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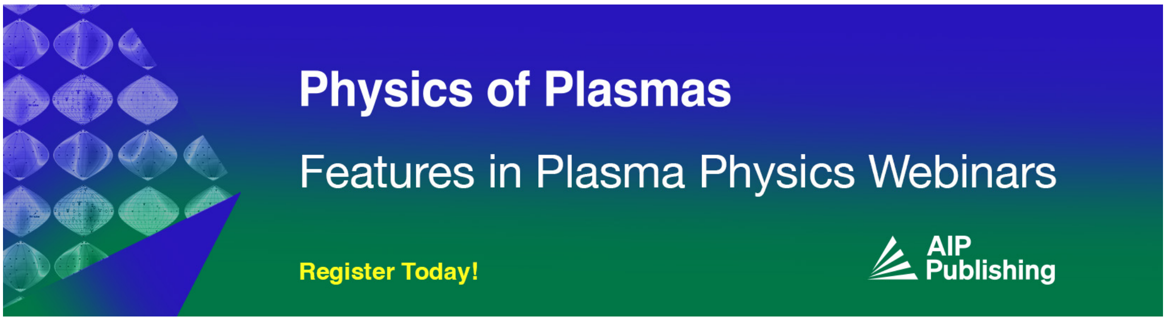

Comment on "Kinetic theory of surface waves in plasma jets" [Phys. Plasmas 9, 701 (2002)]

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It is shown that the dispersion relation of electromagnetic surface waves propagating on the interface between a vacuum and drifting Maxwellian plasmas derived in recent work [Phys. Plasmas **9**, 701 (2002)] is incorrect. Correct electromagnetic and electrostatic dispersion relations are obtained. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2012167]

Propagation of surface waves on the interface between a vacuum (or a dielectric) and moving plasmas has drawn much attention because of its various technological applications related with such devices as plasmatrons and surfatrons operating on surface waves. Recently Shokri[1] wrote down, without actual derivation, a dispersion relation Eq. (19S) (S attached to the equation number will signify equations in Ref. 1) of surface waves propagating on a semi-bounded plasma jet, from the well-known surface wave dispersion relation for a stationary isotropic plasma [Eq. (11S)] by simply redefining the corresponding dielectric permittivities for a moving plasma. It is shown here that this procedure is invalid. Furthermore, his cold plasma dispersion relation obtained in his revised version [Eq. (3) in Ref. 2] which can be rearranged in the form

$$\left(1-\frac{\omega_{\rm pe}^2}{\omega'^2}\right)\sqrt{k_z^2-\frac{\omega^2}{c^2}}-k_z+\frac{(1-(\omega_{\rm pe}^2/\omega'^2))[k_z+\sqrt{k_z^2-(\omega^2/c^2-\omega_{\rm pe}^2/c^2)}]}{1-(\omega_{\rm pe}^2/\omega^2)}=0, \quad (1)$$

is still incorrect. It is well-established in nonrelativistic theory that the cold plasma dispersion relation of electromagnetic surface wave takes the form[3,4]

$$\sqrt{k_z^2-\frac{\omega^2-\omega_{\rm pe}^2}{c^2}}+\left(1-\frac{\omega_{\rm pe}^2}{\omega'^2}\right)\sqrt{k_z^2-\frac{\omega^2}{c^2}}=0. \quad (2)$$

In Eqs. (1) and (2), ω' is the Doppler-shifted frequency. Equations (1) and (2) are not even close, and Eq. (1) reflects the incorrectness of his dispersion relation, Eq. (19S).

In this Comment, the steps leading to correct transverse magnetic (TM) mode surface wave dispersion relation are elucidated by means of kinetic equation. We consider equilibrium distribution $f_{0\alpha}$, which is a drifting Maxwellian

$$f_{0\alpha}(\mathbf{v})=\left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2}\exp\left[-\frac{m_\alpha(\mathbf{v}-\mathbf{u}_\alpha)^2}{2T_\alpha}\right], \quad (3)$$

where α takes e for electrons and i for ions and $\mathbf{u}_\alpha$ is the drifting velocity of the plasma. Assuming a small perturbation (denoted by $f_{1\alpha}$) about a homogeneous equilibrium distribution [Eq. (3)], the Vlasov equation gives

$$\frac{\partial}{\partial t}f_{1\alpha}(\mathbf{r},\mathbf{v},t)+\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}f_{1\alpha}+\frac{e_\alpha}{m_\alpha}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial\mathbf{v}}f_{0\alpha}=0. \quad (4)$$

It should be noted that the last term involving the magnetic field $\mathbf{B}$ vanishes if $f_{0\alpha}$ is isotropic ($\mathbf{u}_\alpha=0$). We obtain from Eq. (4), in terms of Fourier variables,

$$f_{1\alpha}(\mathbf{k},\mathbf{v},\omega)=-\frac{i}{\omega}\frac{e_\alpha}{m_\alpha}\frac{E_j}{\omega-\mathbf{k}\cdot\mathbf{v}}[(\omega-\mathbf{k}\cdot\mathbf{v})\delta_{sj}+k_s v_j]\frac{\partial f_{0\alpha}}{\partial v_s}, \quad (5)$$

where and in the following the repeated Cartesian indexes are summed over unless stated otherwise.

Substituting this equation into

$$\mathbf{J}=\sum e_\alpha\int\mathbf{v}f_{1\alpha}d^3v, \quad (6)$$

the current $\mathbf{J}$ can be written in terms of $\mathbf{E}$, and it is customary to write this relation by the conductivity tensor σ_{ij}: $J_i=\sigma_{ij}E_j$. We need to calculate the dielectric tensor, $\varepsilon_{ij}=\delta_{ij}+(4\pi i/\omega)\sigma_{ij}$ for the moving Maxwellian, Eq. (3). Equations (5) and (6) give[5]

$$\varepsilon_{ij}=\delta_{ij}\left(1-\Sigma\frac{\omega_{p\alpha}^2}{\omega^2}\right)+\Sigma\frac{\omega_{p\alpha}^2}{\omega^2}\int\frac{v_iv_jk_s}{\omega-\mathbf{k}\cdot\mathbf{v}}\frac{\partial f_{0\alpha}}{\partial v_s}d^3v \quad (7)$$

where the velocity integral is performed along the Landau contour. The velocity integral in Eq. (7) can be carried out by transforming, $v_i'=v_i-u_{i\alpha}$ and thus performing integral over isotropic Maxwellian distribution $f_{0\alpha}'(v')$. We have

$$\int\frac{v_iv_jk_s}{\omega-\mathbf{k}\cdot\mathbf{v}}\frac{\partial f_{0\alpha}}{\partial v_s}d^3v=(1-\omega_\alpha' I_{1\alpha})\left(\delta_{ij}+\frac{m_\alpha}{T_\alpha}u_{i\alpha}u_{j\alpha}\right)-\omega_\alpha'(k_iu_{j\alpha}+k_ju_{i\alpha})I_{2\alpha}-\omega_\alpha'2\frac{T_\alpha}{m_\alpha}k_ik_jI_{3\alpha}, \quad (8)$$

where $\omega_\alpha'=\omega-\mathbf{k}\cdot\mathbf{u}_\alpha$ is the Doppler-shifted frequency and

$$I_n = \int \frac{f_0'}{(\omega' - \mathbf{k}\cdot\mathbf{v}')^n} d^3v' \quad (n = 1,2,3).$$

I_n can be written in terms of the plasma dispersion function $Z(\zeta)$,

$$Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-q^2} dq}{q - \zeta} \quad \left(\zeta = \frac{\omega'/k}{\sqrt{2T/m}}\right).$$

We have $I_1 = -\zeta Z/\omega'$, $I_2 = -2\zeta^2(1+\zeta Z)/\omega'^2$, and $I_3 = \zeta^2[\zeta Z - 2\zeta^2(1+\zeta Z)]/\omega'^3$. Substituting Eq. (11) into Eq. (9), we obtain the dielectric tensor of moving Maxwellian plasma,

$$\varepsilon_{ij} = \delta_{ij} + \Sigma \frac{\omega_{p\alpha}^2}{\omega^2}\left[\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\zeta_\alpha Z(\zeta_\alpha) + 2\zeta_\alpha^2(1 + \zeta_\alpha Z(\zeta_\alpha)) \times \left(\frac{k_i k_j}{k^2} + \frac{k_i u_{j\alpha} + k_j u_{i\alpha}}{\omega_\alpha'} + \frac{k^2 u_{i\alpha} u_{j\alpha}}{\omega_\alpha'^2}\right)\right], \tag{9}$$

where

$$\zeta_\alpha = \sqrt{\frac{m_\alpha}{2T_\alpha}} \frac{\omega_\alpha'}{k}$$

and the summation is taken over α, electron, and ion quantities. Equation (9) agrees with the dielectric tensor in a moving medium obtained from Lorentz–Minkowski transform (p. 159 in Ref. 6). When the flow velocity $\mathbf{u}=0$, the dielectric tensor, Eq. (9), reduces to the well-known result for isotropic plasma

$$\varepsilon_{ij} = \varepsilon_T \delta_{ij} - \frac{k_i k_j}{k^2}(\varepsilon_T - \varepsilon_L), \tag{10}$$

where $\varepsilon_{T,L}$ are, respectively, the transverse and the longitudinal dielectric permittivity,

$$\varepsilon_T = 1 - \Sigma \frac{\omega_p^2}{\omega} \int \frac{f_0 d^3v}{\omega - \mathbf{k}\cdot\mathbf{v}} = 1 + \Sigma \frac{\omega_p^2}{\omega^2} \zeta Z(\zeta), \tag{11}$$

$$\varepsilon_L = 1 - \Sigma \omega_p^2 \int \frac{f_0 d^3v}{(\omega - \mathbf{k}\cdot\mathbf{v})^2} = 1 + \Sigma \frac{\omega_p^2}{\omega^2} 2\zeta^2(1 + \zeta Z(\zeta)), \tag{12}$$

where ζ involves the wave frequency, not Doppler-shifted. Shokri's dielectric tensor, Eq. (17S), should be compared with Eq. (9). Apparently the ion contribution is excluded in Eq. (17S). With the identification,

$$\varepsilon'^{\text{tr}} - 1 = \frac{\omega_{\text{pe}}^2}{\omega'^2} \zeta Z,$$

$$\varepsilon'^{l} - 1 = \frac{\omega_{\text{pe}}^2}{\omega'^2} 2\zeta^2(1 + \zeta Z),$$

Eq. (17S) agrees with Eq. (9).

The wave equations for the set of TM mode are written in Ref. 6, which read

$$k_x E_z(k_x) - k_z E_x(k_x) + \frac{\omega}{c} B_y(k_x) = 0, \tag{13}$$

$$ik_z B_y(k_x) - i\frac{\omega}{c} E_x(k_x) + \frac{4\pi}{c} J_x(k_x) = 0, \tag{14}$$

$$ik_x B_y(k_x) + i\frac{\omega}{c} E_z(k_x) - \frac{4\pi}{c} J_z(k_x) = \frac{a}{\pi}, \tag{15}$$

where the constant a/π represents the discontinuity of B_y at $x=0$ given rise to by continuing B_y oddly into the region $x<0$, in conformity with the specular reflection condition.[7] Since the y-coordinate is ignorable, $k_y=0$. Using the dielectric tensor to eliminate the currents $J_{x,z}$, the above equations can be solved for the field components

$$E_x(k_x) = \frac{ica}{\pi\omega\Delta}\left(\varepsilon_{xz} + \frac{c^2}{\omega^2} k_x k_z\right), \tag{16}$$

$$E_z(k_x) = \frac{-ica}{\pi\omega\Delta}\left(\varepsilon_{xx} - \frac{c^2}{\omega^2} k_z^2\right), \tag{17}$$

$$B_y(k_x) = \frac{ia\,c^2}{\pi\Delta\omega^2}(k_z \varepsilon_{xz} + k_x \varepsilon_{xx}), \tag{18}$$

where

$$\Delta = \varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{xz}\varepsilon_{zx} - \frac{c^2}{\omega^2}(\varepsilon_{xx} k_x^2 + \varepsilon_{zz} k_z^2 + 2\varepsilon_{xz} k_x k_z). \tag{19}$$

By Fourier inversion of the above equations, we can obtain the plasma fields in the region $x>0$.

Next we turn to the vacuum solutions in the region $x<0$. We have

$$E_z(x) = \frac{iFc}{\omega} \lambda e^{\lambda x}, \tag{20}$$

$$E_x(x) = \frac{cFk_z}{\omega} e^{\lambda x}, \tag{21}$$

$$B_y(x) = Fe^{\lambda x}, \tag{22}$$

where F is a constant and

$$\lambda = \sqrt{k_z^2 - \frac{\omega^2}{c^2}}. \tag{23}$$

The plasma solutions and the vacuum solutions should be matched at the interface $x=0$ by physical boundary conditions: the continuity of E_z and B_y. To reinstate the x-dependence, the Fourier inversion integral $\int dk_x$ must be performed on Eqs. (19)–(21). The boundary conditions involve two constants, F and a/π, and the solvability condition yields the desired surface wave dispersion relation,

$$i\lambda \frac{c^2}{\omega^2} \int_{-\infty}^{\infty} \frac{k_z \varepsilon_{xz} + k_x \varepsilon_{xx}}{\Delta} dk_x + \int_{-\infty}^{\infty} \frac{\varepsilon_{xx} - (c^2/\omega^2) k_z^2}{\Delta} dk_x = 0. \tag{24}$$

Equation (24) is the general electromagnetic TM mode surface wave dispersion relation. The dielectric tensor elements therein can be computed kinetically as well as fluidly.

We shall consider a cold plasma moving with a flow velocity $\mathbf{u}=\hat{z}u_z$ with ions immobile. In this case an electron surface current exists on the interface and B_y is not continuous.[4,8] Thus, we have to use

$$B_y(x=0^+)-B_y(x=0^-)=\frac{u_z}{c}(E_x(x=0^+)-E_x(x=0^-)). \tag{25}$$

In this case the dispersion relation incorporating the boundary condition (25) takes the form

$$i\lambda\frac{c}{\omega'}\int_{-\infty}^{\infty}\frac{(c/\omega)(k_z\varepsilon_{xz}+k_x\varepsilon_{xx})-\frac{u_z}{c}(\varepsilon_{xz}+(c^2/\omega^2)k_xk_z)}{\Delta}dk_x + \int_{-\infty}^{\infty}\frac{\varepsilon_{xx}-(c^2/\omega^2)k_z^2}{\Delta}dk_x=0. \tag{26}$$

It is emphasized that Eq. (26) can be used only for cold plasma in which ions do not contribute to the surface current. This is true for electromagnetic perturbation. In a cold plasma the dielectric tensor can be easily obtained from Eq. (9) by putting the asymptotic values, $\zeta_\alpha Z(\zeta_\alpha)=-1$ and $2\zeta_\alpha^2(1+\zeta_\alpha Z(\zeta_\alpha))=-1$. We have

$$\varepsilon_{xx}=1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2}, \tag{27}$$

$$\varepsilon_{zz}=1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2}\left(1+\frac{2k_zu_z}{\omega'}+\frac{k^2u_z^2}{\omega'^2}\right), \tag{28}$$

$$\varepsilon_{xz}=\varepsilon_{zx}=-\frac{\omega_{\mathrm{pe}}^2}{\omega^2}\frac{k_xu_z}{\omega'}. \tag{29}$$

Putting Eqs. (27)–(29) into Eq. (19), we obtain for a cold plasma

$$\Delta_{\mathrm{cold}}=\left(1-\frac{\omega_{\mathrm{pe}}^2}{\omega'^2}\right)\left(1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2}-\frac{c^2k^2}{\omega^2}\right). \tag{30}$$

In obtaining Eq. (30) we neglected $k_x^2u_z^2 \ll c^2k^2$. Substitution of Eqs. (27)–(29) into Eq. (26) with neglect of $u_z\omega/c^2k_z$ as compared to unity yields

$$i\lambda\frac{c^2}{\omega^2}\left(1-\frac{\omega_{\mathrm{pe}}^2}{\omega'^2}\right)\int_{-\infty}^{\infty}\frac{k_xdk_x}{\Delta_{\mathrm{cold}}} + \int_{-\infty}^{\infty}\frac{1-(\omega_{\mathrm{pe}}^2/\omega^2)-(c^2/\omega^2)k_z^2}{\Delta_{\mathrm{cold}}}dk_x=0. \tag{31}$$

Carrying out the integrations gives the dispersion relation of cold plasma TM mode surface wave

$$\sqrt{k_z^2-\frac{\omega^2-\omega_{\mathrm{pe}}^2}{c^2}}+\lambda\left(1-\frac{\omega_{\mathrm{pe}}^2}{\omega'^2}\right)=0. \tag{32}$$

Let us return to the general dispersion relation, Eq. (24). For an isotropic dielectric tensor [Eq. (10) or Eq. (16S)], we have $\Delta=\varepsilon_L(\varepsilon_T-c^2k^2/\omega^2)$, $k_z\varepsilon_{xz}+k_x\varepsilon_{xx}=k_x\varepsilon_L$, and $\varepsilon_{xx}-c^2k_z^2/\omega^2=\varepsilon_Lk_x^2/k^2+(\varepsilon_T-c^2k^2/\omega^2)k_z^2/k^2$, giving

$$\pi\left(k_z^2-\frac{\omega^2}{c^2}\right)^{1/2}+\int_{-\infty}^{\infty}\frac{dk_x}{k^2}\left(\frac{k_z^2}{\varepsilon_L}+\frac{k_x^2}{\varepsilon_T-c^2k^2/\omega^2}\right)=0, \tag{33}$$

which is Eq. (11S). Equation (33) is characteristic of isotropic plasma whose dielectric tensor can be written as Eq. (10), as a linear combination of two coordinate-independent basic tensors δ_{ij} and k_ik_j/k^2. In order for Eq. (19S) to be valid, Shokri's dielectric tensor, Eq. (17S), should be isotropic in its form, with the modified dielectric permittivities. Clearly this is not the case because of the terms that depend on the flow velocity $u_{i,j}$. Physically, a moving plasma is intrinsically anisotropic in the laboratory frame in which we dwell, and the dielectric tensor, given by Eq. (17S) or Eq. (9) cannot not be isotropicized into the form of Eq. (16S) with any possible modification of the dielectric constants. Thus Eq. (19S) is invalid.

In a warm plasma flow, the general dispersion relation, Eq. (24), is a very complicated function. Tedious but straightforward algebra reduces it to

$$i\lambda\frac{c^2}{\omega^2}\int_{-\infty}^{\infty}\frac{k_x[1+\Sigma(\omega_{p\alpha}^2/\omega\omega'_\alpha)2\zeta_\alpha^2(1+\zeta_\alpha Z(\zeta_\alpha))]dk_x}{\Delta} + \int_{-\infty}^{\infty}\frac{\varepsilon_{xx}-(c^2/\omega^2)k_z^2}{\Delta}dk_x=0, \tag{34}$$

where

$$\Delta=\Delta'\left(1+\Sigma\frac{\omega_{p\alpha}^2}{\omega^2}\zeta_\alpha Z(\zeta_\alpha)-\frac{c^2k^2}{\omega^2}\right)$$

with

$$\Delta'=1+\Sigma\frac{\omega_{p\alpha}^2}{\omega_\alpha'^2}2\zeta_\alpha^2(1+\zeta_\alpha Z(\zeta_\alpha)).$$

By taking the limit of $c\to\infty$ in Eq. (34), we obtain the electrostatic dispersion relation,

$$\int_{-\infty}^{\infty}\frac{-ik_x[1+\Sigma(\omega_{p\alpha}^2/\omega\omega'_\alpha)2\zeta_\alpha^2(1+\zeta_\alpha Z(\zeta_\alpha))]+k_z}{k^2\Delta'}dk_x=0. \tag{35}$$

This equation should be compared with Eq. (23S). As a matter of fact, Eq. (23S) should not contain any ion contribution since it is obtained from Eq. (19S), which has already excluded ion contributions. Looking at Eq. (23S) as it is, it has an invariant form which is exactly identical to the electrostatic surface wave dispersion relation in stationary isotropic plasma, with the wave frequency replaced by the Doppler-shifted frequency. This is unphysical since the dynamics and the surface boundary conditions in a flowing plasma cannot be conjured up to be invariant with the dispersion relation in a stationary plasma. Also it cannot predict an ion-acoustic instability which can be excited in a moving plasma.

To carry out the integrals in Eqs. (34) and (35) would require considerable amount of algebraic work by expanding the plasma dispersion function asymptotically and performing the contour integrals. The warm plasma dispersion relation thus obtained from Eq. (34) contains the cold plasma dispersion relation [Eq. (32)] as a subset. If the cold plasma limit is taken in the judiciously obtained warm plasma dispersion relation, we can obtain the correct cold plasma dispersion relation, even though different surface boundary conditions, in regard to the matter of presence of surface currents, are employed.[4] This is a pleasant feature of surface wave theory. In conclusion, there is no shortcut to reach the dispersion relation in a plasma jet other than carrying out Eq. (24) with correct dielectric tensor.

[1]B. Shokri, Phys. Plasmas **9**, 701 (2002).
[2]B. Shokri, Phys. Plasmas **10**, 3036 (2003).
[3]E. Benova, S. T. Ivanov, and A. A. Rukhadze, J. Plasma Phys. **63**, 489 (2000).
[4]H. J. Lee and S.-H. Cho, J. Plasma Phys. **58**, 409 (1997).
[5]M. J. Lee and H. J. Lee, Phys. Plasmas **12**, 052104 (2005).
[6]A. F. Alexandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics* (Springer-Verlag, Berlin, 1984).
[7]H. C. Barr and T. J. M. Boyd, J. Phys. A **5**, 1108 (1972).
[8]H. J. Lee and S.-H. Cho, J. Plasma Phys. **61**, 173 (1999).