 = 1000\text{kg/m}^3$. If we use the quantities at 100 mV input: $p_0 = 22.3\,\text{kPa}/2 = 11.2\,\text{kPa}$, $f = 3.5$ MHz, and $z = 20$ cm, we obtain σ is about 0.05. σ is about 0.1 and 0.25 at 200 and 400 mV, respectively. Based on these results, the σ value of less than 0.1 is found to be adequate for the quasilinear condition or the weak nonlinearity to determine acoustic parameters using the data-fitting method proposed in this work. The same value is recommended for choosing the pressure amplitude when measuring the attenuation coefficient.[9]

A nonlinear least squares data-fitting method was described to determine the nonlinearity parameter together with attenuation coefficients and source pressure. The proposed method rests on the quasilinear system of solutions and requires explicit formulas for attenuation and diffraction corrections. The method was validated by extracting the related acoustic parameters in water. Since the proposed method is based on the quasilinear approximation, the use of a suitable source pressure is important and the shock parameter value of less than 0.1 is suggested. The proposed method here can also be applied to solid samples if multiple measurements are possible along the propagation path.

ACKNOWLEDGEMENT

This work was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) (Grant No. 2013-R1A2A2A01016402).

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