

# A Spatial Theory of Rhythmic Resolution

Neil McLachlan

*Thus profound metaphysics is rooted in an implicit geometry which—whether we will or no—confers spatiality upon thought; if a metaphysician could not draw, what would he think?*

—G. Bachelard [1]

This article presents musical structures from percussion ensembles of Africa and Indonesia mapped into cyclic arrays, which I then analyze using concepts from Gestalt psychology, mathematical group theory and psycho-acoustics. This analysis suggests a tentative theory regarding the musical efficacy of certain well-established rhythmic forms.

P. Desain [2,3] has described a connectionist model for rhythmic perception. This model derives relations of event expectancies (e.g. the expectation of a beat at any point along a timeline) mathematically from a rhythmic information stream. These relations are projected into the future and past to evaluate the level of agreement of new data with past events and so decide on the rhythmic significance of variations in the information stream in “real” time. Data from past events may be reconfigured in the light of present information, which in turn creates new expectations of future data.

M.R. Jones [4,5] has described rhythmic awareness as psychological trajectories forming nested hierarchies of patterns in mental space and time. She quantizes temporal durations in music or other activities by defining moments of transition or discontinuity and maps them in tree diagrams of sections and sub-sections. James Tenney [6] has generated a temporal analysis system that is similar to that of Jones but more broadly applicable to music by using Gestalt perceptual theories to quantize as many musical parameters as possible within information hierarchies. More recently, composers working in computer composition, analysis and notation [7] have developed many hierarchical representations of musical structure.

Gestalt principles were originally developed in studies of visual perception, where they are believed to reflect the way that human perceptual systems group sense data in order to identify partially obscured forms. Gestalt principles state that patterns are interpreted so that the resulting structures are as simple as possible (the principle of the most simple representation) [8] and that lines or boundaries will appear to continue in relation to their form until discontinuities or inconsistencies imply alternatives (the principle of good continuance) [9]. In my work, I have mapped certain rhythms in cyclic arrays to apply Gestalt principles to the resultant patterns and so attempt to reveal underlying structural principles in the music.

J. Becker [10] has used a cyclic array to map a traditional Javanese gamelan piece, as the cyclic array more accurately represents the concepts of time described by the gamelan musicians themselves than do the linear models and notations used by Western musicologists. Becker shows how repeating musical forms generate temporal symmetries and balance in accordance with Hindu and Buddhist beliefs.

She also describes the similarity in form between traditional Javanese gamelan music when mapped in this way and Hindu mandalas, while pointing out that both evolved in ritual and meditative practices.

Cyclic arrays have been used in European clock faces for centuries to map the movement of the clock hands through sub-divisions of a circle. A very important advantage of this over a linear array (e.g. the hours of a day displayed along a line) is that the end of one cycle is at the same spatial position as the beginning of the next.

Mathematical group theory finds application to most fields of study in which mathematics is used to analyze arrays of related data. It underlies 12-tone serial composition, in which operations such as inversion and permutation are applied to sets of pitch classes [11]. M. Babbitt [12] has described inherent rhythmic properties of 12-tone serial composition by showing how various permutations and inversions of pitch class sets affect the temporal ordering of pitch intervals. He has proposed that group theory (including set theory) may generate rhythms independent of, or in relation to, permutations of pitch class sets. Jones [13] describes the importance of symmetry groups in mental patterns of time and space in allowing reversibility and conservation of thought.

J. Pressing [14] has used group theory to generate a number of traditional rhythmic patterns and has explored their structural similarity (isomorphism) to various Western scales. Pressing proposes a linear array of identical elements analogous to either the semitone intervals of the chromatic scale or to the largest common factor of the temporal intervals of a given rhythm (as in additive rhythms). Patterns analogous to rhythms and scales are generated by the demarcation of

## ABSTRACT

Cyclic arrays, such as clock faces, have advantages over linear arrays for conceptualizing repetitive rhythmic structures. The author maps rhythms from African and Indonesian musics into cyclic arrays and analyzes them using concepts from Gestalt psychology, mathematical group theory and psycho-acoustics. The perceptual structures thus revealed exist between the different musical parts played on various instruments and contradict the usual processes of auditory segregation according to the physical locations of instrumentation. This prompts a proposal for a theory of musical *despatialization* to explain the psychological efficacy of these rhythms.

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various group sizes along this array of elements.

For example, the Western major scale may be modeled by superimposing six groups of 7 elements each over four groups of 12 elements each, where each repetition of the 12-element group is considered equivalent (as in the octave equivalence of pitch classes). This process is well known to Western musicians as the circle of fifths, where the 7-element group represents the seven semitones in the interval of a fifth, and the 12-element group represents the 12 semitones in an octave.

### MAPPINGS IN CYCLIC ARRAYS OF PERCUSSION ENSEMBLE MUSICS

Visual mapping of mathematical group theory identities is a common aid to conceptualization. Figure 1 shows the patterns generated by joining the points created after first counting out and marking each iteration of all the possible group sizes (groups containing 1–11 elements) within a 12-element cyclic array. The hexagon is produced by groups of 2, the square by groups of 3, and the triangle by groups of 4, when these groups are marked out by counting around the circle starting at the top (element 12). Six-element groups simply produce a line between elements 12 and 6.

Marking out 10-element groups around the circle will also produce a hexagon, because the difference between 10 and 12 is 2 (i.e. a full circle minus 2 elements is traversed each time a 10-element group is counted out). Simi-

larly, marking out 9- and 8-element groups will also produce the square and triangle, respectively. Marking out odd-numbered group sizes (groups of 1, 3, 5, 7, 9 and 11 elements) will mark every element in the cyclic array after 12 iterations unless these groups (or their complements) are factors of 12 (e.g. 3 and its complementary 9-element group). The pattern shown for 7-element groups (and its complementary 5-element group) is generated by first marking out six iterations of 7 elements in a clockwise direction starting at element 5. The marked points are then joined by a line, as shown in Fig. 1. The numbers in brackets in Fig. 1 indicate the successive iterations of 7-element groups required to produce this pattern. The 12-sided dodecahedron that would be produced by iterations of 1- or 11-element groups is not shown for reasons of clarity.

The square in Fig. 1 can represent triplets in a 4/4 meter if one counts out each element around the array while placing accents on those elements that coincide with the points of the square. Similarly, the triangle can represent crotchets in 3/4 time, and if played together the square and triangle represent a 3:4 polymeter. The geometric pattern generated by 7- (or 5-) element groups models the Western major scale as described earlier, as well as a common bell pattern from drum ensemble music of the Ewe people from West Africa [15]. It is the only asymmetric pattern created within this set.

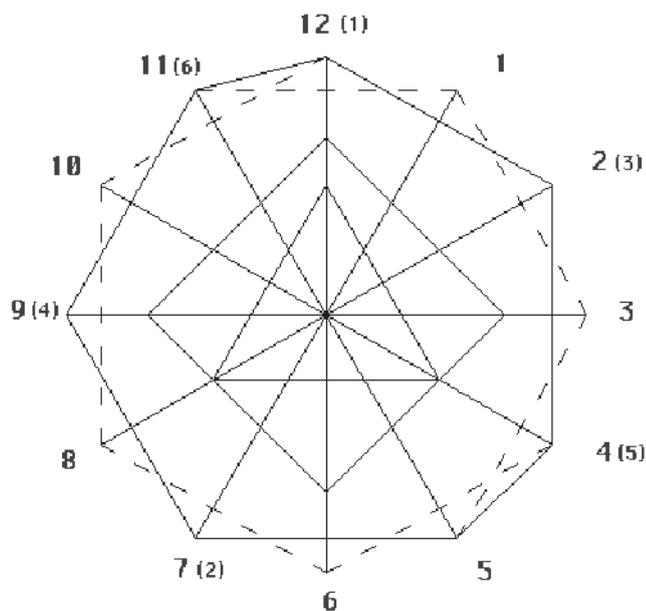
The bell pattern of the Ewe music can be read by first placing a low-pitched

bell strike at element 12 (Fig. 1). Place high-pitched bell strikes on the remaining points of the pattern described by the solid line while counting successive elements at a constant tempo around the circle in a clockwise direction.

Further inspection will reveal that this pattern forms two incomplete hexagons (finished by broken lines) joined at pairs of adjacent elements. Once one cycle of the rhythm is completed and its 12-beat duration established, the first two high-pitched bell strikes strongly imply a regular six-beat pattern. This pattern may be described as one hexagon by application of the Gestalt principle of good continuance (just as in Desain's model, past relationships are projected into the future as expectations). However, the third strike occurs only half a beat later and the remaining beats describe an interlocked hexagon, until the low-pitched bell marks the beginning of a new cycle. Therefore, by the application of Gestalt principles, the perceptual vectors of future expectancy and past reinforcement described by Desain and Jones may be seen to form a closed pattern of two interlocking hexagons. As this rhythm is performed, we are always expecting one hexagon and remembering its complement.

The square in Fig. 1 represents four groupings of 3 elements. These are analogous to hand-clapping parts and the principal accents of set drum parts in the Ewe music featuring the bell pattern described above [16]. This square is also defined by its coincidence with the bell pattern at two points (elements 9 and 12). These two points comprise the minimum information required to imply the complete square according to the Gestalt principle of good continuance within a cyclic array (this effect may be heard by tapping only the first two beats of a  $\frac{3}{4}$  bar). The second of these points at the beginning of the cycle and is given added importance by being played on the low-pitched bell. In generating this implied square, the asymmetric bell pattern may be heard in relation to a regular temporal frame similar to the Western notion of meter.

All possible regular polyhedra that can be created within a 12-element array (and that contain element 12) coincide with at least 2 elements of the bell pattern. These are given musical expression by different drum parts within the music and produce a polymetric effect. Further iteration of groupings of 7 or 5 would eventually result in all 12 elements in the array being marked. Press-



**Fig. 1. Polygons generated by the iteration of all possible group sizes within a 12-element cyclic array. The numbers in brackets refer to the iterations of a 7-element group.**

ing points out that the ratio of marked to unmarked elements is usually about 1:2 in the traditional rhythms he studied. In this example it can be seen that six iterations is the minimum required to define the square, while any more would confuse the resolution of the two hexagons.

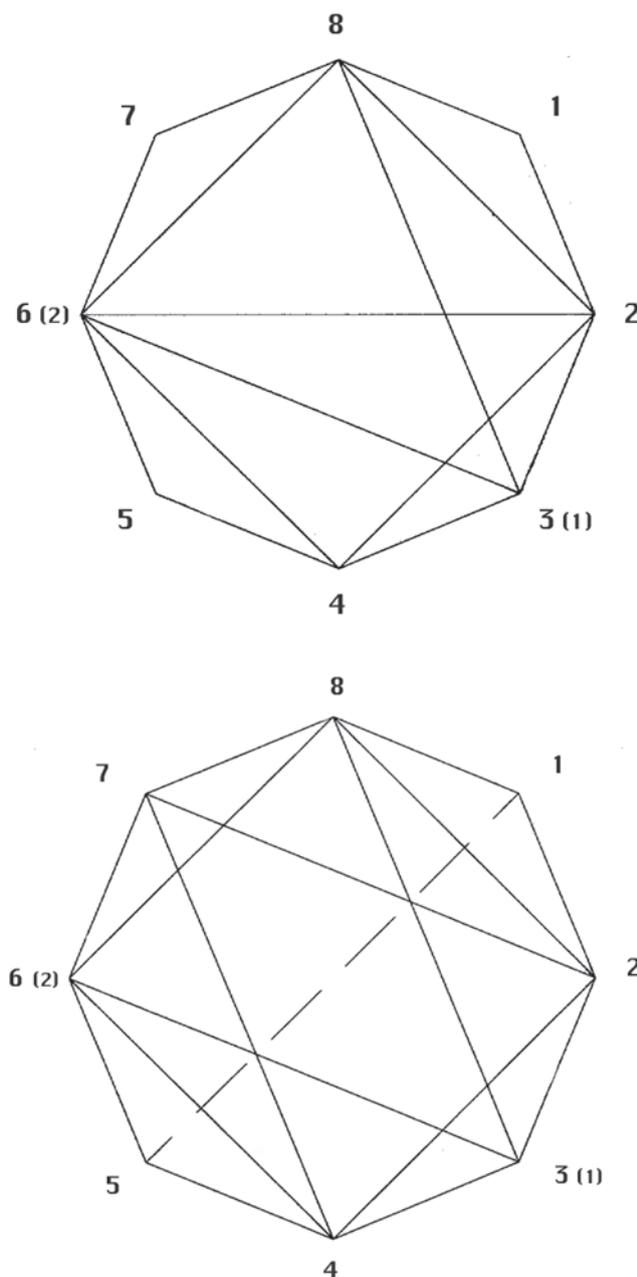
Rhythmic cycles based on even multiples of 4 beats (8, 16, 32) are common in many musical cultures. Rhythmic interest may be created with asymmetric patterns formed by the superimposition of groupings of three over these cycles. In central Javanese gamelan music [17] it is common to find cycles of 8 elements defined by symmetrical patterns of two groups of 4 and four groups of 2, with a non-symmetric pattern of beats generated by two groups of 3 superimposed over the symmetric patterns.

Figure 2a shows all possible group sizes mapped into an 8-element cyclic array. Groups of 1 and 7 produce an octagon, 2 and 6 produce a square, and groups of 4 produce a line. Only two iterations of 3-element groups are shown (numbered in brackets starting at element 8) since it is clear from the figure that further iteration of the 3-element groups would confuse the definition of the first of these groups. Like the bell pattern described in Fig. 1, the final 2 elements of this pattern coincide with two points of a square. Since these two points alone are sufficient to establish a complete square through the Gestalt principle of good continuance, a rhythm derived from this pattern may imply a 4-beat meter.

Reflecting the pattern created by the 3-element groups across a line between elements 1 and 5 produces an interlocked pattern that combines to form a square. This pattern is shown in Fig. 2b, and with the inclusion of beats on elements 1 and 5 it represents rhythmically interlocking parts found in Kecak chants in Bali.

At this point it is necessary to introduce the psycho-acoustic concept of auditory streams. A.S. Bregman [18] has described how the application of Gestalt and other perceptual principles may be employed to explain people's natural ability to differentiate sound sources occurring in the world about them. The complex mixtures of spectral envelopes that are commonly sensed by the ear are grouped into auditory streams relating to individual sources through the analysis of similarities, continuities and discontinuities in spectral, temporal and binaural sensory data. These *spatialized*

Fig. 2. (a, top) Iteration of all possible groups in an 8-element array. (b, bottom) Reflection through points 1 and 5 of 3-element group in Fig. 2a.



auditory streams generate an auditory scene or field, described by D. Ihde [19] as a specific form of opening into the physical world with its own center and horizon, independent but related to other sensory fields.

In the Kecak chant described above, two spatially and temporally distinct musical parts combine to form a structure (perceptual or auditory stream) that is powerfully cohesive due to its simplicity relative to its components. This perceptual grouping contradicts the auditory streams relating to individual sources that define the spatialized auditory field and so may lead the listener to experience the music as a *despatialized* entity. This may then contribute to the experi-

ences and emotions of removal from normal acoustic time and space reported by performers of or listeners to this music [20]. Other common musical devices, such as the simultaneous sounding of different instruments or musical harmony creating spectral fusion, may also create despatialized auditory streams and have similar effects [21].

The rhythms described in this paper evolved within ritual performances. In a recent paper, Becker [22], using connectionist theories of mind, has proposed that particular neuronal maps of trance states may be produced in performers' minds by the repetition of religious rituals. These maps contain a rich tapestry of remembered associations



music [24]. The rhythm may be described as beats on elements 3, 6, 10, 12 and 16 in a 16-element array. Figure 5 is a geometric form generated as a model for this rhythm by reflecting a 12-element cyclic array (the circle on the right), containing 3- and 4-element groupings (the square and triangle respectively), across a line through elements 8 and 16. Counting around the perimeter of the resultant pattern generates a 16-element array, with four groups of 4 elements marked by the reflected triangles.

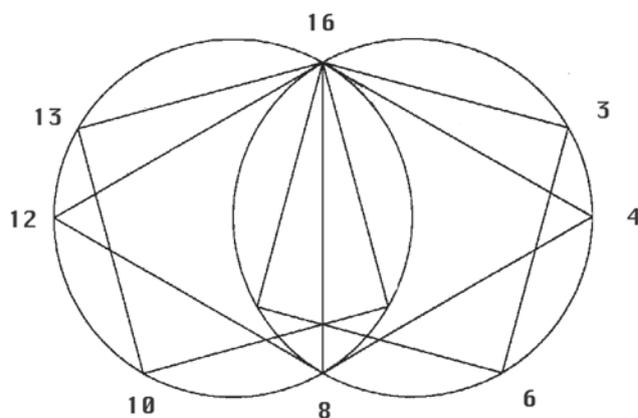
The asymmetric rhythm described above is defined by the elements marked by the points where the reflected squares coincide with the perimeter of the figure, except that element 12 (which is marked by a triangle), replaces element 13 (which is marked by a square). If this accent were not shifted, the rhythm would be symmetrical and perceived as a reflected 8-beat cycle, instead of a 16-beat cycle.

The accents on elements 12 and 16 also coincide with the groupings of four. These points provide the minimum information required to define one triangle through the principle of good continuance, the other being generated by the reflection through the line between elements 8 and 16. Like the rhythms described by Figs 1–3, a 4-beat meter (accented by other musical parts) follows from the asymmetric pattern.

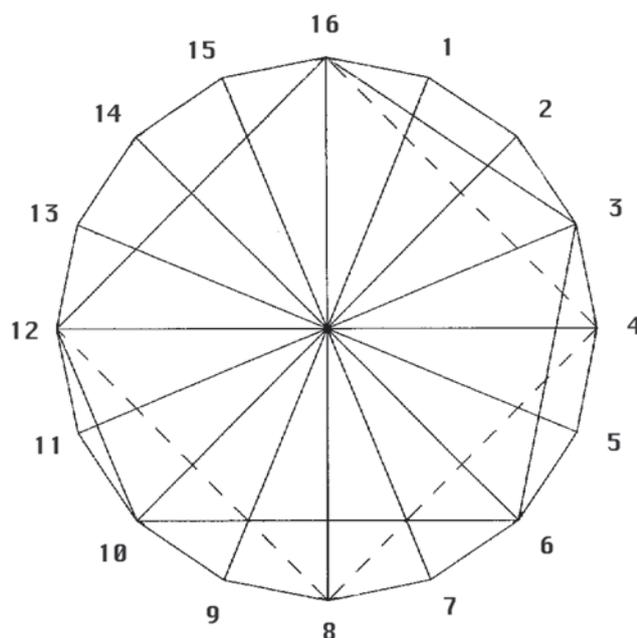
I have mapped this rhythm into a 16-beat cyclic array to ascertain which representation best accords with the Gestalt principle of the simplest possible representation (interpreted here as requiring the least amount of information to reproduce). The 16-element array only generates a symmetry across a line through elements 3 and 11 (see Fig. 6). The broken lines in Fig. 6 complete a square in accord with the principle of good continuance. The greater prevalence of regular polyhedra in symmetrical relationships with each other in Fig. 5 suggests a greater redundancy of information in this representation of the rhythmic structure (hence requiring less information to reproduce).

In its musical context, with other parts accenting the 4-element groups, the rhythm begins with the superimposition of two groups of 3 elements each over two groups of 4 each. This is the minimum information required to imply a 12-element array as shown in Fig. 5, since this is the simplest resolution of the musical information presented up to element 8 (12 being the lowest common multiple of 3 and 4).

**Fig. 5. Reflection of a 12-element cyclic array containing 3- and 4-element groups through a line between points 8 and 16 to produce a 16-element array.**



**Fig. 6. Mapping of the clave rhythm in a 16-element cyclic array.**



Once this expectation is established, it is suspended by the creation of a 16-beat rhythm through the reflection of the pattern after element 8 in Fig. 5. In this music, the auditory stream created as the composite of the two principal instrumental parts undergoes an abstract spatial reconfiguration during each cycle of the rhythm. This may have the effect of constantly refocusing the listeners' attention on the despatialized composite perceptual stream rather than the individual (physically distinct) instrumental parts.

Figure 5 may be expanded by reflecting its 12-element array six times to complete the larger hexagon shown in Fig. 7. A pattern very similar to this may be found in Arabic mosaics [25]. To rhythmically express the larger hexagonal geometry, as well as the 12-element array that generated it, we need to introduce a model of heliocentric motion (motion about more than one center).

In this static representation, heliocentric motion may be imitated by tracing a circular 12-element array starting at the center of the figure. Upon return to the center, shift to trace the next overlapping cyclic array and so on around the complete figure. A 16-element array can be traced in a similar fashion by describing the perimeter of two triangles; a 20-element array with three triangles; and so on up to 32 elements. If each face of the triangle contains four elements, simply describing the perimeter of the figure produces a 24-element array. The rhythmic relationship that this array has to those described by groupings of triangles may be defined by a composer. If the same tempos were adopted in the above example, the array described by the perimeter of the figure would occur twice for every three cycles of the 16-element array, with a high coincidence of beats.

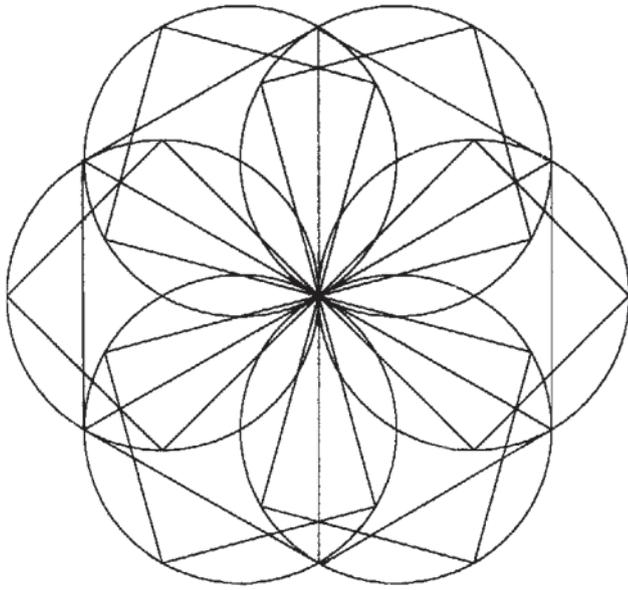


Fig. 7. Four iterations of the reflection shown in Fig. 5.

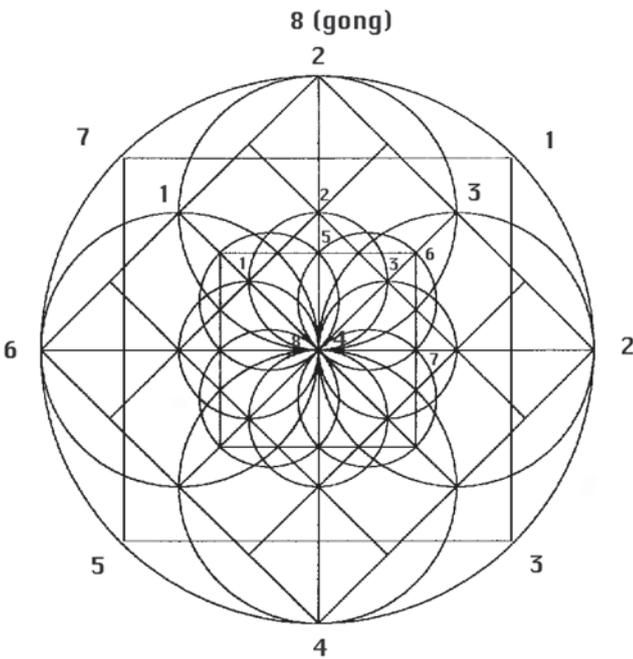


Fig. 8. (a, left) Mapping of the Javanese gamelan piece “Udon Mas” in 4- and 8-element cyclic arrays scaled by a grid. (b, below) Cipher notation for “Udon Mas.”

Figure 8a explores the heliocentric model further by creating a geometric model of stratified gong rhythms used in Javanese gamelan. An example of one gong cycle (repeating section) typical of the form described by Fig. 8a is given in cipher notation in Fig. 8b [26].

The parts for *kempul* and *kenong* (large gongs) mark out two interlocked squares in an 8-element cyclic array (the perimeter of the figure). The *balungan* plays 4 beats within each beat of the *kenong*. These are defined by the points in the diagram where grid intersections coincide with the four mid-sized circles (the first cycle is numbered 1–4). The *bonang barung* (higher pitched gong) plays twice as fast as the *balungan*, and its part is similarly described by the eight small circles at the center of the figure (the first two cycles are numbered 1–8 in a smaller font). The *bonang penerus* (highest pitched gongs) plays twice as fast as the *bonang barung* and could also be mapped. However, this is not shown to avoid visual confusion in the figure drawn at this scale.

This figure successfully maps three rhythmic strata in relation to each other by use of a grid, given equal temporal intervals between grid intersections. A composer may now explore the stratification of sets of geometrically modeled rhythms through the application of appropriate grids. Figure 9 shows rhythmic stratification of the patterns in Fig. 6 by application of a triangular coordinate grid. Grids using prime numbers of coordinates greater than 3 will require the application of curved surfaces.

### DISCUSSION

Once relationships between forms such as those in Fig. 9 are established, variations such as the rotation of the squares in varying relation to the triangles may be explored. This allows

B. PENERUS	1	5	1	1	5	1	5	6	1	6	6	1	6	1	5	6	5	5	6	5	6	1	2	1	1	2	1	2
B. BARUNG		1		5		1		5		6		1		6		1		5		6		5	6	1	2	1	2	2
BALUNGAN			1			5				6				1			5			6				1				2
KENONG														1														2
KEMPUL																												
B. PENERUS	2	1	2	2	1	2	1	6	5	6	6	5	6	5	6	1	6	6	1	6	1	6	5	6	6	5	6	5
B. BARUNG		2		1		2		1		6		5		6		5		6		1		6	1	6	5	6	5	5
BALUNGAN					2			1				6			5		6				1			6				5
KENONG														5														5
KEMPUL							1																1					G

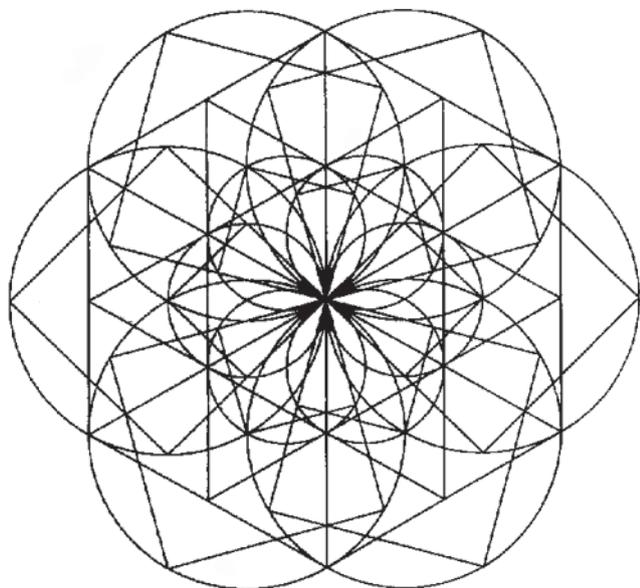


Fig. 9. Diagram in Fig. 6 scaled by use of a triangular coordinate grid.

the conceptualization of polymeters that do not contain coincident beats. For example, in a rhythm described by a square and triangle drawn with one side in parallel: if the points of the forms are defined by a varying musical parameter such as pitch or timbre, then they can be inverted, reversed, rotated or reflected in a way similar to tone rows in 12-tone composition.

Becker and Desain both discuss connectionist theories in relation to cognitive processes surrounding the perception of rhythmic structure. These models operate at sub-conceptual levels (representing processes believed to occur prior to, and in order to generate, a perceptual concept), utilizing numerical calculations on large groups of sub-symbolic entities rather than traditional logic computations of discrete symbols. A range of strategies has been adopted to fit connectionist models to what we know of human cognitive systems, often using representational features from preexisting theoretical models [27]. Thagard's theory of explanatory coherence [28] has many features similar to the present analysis (since it is based on mathematical vectors and rules of coherence, simplicity and analogy). The mappings discussed in this paper can all be described using mathematical identities from group theory, so it should be possible to use them to extend the models developed by Desain.

In this paper I have proposed that rhythmic structures may operate to *de-segregate* auditory streams—that is, to create ordered structures between musical parts that contradict the segregation of these parts according to physical location. The

effect of this despatialization is to place musical experiences outside of the range of normal physical experiences of sound. As the auditory space between discrete sounding objects in the physical world appears less defined, so may the space between these objects and the listening self. Under these conditions, music, musicians and the acoustic sense of self may appear to cohabit the same abstract space, a space that is not entirely interior or exterior to the listening individual. Such dissolution of the individual self could be expected to produce powerful emotional responses as sometimes occur when playing or listening to music.

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*After studying the physical sciences, Neil McLachlan decided to pursue his fascination with music and sound by working as a composer and performer with dance and theater companies in Melbourne, Australia. In 1989 he formed a company with dancers, craftspeople and other musicians to collaboratively design and build just-tuned percussion ensembles specifically for movement-based music theater. This work was greatly enhanced by musical structures learned from Indonesian, Indian and West African performance genres. McLachlan is still active in designing musical instrument ensembles and teaches acoustic design at RMIT University.*