

**Discussion: “On the Thermodynamical Admissibility of the Triphasic Theory of Charged Hydrated Tissues” (Huyghe, J. M., Wilson, W., and Malakpoor, K., ASME J. Biomech. Eng., 2009, 131, p. 044504)**

**Van C. Mow**

Stanley Dicker Professor of Biomedical Engineering and Orthopedic Department of Biomedical Engineering, Columbia University, New York, NY 10027

**W. Michael Lai**

Emeritus Professor of Mechanical Engineering Department of Mechanical Engineering, Columbia University, New York, NY 10027

**Lori A. Setton**

Mary Milius Yoh and Harold L. Yoh, Jr. Bass Professor of Biomedical Engineering Associate Professor of Orthopaedic Surgery Duke University, Durham, NC 27708

**Weiyong Gu**

Associate Professor Department of Biomedical Engineering, College of Engineering, University of Miami, Coral Gables, FL 33124

**Hai Yao**

Assistant Professor Department of Bioengineering, Clemson University, Clemson, SC 29634

**Leo Q. Wan**

**X. Lucas Lu**

Department of Biomedical Engineering, Columbia University, New York, NY 10027

We feel it is necessary for us to point out some critical mathematical errors and erroneous statements that Huyghe et al. made in the manuscript listed above (hereafter, referred to as the Huyghe et al.’s paper). As shown in detail below, this manuscript is misleading and contains fatal flaws of mathematics and logic. Therefore we request a retraction of their statement that “...all

results [10-13,16] obtained with the theory of Lai et al. [5] should be distrusted” as stated in the main conclusion of their paper (last paragraph). These errors are detailed as follows.

- (1) In Huyghe et al. paper, the authors ignored the fact that the general formulation of the triphasic theory by Lai et al. [1] (i.e., Ref. [5] of their paper) is directly derived from the entropy inequality; see Eqs. (45)–(49) on page 251 in Ref. [1]. Therefore the constitutive equations are fully compatible with the second law of thermodynamics.
- (2) The authors ignored the fact that the specific formulation for ion (electro)chemical potential is, in general, strain-dependent (see the footnote on page 252 in Ref. [1]). These oversights led them to reach erroneous statements in their paper.
- (3) The authors did their analysis in their “thought experiments” based on the mistakes stated above (points 1 and 2), and therefore their results and conclusion are generally invalid.
- (4) The authors also made several fundamental and mathematical errors in their analysis and derivations. For example, their Eq. 14  $((\partial\phi^w/\partial E)_{c_e}=1)$  is incorrect; it should be  $(\partial\phi^w/\partial E)_{c_e}=(1-\phi_0^w)/(1+E)^2\approx 1-\phi^w$ . This fundamental error propagates throughout their derivation, resulting in a total of seven incorrect equations, including Eqs. 14, 17, 18, 20, 23, 25, and 27. For example, Eq. 20 should be

$$\left(\frac{\partial(\phi^w c^+)}{\partial E}\right)_{c_e} = -\phi^w c^+ + \frac{2c^+ c^-}{c^+ + c^-}$$

$$\left(\frac{\partial(\phi^w c^-)}{\partial E}\right)_{c_e} = -\phi^w c^- + \frac{2c^+ c^-}{c^+ + c^-}$$

instead of their erroneous results:

$$\left(\frac{\partial(\phi^w c^+)}{\partial E}\right)_{c_e} = \left(\frac{\partial(\phi^w c^-)}{\partial E}\right)_{c_e} = \frac{2c^+ c^-}{c^+ + c^-}$$

- (5) The authors used a first order approximation (i.e., linearization) for most of their analysis, but in the end they presented their result with a “positive” small value at the second order. The authors further made all of their conclusions based on the fact that this second order small term is not zero or negligible. This inconsistent mathematical procedure is problematic and results in their incorrect conclusions.
- (6) Based on their analysis, the authors then conclude with misleading statements and incorrect conclusions on the  $T_c$  term in the triphasic theory (see first paragraph in Results of Huyghe et al. paper.) and on the “concentration dependent Lamé constants” introduced in Ref. [2] (i.e., Ref. [19]) in the last paragraph of their paper.
- (7) Finally, based on the second law of thermodynamics, the total mixture stress must depend on solute concentration if the (electro)chemical potential of solute depends on the solid strain, or vice versa. In the triphasic theory, it is assumed that the stress is dependent on solute concentration (e.g., the  $T_c$  term) and the solute chemical potential is dependent on strain; see Eqs. (46)–(49) in Ref. [1]. Due to the lack of experimental data for solute (electro)chemical potential in cartilage, it was assumed that the value of the activity coefficient to be constant and the value of  $B_i$  to be zero in examples of numerical calculations in literature (Refs. [5] (Ref. [1]), [10]–[13] (Refs. [3–6]), [16] (Ref. [7]), and [19] (Ref. [2])). These approximations do not necessarily mean that  $T_c$  does not exist nor that the Lamé coefficients should not depend on concentration. One may argue whether these were good assumptions if we have any experimental data, but these approximations in no way invalidate our triphasic theories themselves where the chemi-

Contributed by the Bioengineering Division of ASME for publication in the JOURNAL OF BIOMECHANICAL ENGINEERING. Manuscript received March 18, 2009; final manuscript received March 19, 2009; published online August 5, 2009. Review conducted by Michael Sacks.

cal potentials are functions of strain. In some of our later papers [8–11] to simplify the analysis, we also assume  $T_c$  to be zero in addition to these approximations. Again, these approximations are consistent with our basic assumptions used to develop the triphasic theory, and they do not invalidate the full triphasic theory developed by Lai et al. [1].

While we are pleased that Professor Huyghe and colleagues continue to show interest in our work, as they have over the years, but we must point out that their present study contains numerous errors and their conclusions are misleading. These arguments are the basis that we request Professor Jacques M. Huyghe and co-workers retract their statement “all results [10-13, 16] obtained with the theory of Lai et al. [5] should be distrusted,....”

## References

- [1] Lai, W. M., Hou, J. S., and Mow, V. C., 1991, “A Triphasic Theory for the Swelling and Deformation Behaviors of Articular Cartilage,” *ASME J. Biomech. Eng.*, **113**, pp. 245–258.
- [2] Gu, W. Y., Lai, W. M., and Mow, V. C., 1998, “A Mixture Theory for Charged-Hydrated Soft Tissues Containing Multi-Electrolytes: Passive Transport and Swelling Behaviors,” *ASME J. Biomech. Eng.*, **120**, pp. 169–180.
- [3] Lai, W. M., Gu, W. Y., Setton, L. A., and Mow, V. C., 1991, “Conditional Equivalence of Chemical Loading and Mechanical Loading on Articular Cartilage,” *Adv. Bioeng.*, **20**, pp. 481–484.
- [4] Setton, L. A., Lai, W. M., and Mow, V. C., 1993, “Swelling Induced Residual Stress and the Mechanism of Curling in Articular Cartilage,” *Adv. Bioeng.*, **26**, pp. 59–62.
- [5] Gu, W. Y., Lai, W. M., and Mow, V. C., 1993, “Transport of Fluid and Ions Through a Porous-Permeable Charged-Hydrated Tissue, and Streaming Potential Data on Normal Bovine Articular Cartilage,” *J. Biomech.*, **26**, pp. 709–723.
- [6] Gu, W. Y., Lai, W. M., and Mow, V. C., 1997, “A Triphasic Analysis of Negative Osmotic Flows Through Charged Hydrated Soft Tissues,” *J. Biomech.*, **30**, pp. 71–78.
- [7] Sun, D. N., Gu, W. Y., Guo, X. E., Lai, W. M., and Mow, V. C., 1999, “A Mixed Finite Element Formulation of Triphasic Mechano-Electrochemical Theory for Charged, Hydrated Biological Soft Tissues,” *Int. J. Numer. Methods Eng.*, **45**, pp. 1375–1402.
- [8] Mow, V. C., Ateshian, G. A., Lai, W. M., and Gu, W. Y., 1998, “Effects of Fixed Charge Density on the Stress-Relaxation Behavior of Hydrated Soft Tissues in a Confined Compression Problem,” *Int. J. Solids Struct.*, **35**, pp. 4945–4962.
- [9] Yao, H., and Gu, W. Y., 2004, “Physical Signals and Solute Transport in Cartilage Under Dynamic Unconfined Compression: Finite Element Analysis,” *Ann. Biomed. Eng.*, **32**, pp. 380–390.
- [10] Yao, H., and Gu, W. Y., 2006, “Physical Signals and Solute Transport in Human Intervertebral Disc During Compressive Stress Relaxation: 3D Finite Element Analysis,” *Biorheology*, **43**, pp. 323–335.
- [11] Huang, C. Y., and Gu, W. Y., 2007, “Effects of Tension-Compression Nonlinearity on Solute Transport in Charged Hydrated Fibrous Tissues Under Dynamic Unconfined Compression,” *ASME J. Biomech. Eng.*, **129**, pp. 423–429.