Discussion: "On the Thermodynamical Admissibility of the Triphasic Theory of Charged Hydrated Tissues" (Huyghe, J. M., Wilson, W., and Malakpoor, K., ASME J. Biomech. Eng., 2009, 131, p. 044504)

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We feel it is necessary for us to point out some critical mathematical errors and erroneous statements that Huyghe et al. made in the manuscript listed above (hereafter, referred to as the Huyghe et al.'s paper). As shown in detail below, this manuscript is misleading and contains fatal flaws of mathematics and logic. Therefore we request a retraction of their statement that "…all results [10-13,16] obtained with the theory of Lai et al. [5] should be distrusted" as stated in the main conclusion of their paper (last paragraph). These errors are detailed as follows.

(1) In Huyghe et al. paper, the authors ignored the fact that the general formulation of the triphasic theory by Lai et al. [1] (i.e., Ref. [5] of their paper) is directly derived from the entropy inequality; see Eqs. (45)–(49) on page 251 in Ref. [1]. Therefore the constitutive equations are fully compatible with the second law of thermodynamics.

(2) The authors ignored the fact that the specific formulation for ion (electro)chemical potential is, in general, strain-dependent (see the footnote on page 252 in Ref. [1]). These oversights led them to reach erroneous statements in their paper.

(3) The authors did their analysis in their "thought experiments" based on the mistakes stated above (points 1 and 2), and therefore their results and conclusion are generally invalid.

(4) The authors also made several fundamental and mathematical errors in their analysis and derivations. For example, their Eq. 14 ($(\partial\phi^w/\partial E)_{c_e}=1$) is incorrect; it should be $(\partial\phi^w/\partial E)_{c_e}=(1-\phi_0^w)/(1+E)^2\approx 1-\phi^w$. This fundamental error propagates throughout their derivation, resulting in a total of seven incorrect equations, including Eqs. 14, 17, 18, 20, 23, 25, and 27. For example, Eq. 20 should be

$$\left(\frac{\partial(\phi^w c^+)}{\partial E}\right)_{c_e}=-\phi^w c^+ + \frac{2c^+c^-}{c^+ + c^-}$$

$$\left(\frac{\partial(\phi^w c^-)}{\partial E}\right)_{c_e}=-\phi^w c^- + \frac{2c^+c^-}{c^+ + c^-}$$

instead of their erroneous results:

$$\left(\frac{\partial(\phi^w c^+)}{\partial E}\right)_{c_e}=\left(\frac{\partial(\phi^w c^-)}{\partial E}\right)_{c_e}=\frac{2c^+c^-}{c^+ + c^-}$$

(5) The authors used a first order approximation (i.e., linearization) for most of their analysis, but in the end they presented their result with a "positive" small value at the second order. The authors further made all of their conclusions based on the fact that this second order small term is not zero or negligible. This inconsistent mathematical procedure is problematic and results in their incorrect conclusions.

(6) Based on their analysis, the authors then conclude with misleading statements and incorrect conclusions on the T_c term in the triphasic theory (see first paragraph in Results of Huyghe et al. paper.) and on the "concentration dependent Lamé constants" introduced in Ref. [2] (i.e., Ref. [19]) in the last paragraph of their paper.

(7) Finally, based on the second law of thermodynamics, the total mixture stress must depend on solute concentration if the (electro)chemical potential of solute depends on the solid strain, or vise versa. In the triphasic theory, it is assumed that the stress is dependent on solute concentration (e.g., the T_c term) and the solute chemical potential is dependent on strain; see Eqs. (46)–(49) in Ref. [1]. Due to the lack of experimental data for solute (electro)chemical potential in cartilage, it was assumed that the value of the activity coefficient to be constant and the value of B_i to be zero in examples of numerical calculations in literature (Refs. [5] (Ref. [1]), [10]–[13] (Refs. [3–6]), [16] (Ref. [7]), and [19] (Ref. [2]). These approximations do not necessarily mean that T_c does not exist nor that the Lamé coefficients should not depend on concentration. One may argue whether these were good assumptions if we have any experimental data, but these approximations in no way invalidate our triphasic theories themselves where the chemi-

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cal potentials are functions of strain. In some of our later papers [8–11] to simplify the analysis, we also assume T_c to be zero in addition to these approximations. Again, these approximations are consistent with our basic assumptions used to develop the triphasic theory, and they do not invalidate the full triphasic theory developed by Lai et al. [1].

While we are pleased that Professor Huyghe and colleagues continue to show interest in our work, as they have over the years, but we must point out that their present study contains numerous errors and their conclusions are misleading. These arguments are the basis that we request Professor Jacque M. Huyghe and coworkers retract their statement "all results [10-13, 16] obtained with the theory of Lai et al. [5] should be distrusted,…."

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