on the other hand. Indeed, the first term contains the factor $i^2K_0$ which is the moment of inertia of the system due to the self-field of the particle. That the second term represents the effect of damping can be seen by tracing our course of calculation. One sees then that this term has appeared in connection with the term $i\pi\delta(K-K_0)$ in (1.37). This means that this second term is connected directly with our boundary condition saying that there exist only outgoing waves at infinity. That the second term corresponds to the damping reaction of the field is implied also by the fact that (1.47) has the form of a dispersion formula and thereby this term plays the role of the width of the resonance level.

It is here to be noted that the second term in the denominator of (1.47) is small compared with the first term, so far as $K_0 \ll K$ holds. This means that in such a case the effect of the damping is small compared with the effect of the inertia. This shows in turn that the amplitude of the forced vibration of $\tau$ is small compared with unity, because if it were not the case, the damping reaction would be necessarily large so that the second term would predominate the first. Our assumption $\epsilon \ll 1$ can thus be justified.

In concluding the classical treatment, we shall notice that every result obtained in this paragraph agrees perfectly with the corresponding result obtained quantum-theoretically by means of Wentzel's method. This will be shown in the next paragraph. The physical consequences of our results will be described in the last paragraph.

(to be continued).

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A Note on the Method of Radiation Damping and its Application to the Photo-mesonic Process.*

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1. As has been shown by Heitler, Wilson and others,1) a part of the converging difficulties, which is connected with the special form of the interaction energies, in the calculation of the transition probabilities can be removed by the method of 'radiation damping,' which was first introduced by Waller2) into quantum mechanical calculations. In the scattering processes, the cross sections increase unlimitedly when the interaction energies between the scattering particles and incoming (or outgoing) fields increase with the increasing energies of incoming quanta. But, in such cases the transition probabilities of the inverse processes also tend to infinity according as the probabilities of the scattering processes diverge. And, we can expect that these inverse processes suppress the increasing of the scattering probabilities as in the case of the classical theory of radiation damping. The calculations made by the authors cited above have shown that this is the case for the scattering of mesons by nucleons. But, we think the physical meaning of this method is somewhat obscurely shown in such an example. In this paper, we shall inquire into the reason why this method generally leads to the converging results and illustrate it with the application to the photo-mesonic process where the final state is different in kind from the initial state.

2. Let us consider the transition process between the initial state \( i \) and the final state \( f \), and denote the matrix element connecting these two states by \( H_{if} \). There may be many kinds of states \( f' \) which are connected with the initial state \( i \) by non vanishing matrix elements \( H_{if'} \). One must solve the simultaneous equations for the probability amplitudes of these states in order to calculate the transition probabilities from the initial state to these

* The contents of this paper were reported at the semi-annual meeting of I.P.C.R. held June 1943, but the publication has been postponed on account of the war.

   A. H. Wilson; ibid., 38 (1941), 301.

2) I. Waller: Zeits. ftr Physik, 88 (1934), 436.
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final states. Here, we must take into account all these final states even if whose energies are different from that of the initial one. As is well known, the summation (or integration) of the transition probabilities over these states which are different in energy leads to the approximate conservation of energy. On the other hand, if we demand that the inverse processes shall be treated on an equal footing with the transition processes in order to take the reaction effects into account, the transitions from final states to the states \( i' \) which are connected with final states \( f' \) (including \( f \)) by \( H_{fu} \) and belong to the same class as the initial one but different from it in energy must be involved in the simultaneous equations, and we must perform the summation (or integration) over these states. One will see in the following formulations that this summation just secures the convergence of the transition probability for any process caused by the interaction terms which increase unlimitedly with increasing energy.

When initial and final states are of the same kind as in the case of pure scattering (where the incoming and outgoing particles are of the same kind) which has been picked up by Heitler and Wilson as an example, the distinction between these two kinds of summations disappears, and this circumstance confuses the physical interpretation of the converging effect of the method of radiation damping.

3. Next, we shall formulate this method according to Wilson's paper, but making distinction between two cases mentioned above, i.e. case 1. the initial state is different in kind from the final states and case 2. they are of the same kind.

In case 1, the simultaneous equations for the probability amplitudes are given as follows

\[
-i\hbar \dot{a}_i = \sum_{f'} H_{fi'} a_{f'} e^{i (E_{f'} - E_i) t / \hbar} \\
-i\hbar \dot{a}_\nu = \sum_{f'} H_{f'i} a_{f'} e^{i (E_{f'} - E_\nu) t / \hbar} \\
-i\hbar \dot{a}_{f'} = H_{f'f} a_i e^{i (E_f - E_{f'}) t / \hbar} + \sum_{f''} H_{f'f''} a_{f''} e^{i (E_{f''} - E_{f'}) t / \hbar}
\]

(1)

\[
(H_{fi'} = \tilde{H}_{fi'}, \quad H_{f'f} = \tilde{H}_{f'f})
\]

where \( a_i \), \( a_\nu \) and \( a_{f'} \) are the probability amplitudes of the initial state \( i \) with energy \( E_i \), those of states \( i \) with energies \( E_\nu \) which are of the same kind as the initial state, and those of the different final states \( f' \) (including
with energy $E_n$ respectively, and $\dot{a}_i, \ldots$ denote the differential coefficients of $a_i, \ldots$ with respect to time $t$.

In case $2$, the simultaneous equations (1) reduced to

\[
\begin{align*}
-i\hbar \dot{a}_i &= \sum_{j} H_{ij} a_j e^{(E_j - E_i)it}, \\
-i\hbar \dot{a}_j &= H_{ij} a_j e^{(E_j - E_i)it} + \sum_{j} H_{jj'} a_{j'} e^{(E_{j'} - E_j)it},
\end{align*}
\]

(2)

where $\sum_{j}$ denotes the summation which is taken all over other final states (or, strictly speaking, initial and final states) $j''$ than $j'$. These equations are identical with those obtained by Wilson and others. Here, it must be noted that the appearance of the matrix elements $H_{jj''}$ which connect two final states is due to the accidental circumstance that the initial and final states are of the same kind in this case.

4. When the process takes place through the intermediate states $n, n', \ldots$, we have to comprehend that the matrix elements $H_{ij}$ (or $H_{jj''}$) represent those composed by performing the summation $\sum_{n} \frac{H_{in} H_{jn'}}{E_n - E_n}$.

Then, they are given in the form

\[
H_{jj'} = H_{ij'} \text{(direct)} + \sum_{n} \frac{H_{in} H_{jn'}}{E_n - E_n} + \sum_{n'} \frac{H_{in} H_{in'}}{(E_n - E_n)(E_n - E_{n'})} + \\
\ldots
\]

(3)

For these summations with respect to the intermediate states, we must content ourselves to obtain the first non vanishing terms and omit the diverging summations which correspond to the self-energies of particles concerning in transition processes. Then, as will be shown in the following discussions, in no case does the transition probability diverge as long as the inverse processes are taken into account correctly. This is very satisfactory result, because as is generally believed and discussed by us in other places those diverging terms rising from the fact that the self-forces of elementary particles become infinity when the point models are taken for these particles must be removed entirely from the expression for the energy of any system.

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(3) M. Kobayasi and E. Kanai: Progress of Theoretical Physics, 1 (1946), 21, and other papers shortly published by us in this journal.
5. In order to solve the simultaneous equations (1), we put

\[
\begin{align*}
a_{ii} &= e^{-\frac{1}{2}i\gamma} \\
a_{ii} &= A_{ii} \frac{e^{i(E_f - E_i - \frac{1}{2}i\gamma)/\hbar}}{E_i - E_f + \frac{1}{2}i\gamma} \\
a_{iu} &= A_{iu} \frac{e^{i(E_f - E_i - \frac{1}{2}i\gamma)/\hbar}}{E_i - E_u + \frac{1}{2}i\gamma} \end{align*}
\]

in accordance with initial conditions

\[a_{ii} = 1, \quad a_{iu} = 0 \quad \text{for} \quad i = 0.\] (5)

and substitute these expressions in (1). Then we obtain the equations to determine the unknown quantities \(\Gamma, A_{ii}, \) and \(A_{iu}\) in (4). By replacing the summations with respect to states \(f'\) and \(i'\) by integrals in these equations, and using the well-known formula

\[
\int f(E) \frac{1 - e^{i(E_f - E_i + \frac{1}{2}i\gamma)/\hbar}}{E_i - E_f + \frac{1}{2}i\gamma} dE \approx \text{inf} f(E) \quad \text{(4)}
\]

we get

\[
\frac{1}{2} \Gamma = \frac{\pi}{\hbar} \left[ \int H_{fi} A_{ii} \rho_{fi} d\Omega_{fi} \right]_{E_f = E_i}
\]

\[
A_{ii} = H_{fi} + (i\gamma) \left[ \int H_{fi} A_{ii} \rho_{fi} d\Omega_{fi} \right]_{E_f = E_i}
\]

\[
A_{iu} = i\gamma \left[ \int H_{fi} A_{iu} \rho_{fi} d\Omega_{fi} \right]_{E_f = E_i}
\]

where \(\rho_{fi}\) and \(\rho_{iu}\) represent the densities of final and initial states with respect to energy, and \(\int d\Omega_{fi}\) and \(\int d\Omega_{iu}\) denote the integrals over other freedoms of states than energy, for example the solid angles into which the particles are scattered. For case 2, \(A_{ii}\) is given simply by

\[
A_{ii} = H_{fi} + i\gamma \left[ \int H_{fi} A_{ii} \rho_{fi} d\Omega_{fi} \right]_{E_f = E_i}
\]

in accordance with Wilson's result.

By solving these integral equations for \(A_{ii}\), we get for the transition probabilities or the differential cross-sections for the scattering processes.
For the pure scattering where the incident and scattered particles are of the same kind, we can use the integral equation (7) for $A_J$. But, when the incident particle is different from the outgoing particle, as in the case of photo-mesonic process where the incident photon is absorbed by nuclear particles with subsequent emission of charged meson, we must substitute for $A_J$ the expression which is given by solving the equation (6)

As is easily seen from the form of the integral equations (6) and (7), the results thus obtained always converge in both cases even when matrix elements $H_J$ diverge with increasing energy.

Now, we shall apply above results for the case to the photo-mesonic effect which was already treated by Heitler\(^{(4)}\) and by us\(^{(5)}\) by making use of the ordinary perturbation method, and the results obtained in this way strongly diverge contrary to the cosmic ray evidences. In the previous paper, we have avoided this difficulty by taking recourse to cutting-off prescription. But, when the method of radiation damping is applied the cross-section for this process decreases rapidly and one of the discrepancies between the meson theory and experiments disappears as a reasonable consequence.

First, we adopt the pseudo-scalar wave functions for mesons. Then the matrix element for this transition process is approximately given by

$$H_J = \frac{2\mu f \hbar}{\mu c^3} \frac{\vec{k} \cdot \vec{e}_0}{\sqrt{E_0 E_k}} u(0)(\sigma \cdot \vec{e}_0) u(\vec{k})$$

as long as the mass of nucleon is assumed to be infinite, where $u(\vec{k})$ denote the wave function of the nucleon with momentum $\vec{k}$, $\vec{e}_0$ is the unite vector in the direction of polarization of incident photon and $\sigma$ is the spin operator of nucleon. Further, $f$ is the interaction constant between nucleon and meson field, $\mu$ the meson mass, and $E_0$, $E_k$ are the energies of incident photon and emitted meson respectively. The solutions of the integral equation (6) are given by the form

$$A_J = a u(\vec{k})(\vec{e}_0 \cdot \sigma) u(0),$$

and by making use of equations (6) and (8) we obtain

---


\[ a = \frac{A_i^0}{1 + 32\pi \rho_i \rho_f |A_f^0|^2} \]  
\[ A_i^0 = -2\pi \frac{ef}{\mu^2} \left( \frac{\hbar c}{E_0 E_h} \right)^2, \]  
(11)

and

\[ |A_f^0|^2 = \frac{|A_i^0|^2}{(1 + 32\pi \rho_i \rho_f |A_f^0|^2)^2}, \]  
(12)

where \( \rho_f \sim \rho_i = \frac{\hbar^2}{(2\pi)^2 \hbar c} \), and \( \hbar \left( \frac{\hat{p}_f}{\mu c} \right) \) is the momentum of incident photon. The differential cross-section for the emission of meson in the solid angle \( d\Omega \) is finally given by

\[ d\phi_f = \frac{R^2}{(1 + 2R^2 h^2)^2} d\Omega \]  
(13)

with

\[ R^2 = \left( \frac{\hbar c}{\mu^2} \right)^2 \]

And, the total cross-section for photo-mesonic effect is given by

\[ \phi_f = \frac{4\pi R^2}{(1 + R^2 h^2)^2} \]  
(14)

It is remarkable that this cross-section decreases with increasing energy \( E_0 \) of incident photon as \( 1/E_0^2 \) in the high energy region, whereas in the example belongs to case 2 it decreases as \( 1/E_0^3 \).

By using our formulations, we can also calculate the probability for the transition process from the initial state \( i \) to other states \( i' \) which belong to the same class as the initial one. In the present example, this transition corresponds to the pure scattering of photon by nucleon caused by the interaction between photons and virtually emitted mesons. The cross-section for this process is calculated by making use of \( A_{i'} \) and given by

\[ d\phi_{i'} = \frac{2\pi}{\hbar c} \rho_{i'} A_{i'}^* d\Omega = R^2 f d\phi_f \]  
(15)

and

\[ \phi_{i'} = R^2 \phi_f. \]  
(16)

For the vector meson theory, we can calculate the cross-section for the photo-mesonic process in the similar way to the above case by making use of the matrix elements obtained by us in the previous paper. Here, we omit the detailed expressions for the matrix elements in this case and
write down the results obtained. Though the integral equations can not
be solved exactly in this case, we can estimate the value \( A_j \) by replacing
\( |H_y| \) by the average value \( |\overline{H}_y| \). Then \( A_j \) is given approximately by

\[
A_j = \frac{H_y}{1 + 16\pi \alpha |\overline{H}_y|^2}.
\]

For the cross-section we obtain

\[
\phi_j = \frac{\phi_0}{\left[1 + (\alpha/4\pi)\phi_0 \phi_{\overline{H}_y}^2\right]}, \tag{17}
\]

where \( \phi_0 \) denotes the cross-section obtained by neglecting the radiation
damping, and \( \alpha = 1 \) or 2 according as the direction of polarization of the
emitted meson parallel or perpendicular to its momentum. As has been
shown in the previous paper, \( \phi_0 \) itself increases unlimitedly with the energy
of incident photon, but the expression (17) always converges with increasing
energy \( E_0 \) as \( 1/E_0^4 \). This is a general law which holds for the scattering
processes where the incident and scattered particle are different in kind.

From the above calculations, we can conclude that when the reactions
of fields on the particles are taken into account correctly, the diverging
difficulties concerning with the transition probability for any process dis­
appear as long as the infinite self-energies of elementary particles are laid
asleep.

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