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The Importance of Refreezing on the Diurnal Snowmelt Cycle with Application to a Northern Swedish Catchment

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A method for including night-time refreezing of the top layer of a snowpack in the degree-day method for computing daily snowmelt rates is presented. It is found that during days of large diurnal temperature variations the daily melt is more determined by the day-time conditions than by the daily mean conditions. Applications are made to an open area and a forested area. The refreezing-degree-day method is found to describe the snowmelt process very well, whereas the simple degree-day method shows a too fast snowmelt rate.

Introduction

Daily snowmelt rates are usually calculated with the degree-day method using the daily mean temperature. However, if the temperature over the day deviates several degrees from the daily mean temperature, the effective daily melt rate is different from what it would be, if the temperature is constant at the mean temperature over the whole day.

Run-off can not take place from a snowpack until the snowpack is at irredusible liquid saturation. When a day of snowmelt is succeeded by a night of air temperature below freezing, the liquid water of the upper part of the snowpack refreezes. This part of the snowpack must again be saturated to its irredusible water content, before meltwater can pass from the surface through this layer.

This paper discusses the refreezing of the snowpack and its effect on the run-off. A method of how to extend the degree-day method approach so that the refreezing is accounted for is suggested. Applications are made to a forested area and an open field in northern Sweden.

Calculation of the Refreezing Depth

As long as snowmelt takes place the temperature of the snowpack is at the freezing point. When the air temperature drops below freezing and there is no solar radiation, heat is transported from the snowpack to the atmosphere. The heat flow through the snowpack takes place mainly as pure conduction. Convection may contribute if the wind is strong and the snow has a low density. Some of the heat is released by reduction of the snow temperature, but most of the heat lost from the snowpack is from production of ice. The refreezing process is described by the equation

$$k \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} + \frac{\rho F}{\rho_s c_i} \frac{\partial S'}{\partial t} \tag{1}$$

where

- T – snow temperature at depth z,
- z – distance from the snow surface,
- k – thermal diffusivity of the snow,
- ρ – density of water,
- ρ_s – density of snow,
- F – latent heat of fusion,
- c_i – specific heat capacity of ice,
- S' – liquid water content of the snowpack at depth z.

At a level z one of the terms on the right hand side of Eq. (1) is always zero. Where the free liquid freezes, the temperature is at the freezing point, and after the liquid has frozen the only contribution from the right hand side of the equation to the heat flow is due to reduction of snow temperature. It is also clear that there can be no transport of heat from below from the snow at 0°C through the downward penetrating freezing front. When Eq. (1) is integrated over the refreezing depth, the penetrating rate of the freezing front is obtained as

$$\frac{\rho F}{\rho_s c_i} S \frac{\partial z_f}{\partial t} = \int_0^{z_t} \frac{\partial T}{\partial t} dz + k \left(\frac{\partial T}{\partial z} \right)_{z=0} \tag{2}$$

where

- S – irredusible liquid content of the snow,
- z_f – depth of freezing front.

The boundary value at the surface, which is the outgoing heat flux, is determined from the temperature gradient. The parameter a is introduced as

$$a = \frac{\rho F}{\rho_s c_i} S \tag{3}$$

The penetration of the freezing front is shown in Fig. 1. By determining separately

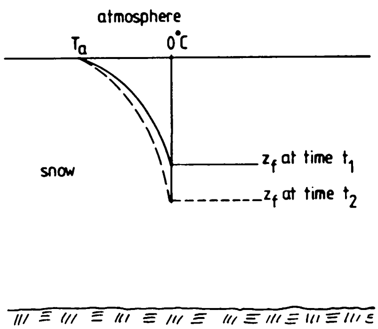

Fig. 1. Development of temperature profile in the snowpack during the downward penetration of the freezing front.

the heat loss due to the development of the temperature profile, the "stretching" of the temperature profile when the freezing front progresses downward, and the ice production at the freezing front, the freezing depth is found from a Fourier series expansion of Eq. (2) as

$$\frac{\partial z_f^2}{\partial t} = \frac{2k(-T_a)\{1+\sum_1^\infty e_n(1+(-1)^n)\}}{a-\frac{T_a}{2}+T_a\sum_1^\infty \frac{2}{n^2\pi^2}e_n(1-(-1)^n)} \tag{4a}$$

where

$$e_n = \exp(-n^2\pi^2\frac{kt}{z_f^2}) \tag{4b}$$

and T_a – the snow surface temperature, which is assumed to equal the air temperature.

The thermal diffusivity is typically 0.002-0.006 cm^2/sec, and the time, during which the heat flux is directed out of the snowpack, is at least several hours. Even for such short time as 6 hours and as large depths as 200 mm only terms of $n=1$ are of importance. Eq. (4a) is then

$$\frac{\partial z_f^2}{\partial t} = \frac{2k(-T_a)}{a-\frac{T_a}{2}+\frac{4}{\pi^2}T_a e_1} \tag{4c}$$

When t is 12 hours and z_f is less than 200 mm, the last term in the denominator is less than 1% of the second term in the denominator. This means that the penetration rate of the freezing front approximately can be obtained from a linear temperature distribution assumption. For small negative T_a Eq. (4) is further simplified to

$$\frac{\partial z_f^2}{\partial t} = \frac{2k}{a}(-T_a) \tag{4d}$$

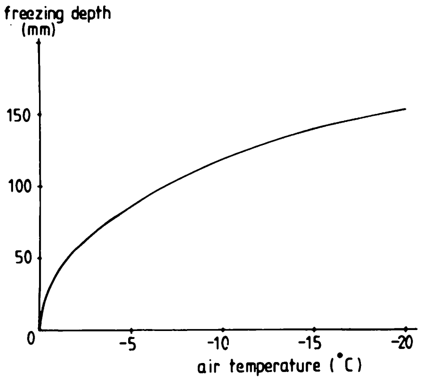

Fig. 2. Freezing depth after 12 hours of negative air temperatur as shown on the axis. Thermal diffusivity 0.004 cm^2/sec, water holding capacity 0.04 by volume, snow density 300 kg/m^3.

Eq. (4) shows that the refreezing depth over a fixed time is proportional to the square root of the negative air temperature for temperatures close to the freezing point and approaches a limiting value for very low air temperatures. The refreezing depth over 12 hours is shown in Fig. 2 as a function of air temperature.

The negative energy in the snowpack between the snow surface and the freezing front is called the cold content. It is determined by a Fourier series expansion of Eq. (1) followed by an integration over depth z_f as

$$CC = \rho_s c_i (-T_a) z_f \{0.5 - \sum_{1}^{\infty} \frac{2}{n^2 \pi^2} e_n (1-(-1)^n)\} \tag{5}$$

where CC is cold content and e_n is explained in Eq. (4b). As before it suffices to consider the linear part of Eq. (5).

The Effect of Refreezing on the Run-off

A cold night is followed by daytime hours of snowmelt. The snow surface adjusts fast to the atmospheric conditions, the surface temperature increases to the freezing point, and snowmelt starts at the surface. The first apparent snowmelt is needed to overcome the cold content of the snowpack. It is

$$m_{CC} = \frac{CC}{\rho F} \tag{6}$$

Next, or partly simultaneously with the decreasing of the cold content, the liquid content of the snowpack must be increased to the irreducible value. The amount of meltwater needed for this is

$$m_{WH} = S z_f \tag{7}$$

The apparent meltwater loss in the snowpack is then, using the linearized form of Eq. (5),

$$m_{-} = m_{WH} + m_{CC} = S \frac{z_f(1+(-T_a)}{a} \tag{8}$$

where T_a is the air temperature °C of the previous period.

Not until the snowpack is at its irreducible liquid content can the vertical transport of water through the snowpack to the ground begin. The percolation of meltwater through homogeneous unsaturated snow was described by Colbeck (1972) by application of Darcian flow. Since the permeability available to the liquid phase increases rapidly with increasing water saturation, intense meltwater flux moves faster than low flux. When reasonable coefficients are inserted into the formula suggested by Colbeck for meltwater wave propagation, it is found that the time necessary for a meltwater wave to travel through a snowpack of 0.5 m, which previously has been affected by snowmelt, is less than one hour. This is also supported by measurements by Colbeck and Davidsson (1973) and by recent measurements at WREL. The observations at WREL have also shown that the vertical transport of meltwater usually takes place through vertical draines with diameter about 0.5 m spaced at an equidistance of about 1 m. It can be concluded that once a snowpack has been affected by snowmelt, the vertical transport of meltwater is fairly fast.

The rate at which meltwater reaches the base of the snowpack usually differs from the meltrate at the surface and may exceed the surface melt rate. Once the meltwater has reached the base, the run-off continues at a rate, which is not dependent on previous refreezing conditions. The amount of water that leaves a snowpack initially at irreducible saturation during a diurnal snowmelt cycle, exceeds the amount of surface melt by a relative value corresponding to the irreducible liquid content by weight, which may be about 10-15%.

The Effect of Temperature Variations over the Day on the Daily Total Melt

When snowmelt is calculated by the degree-day method

$$m' = C\,T_a \tag{9}$$

where

m' – melt rate,

C – degree-day coefficient,

T_a – mean positive air temperature in °C over a period (usually one day),

calculations show the same mean melt rate, if the temperature is constant over the day, as if the temperature varies within a large range, but has the same mean value. As can be seen from Fig. 2 or Eq. (4) the refreezing depth does not, however, increase linearly with negative air temperature and is not much affected

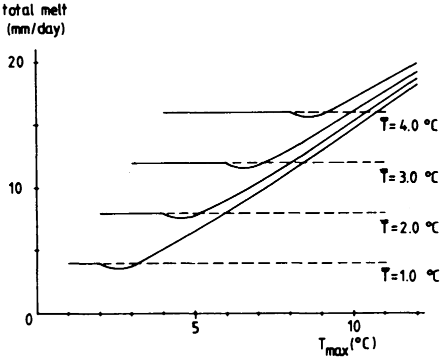

Fig. 3. Total daily melt calculated with the refreezing-degree-day method for days of equal daily mean temperature versus daily maximum temperature (constant over 12 hours), C=mm/°C-day, ϱ_s/ϱ=0.3, k=0.004 cm^2/sec, S=0.05. The dashed lines show the results of a straightforward degree-day approach.

by very low air temperatures, whereas the melt rate increases linearly with positive air temperature. Consequently, large temperature variations give rise to more snowmelt than what small air temperature variations with the same mean daily temperature do.

Fig. 3 shows the total melt over 24 hours for different daily mean temperatures and different temperature variations. The depth of refreezing of a top layer at irreducible saturation exposed to 12 hours of constant negative air temperature has been determined from Eq. (4). The liquid water that can be added and still held within the snowpack, is the irreducible liquid content by volume times the refreezing depth. This amount of water and the meltwater needed for overcoming the cold content are subtracted from the surface snowmelt determined for 12 hours of positive constant air temperature by the degree-day Eq. (9) to find the true amount of melt produced over a day. It is interesting to find that for high maximum air temperatures the total melt is more related to the maximum temperature than to the mean daily temperature.

It is shown in Fig. 4 that the degree-day method applied on the mean daily temperature determines the daily snowmelt quite accurately, if the degree-day coefficent has a value of 2 mm/°C-day, which is valid for forested areas, c.f. Bengtsson (1981). For higher degree-day coefficients and high daily maximum temperatures the degree-day method applied on the mean daily temperature underestimates the total daily melt. This could be accounted for by increasing the coefficient, but then the melt rate at small daily temperature variations would subsequently be overestimated.

Summarizing the Approach

Computations are performed on a 12 hour basis. When the air temperature is positive Eq. (9) is used for determining the surface melt rate.

Not until the entire snowpack is above irreducible saturation can run-off start.

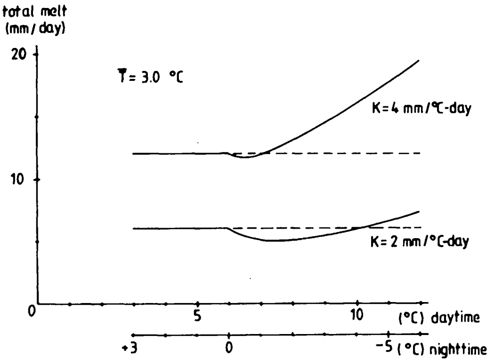

Fig. 4. Total daily melt computed with the refreezing-degree-day method for a day of mean temperature +3.0°C for different daytime and nighttime temperature of the day. $\varrho_s/\varrho=0.3$, $k=0.004$ cm²/sec, $S=0.05$. The dashed lines show the results of simple degree-day computations.

When the air temperature is below freezing Eq. (4) is used for determining the refreezing depth. When again the air temperature is positive, the time for the top layer to reach irreducible water saturation is calculated for determining the run-off. The amount of water necessary for raising the liquid content of the snowpack to its irreducible content and the apparent amount of meltwater needed for over-coming the cold content of the snowpack, together determined from Eq. (8), are subtracted from the calculated surface melt.

The suggested approach can be used only, if the coefficients of Eqs. (3), (4), and (9) are known. The degree-day coefficient C varies especially with solar radiation and state of the snow. It can be shown, c.f. Bengtsson (1976), that it is almost a true constant for forested areas but varies for open areas. Typical degree-day coefficients for open areas in Sweden are 3 mm/°C-day in the southern parts and 4-5 mm/°C-day in the north. The problem of choosing appropriate values of the degree-day coefficient is, however, a problem also in the traditional degree-day method.

The irreducible liquid content is during snowmelt periods 4-5% by volume as found by Lemmelä (1973), by investigations at WREL, or as stated in Snow Hydrology (1956). For initially dry snow this irreducible value is higher.

The thermal diffusivity of snow of relative density about 0.3, which is typical for the melting period in northern Sweden, is according to Snow Hydrology and Yen (1969) in the range 0.002-0.006 cm²/sec, which is consistent with Swedish hand-books. For high wind speeds the heat flux in the top surface layer is affected by the wind, thereby increasing the effective thermal diffusivity. This effect is more pronounced for fresh snow, where the air-filled pore space is 80-90%, but may be significant also for snow which has been affected by snowmelt. From measure-ments of Reimer (1980), which seem to be for snow of relative density 0.20-0.24, the effective thermal diffusivity was found to be unaffected by the wind until the wind speed exceeded about 4-5 m/sec, when it increased by almost one order of magnitude. Preliminary results from measurements at WREL show that the wind effect is considerably less on snow of relative density 0.3. Still, very close to the snow surface the effective thermal diffusivity is higher in nature than what is reported from laboratory measurements, unless the convection is explicitly accounted for.

Application to the Bensbyn Catchment Area

The suggested method of computing snowmelt on a 12 hour basis was applied for 1980 to a forested area and an open area of the Bensbyn catchment outside the city of Luleå in northern Sweden. The degree-day coefficient 4 mm/°C-day was used for the open field and 2 mm/°C-day for the forest. The water holding capacity was assumed to be 0.05 by volume for ripe snow and 0.07 for the initially dry snow. The thermal diffusivity was taken as 0.004 cm^2/sec through the entire snowpack. The air temperature was assumed to be constant over the daytime and the nighttime hours, respectively. The results of the computations are shown in Tables 1 and 2. Note that there may be many frozen layers interchanged with layers at irreducible liquid saturation.

The calculated and measured ablation of the snow cover is shown for the forested area in Fig. 5 and for the open area in Fig. 6. Also the disappearance of the snow cover calculated with the degree-day method based on the mean daily temperature is shown. These degree-day calculations have been somewhat improved compared to the usual degree-day approach, since the time for initially raising the snowpack to irreducible liquid saturation has been accounted for.

The snow water equivalents calculated by the refreezing-degree-day approach for the forest agree very well with observations, whereas the simple degree-day

Table 1 – Computed snowmelt, run-off to the base of the snowpack, and refreezing depth on a 12 hours basis for the snowpack in *the forested area* of Bensbyn, 1980. – Depth of top layer saturated to its irreducible value = h_w, refreezing depth = h_f, depth of snowpack = h_s (initially 430 mm). Irreducible water content of initially dry snow 0.07, of ripe snow 0.05, thermal diffusivity 0.004 cm^2/sec, relative snow density 0.3, degree-day coefficient 2 mm/°C-day.

date		temp. °C	surface melt mm	depth mm	run-off period hours	run-off intensity mm/day	run-off mm
30 March	day	2.5	2.5	h_w = 35			
	night	0.0					
31	d	1.0	1.0	h_w = 50			
	n	0.0					
1 April	d	1.0	1.0	h_w = 64			
	n	−1.2		h_f = 46			
2	d	1.3	1.3	h_w = 26			
	n	−0.5		h_f = 47			
3	d	2.4	2.4	h_w = 64			
	n	−1.5		h_f = 50			
4	d	3.3	3.3	h_w = 75			
	n	−8.0		h_f = h_s			
5	d	8.0	8.0	h_w = 135			
	n	−2.5		h_f = 64			

cont.

date		temp. °C	surface melt mm	depth mm	run-off period hours	run-off intensity mm/day	run-off mm
6	d	10.7	10.7	h_w = 242			
	n	1.7	1.7	h_w = 276			
7	d	7.9	7.9	h_w = 388			
	n	-0.3		h_f = 23			
8	d	2.9	2.9	h_w = h_s	–	–	–
	n	-3.4		h_f = 74			
9	d	2.0	2.0	h_w = 40			
	n	-11.0		h_f = 123	10 mm snow precipitation		
10	d	0.9	0.9	h_w = 5			
	n	-5.6		h_f = 159			
11	d	5.2	5.2	h_w = 104			
	n	-8.6		h_f = 164			
12	d	7.5	7.5	h_w = 150			
	n	-3.4		h_f = 74			
13	d	6.7	6.7	h_w = h_s	15-19	15.7	2.33
	n	-1.7		h_f = 55			
14	d	5.0	5.0		14-19	11.7	2.52
	n	-2.4		h_f = 64			
15	d	7.8	7.8		12-19	18.3	5.20
	n	-6.3		h_f = 98			
16	d	5.9	5.9		18-19	13.8	0.45
	n	-0.7		h_f = 35			
17	d	0.0					
	n	-3.8		h_f = 85			
18	d	1.4	1.4	h_w = 21			
	n	-0.4		h_f = 87			
19	d	4.5	4.5	h_w = h_s	–	–	–
	n	-1.7		h_f = 53			
20	d	5.3	5.3		13-19	12.4	3.00
	n	-3.2		h_f = 73			
21	d	7.2	7.2		13-19	16.9	3.88
	n	-3.3		h_f = 75			
22	d	7.3	7.3		13-19	17.1	3.86
	n	-2.7		h_f = 67			
23	d	5.8	5.8		14-19	13.6	2.65
	n	-0.3		h_f = 23			
24	d	7.9	7.9		9-19	18.4	7.90
	n	-0.8		h_f = 38			
25	d	6.7	6.7		10-19	15.7	5.62
	n	-7.0		h_f = 100			
26	d	9.0	9.0		15-19	21.0	3.86
	n	-4.9		h_f = 86			
27	d	9.8	9.8		13-19	22.9	5.92
	n	0.6	0.6		19- 7	1.4	0.70
28	d	3.0	3.0		7-19	7.0	3.51
	n	1.1	1.1		19- 7	2.6	1.29
29	d	5.1	5.1		7-19	12.0	5.97
	n	-1.2		h_f = 45			
30	d	10.5	10.5		10-19	24.6	9.59
1 May	d	7.6	7.6		7-19	17.8	8.89
	n	-1.2		h_f = 45			
2	d	8.9	8.9		10-19	23.2	7.72
	n	0.6	0.6		19- 7	1.4	0.70
3	d	8.1	8.1		7-19	18.9	9.48
	n	-2.6		h_f = 66			
4	d	13.6	13.6		10-19	31.8	11.84
	n	-2.5		h_f = 65			
5	d	11.0	11.0		11-19	25.7	8.89
	n	-0.9		h_f = 40			
6	d	11.0	11.0		9-19	25.7	10.54
	n	-4.6		h_f=h_s= 10			
7	d	11.6			8-11	27.1	3.00

Table 2 – Computed snowmelt, run-off to the base of the snowpack, and refreezing depth on a 12 hour basis for the snowpack in *the open area* of Bensbyn 1980. – Depth of top layer saturated to its irreducible value = h_w, refreezing depth = h_f, depth of snowpack = h_s (inititally 530 mm). Irreducible water content of initially dry snow 0.07, of ripe snow 0.05, thermal diffusivity 0.004 cm^2/sec, relative snow density 0.3, degree-day coefficient 4 mm/°C-day.

date	temp. °C	surface melt mm	depth mm	run-off period hours	run-off intensity mm/day	run-off mm	
30 March	day	2.5	5.0	$h_w = 71$			
	night	0.0					
31	d	1.0	2.0	$h_w = 100$			
	n	0.0					
1 April	d	1.0	2.0	$h_w = 129$			
	n	-1.2		$h_f = 46$			
2	d	1.3	2.6	$h_w = 133$			
	n	-0.5		$h_f = 29$			
3	d	2.4	4.8	$h_w = 180$			
	n	-1.5		$h_f = 50$			
4	d	3.3	6.6	$h_w = 240$			
	n	-8.0		$h_f = 107$			
5	d	8.0	16.0	$h_w = 392$			
	n	-2.5		$h_f = 64$			
6	d	10.7	21.4	$h_w = h_s$	14-19	50.0	9.75
	n	1.7	3.4		19- 7	8.0	3.98
7	d	7.9	15.8		7-19		18.49
	n	-0.3		$h_f = 23$			
8	d	2.9	5.8		9-19	13.6	5.44
	n	-3.4		$h_f = 74$			
9	d	2.0	4.0	$h_w = h_s$	–	–	–
	n	-11.0		$h_f = 123$	18 mm snow precipitation		
10	d	0.9	1.8	$h_w = 9$			
	n	-5.6		$h_f = 158$			
11	d	5.2	10.4		18-19	24.3	0.50
	n	-8.6		$h_f = 108$			
12	d	7.5	15.0		12-19	35.1	10.15
	n	-3.4		$h_f = 74$			
13	d	6.7	13.4		11-19	31.4	11.05
	n	-1.7		$h_f = 55$			
14	d	5.0	10.0		10-19	23.4	8.37
	n	-2.4		$h_f = 64$			
15	d	7.8	15.6		10-19	36.5	14.36
	n	-6.3		$h_f = 98$			
16	d	5.9	11.8		13-19	27.6	7.36
	n	-0.7		$h_f = 35$			
17	d	0.0					
	n	-3.8		$h_f = 85$			
18	d	1.4	2.8	$h_w = 50$			
	n	-0.4		$h_f = 26$			
19	d	4.5	9.0		11-19	21.1	6.96
	n	-1.7		$h_f = 53$			
20	d	5.3	10.6		10-19	24.8	9.17
	n	-3.2		$h_f = 73$			
21	d	7.2	14.4		10-19	33.7	12.28
	n	-3.3		$h_f = 75$			
22	d	7.3	14.6		10-19	34.2	12.40
	n	-2.7		$h_f = 67$			
23	d	5.8	11.6		11-19	27.1	9.42
	n	-0.3		$h_f = 23$			
24	d	7.9	15.8		8-19	37.0	17.14
	n	-0.8		$h_f = 38$			
25	d	6.7	13.4		9-19	31.4	13.46
	n	-7.0		$h_f = h_s = 28$			
26	d	9.0	18.0		8-13	42.1	7.60

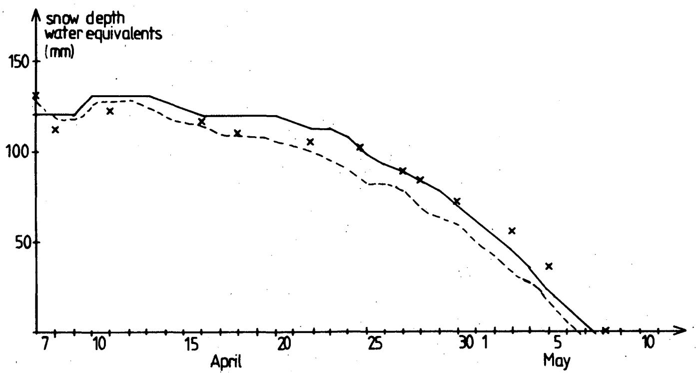

Fig. 5. Snow depth as water equivalents April 1980 for the forested area of the Bensbyn catchment. —— = refreezing-degree-day method. S=5%, k=0.004 cm^2 sec, ϱ_s/ϱ=0.3, C=2mm/°C-day; ---- = degree-day method with account taken of the water holding capacity of the snow; X=observations.

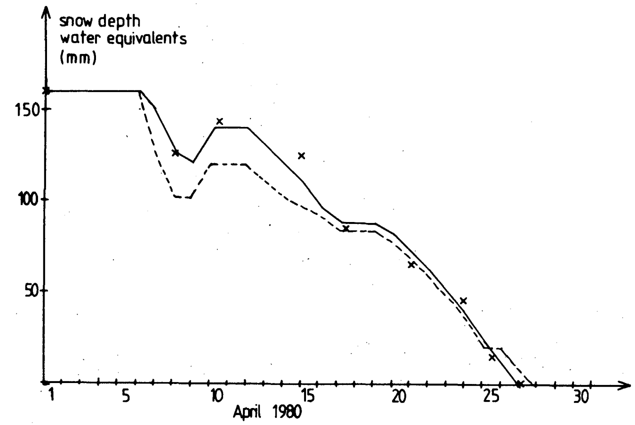

Fig. 6. Snow depth as water equivalents April 1980 for the open area of the Bensbyn catchment. —— = refreezing-degree-day method. S=5%, k=0.004 cm^2/sec, ϱ_s/ϱ=0.3, C=4mm/°C-day; ---- = degree-day method with account taken of the water holding capacity of the snow; X=observations.

method gives a too rapid snowmelt, although account is taken of the initial melting of the snowpack. Also for the open field calculations by the refreezing-degree-day method agree with observations of snow depth expressed as water equivalents except maybe for one occasion. The snowmelt rate calculated with the simple degree-day method is too fast in the beginning of the snowmelt period and too slow during the end of the snowmelt period. The only snow precipitation during the melt period fell on April 9 and was estimated from the snow surveys.

Conclusions

Refreezing of the top layer of the snowpack is an important part of the diurnal snowmelt process. For days with considerable temperature variations the total snowmelt is not calculated correctly with the degree-day method, unless the refreezing during the night is accounted for. For such days the daytime conditions more than the mean conditions over the whole day determine the snowmelt.

The suggested approach of handling day-and-nighttime separately and including refreezing in the degree-day method should be easy to incooperate into conceptual mathematical run-off models.

References

Bengtsson, L. (1976) Snowmelt estimated from energy budget studies. *Nordic Hydrology, 7,* pp. 3-18.

Bengtsson, L. (1981) Snowmelt generated run-off in urban areas. Proc. 2:nd Int. Conf. Urban,storm Drainage, Urbana, Illinois, pp. 444-451.

Colbeck, S. C. (1972) A theory of water percolation in snow. *J. Glaciol., 11,* pp. 369-385.

Colbeck, S. C., and Davidsson, G. (1973) Water percolation through homogeneous snow. Proc. Banff Symp. 1972, pp. 242-257. UNESCO-WMO-IASH publ. The role of snow in hydrology.

Lemmelä, R. (1973) Measurements of evaporation-condensation and melting from a snow cover. Proc. Banff Symp. 1972, pp. 670-679. UNESCO-WMO-IASH publ. The role of snow in hydrology.

Reimer, A. (1980) The effect of wind on heat transfer in snow. *J. Cold Regions Science and Technology, 3,* pp. 129-137.

Snow Hydrology (1956). Summary report of the snow investigations by U.S. Army Corps of Engl., North Pacific Div., Portland, Oregon.

Yen, Y. C. (1969) Recent studies on snow properties. *Advances in Hydroscience, 5,* pp. 173-214.

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