Nordic Hydrology, 23, 1992, 1-12

Uniform Horizontal Groundwater Flow against Dispersion in a Shallow Aquifer: Two Analytical Models

Ram Raj Vinda, Raja Ram Yadava and Naveen Kumar

Dept. of Mathematics, Faculty of Science,
Banaras Hindu Univ., Varanasi – 221005, India

Analytical solutions converging rapidly at large and small values of times have been obtained for two mathematical models which describe the concentration distribution of a non reactive pollutant from a point source against the flow in a horizontal cross-section of a finite saturated shallow aquifer possessing uniform horizontal groundwater flow. Zero concentration or the conditions in which the flux across the extreme boundaries are proportional to the respective flow components are applied. The effects of flow and dispersion on concentration distribution are also discussed.

Introduction

Groundwater pollution may be defined as the artificially induced degradation of natural groundwater quality. The sources and causes of groundwater pollution are closely associated with human use of water. With the growing recognition of the importance of groundwater resources, efforts are increasing to prevent, reduce, and eliminate groundwater pollution. In contrast to surface water pollution, subsurface pollution is difficult to detect, is even more difficult to control, and may persist for decades. These efforts can be mobilised by predicting the attenuation of possible pollution with time and with distance travelled away from the source as the pollutant is transported along the flow till a harmless or very low concentration is reached. Such predictions can be made with the help of analytical or numerical treatment of the particular dispersion problem mathematically modelled. But the lack of sufficient data available for mathematicians and engineers to justify the use of spatially varied aquifer properties, such as porosity, dispersion coefficients, and others, results in the use of constant parameter values. In such cases the problems are studied by an appropriate analytical model. Analytical results are valuable in checking the complicated numerical simulations based on finite difference and finite element methods.

Analytical solutions in one-dimensional analysis to quantitatively describe the dispersive solute transport along steady/unsteady flow through infinite/semi-infinite or finite homogeneous saturated porous media have been presented by several researchers. A non-exhaustive list of references must include the works of Bastian and Lapidus (1956), Banks and Jerasate (1962), Rumer (1962), Banks and Ali (1964), Ogata (1970), Marino (1974), VanGenuchten (1981), Kumar (1983), and Yadava *et al.* (1990). Most of these works have included the effects due to adsorption, first order radio-active decay and/or chemical reactions. Extending to more than one space dimension, useful analytical solutions of the solute dispersion problems through semi-infinite or infinite porous media assuming concentration at infinity being zero and a step function or of varying source concentration being introduced at the origin, have been investigated (Harleman and Rumer 1963; Bruce and Street 1966; Shen 1976; Hunt 1978; Carnahan and Remer 1984; Latinopoulos *et al.* 1988). But in all the two or three dimensional studies only axial or longitudinal flow has been considered. Thus the partial differential equations of parabolic type describing the concentration distribution though contain two/three component terms for solute transport due to dispersion, yet only one term (longitudinal) for that due to convection exists, neglecting the other components.

This paper is concerned with the analytical study of two mathematical models describing control of dispersion in a shallow saturated finite homogeneous aquifer, by two-dimensional uniform horizontal flow towards the region of higher concentration. Such situation often occurs in practice when poor quality water is prevented from spreading by a flow of fresh water. In a shallow aquifer lateral component of the groundwater is also significant, which is negligible in an aquifer of substantial depth. In the first model, the concentration at the origin of the porous medium is zero, while in the second one, the conditions which allow some concentration to be transported at such boundaries as well as at the origin, are imposed. Restricted to one space dimension steady/unsteady flow against dispersion in finite porous media has been outlined by Al-Niami and Rushton (1977), Marino (1978), and Kumar (1983).

Mathematical Formulation and Analytical Solution

We consider a horizontal, saturated, homogeneous, and finite porous medium, possessing uniform two-dimensional flow (Fig.1). The direction of the flow is towards the vertical, parallel to the z-axis (along the depth) through $x \equiv L$ and $y \equiv H$. This vertical axis is the source of non-reactive pollutant of uniform strength.

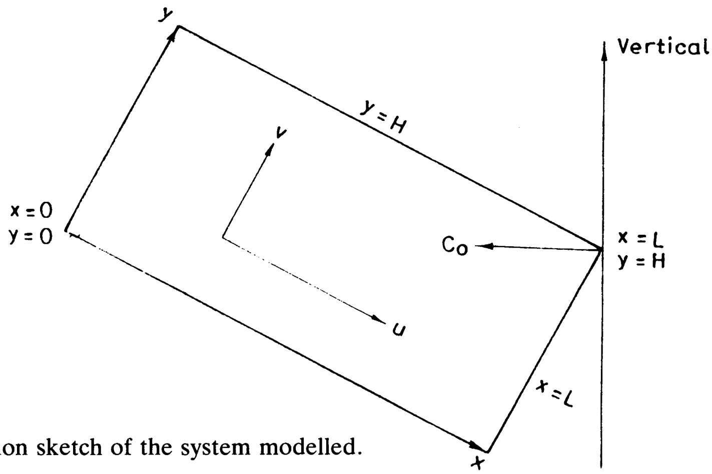

Fig. 1. Definition sketch of the system modelled.

The direction of the flow is towards this axis of source concentration. Let C_0 be the source concentration entrained into a horizontal cross-section of the porous medium through its point of intersection with the vertical axis. The two flow components be u and v along x and y axes respectively. The dispersion coefficients are denoted by D_x and D_y along the two axes respectively. Initially the groundwater is supposed to be solute free.

Case – 1: Concentration at the origin $x = 0$, $y = 0$ is held at zero.

This type of the dispersion problem can be defined mathematically by the partial differential equation describing concentration distribution in the horizontal cross-section, along with initial and boundary conditions as follows

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} - u \frac{\partial C}{\partial x} - v \frac{\partial C}{\partial y} \qquad (1)$$

$$C = 0; \quad x > 0, \quad y > 0, \quad t = 0 \qquad (2)$$

$$C = C_0; \quad x = L, \quad y = H, \quad t > 0 \qquad (3)$$

$$C = 0; \quad x = 0, \quad y = 0, \quad t \geq 0 \qquad (4)$$

Using the transformation

$$C(x,y,t) = K(x,y,t) \exp[h_1 x + h_2 y - \alpha t] \qquad (5)$$

where $h_1 = u/2D_x$, $h_2 = v/2D_y$, $\alpha = D_x h_1^2 + D_y h_2^2$ on Eqs. (1) - (4) and then applying the Laplace transformation on the resulting equations, we can get

$$D_x \frac{\partial^2 \overline{K}}{\partial x^2} + D_y \frac{\partial^2 \overline{K}}{\partial y^2} = p\overline{K} \qquad (6)$$

$$\overline{K} = \frac{C_0}{p-\alpha} \exp -(h_1 L + h_2 H) \; ; \quad x = L, \quad y = H \qquad (7)$$

$$\overline{K} = 0; \quad x = 0, \quad y = 0 \qquad (8)$$

where

$$\overline{K} = \int_0^\infty K(x,y,t)\, e^{-pt}\, dt$$

Introducing a new space variable

$$\eta = x + y\left(\frac{D_x}{D_y}\right)^{\frac{1}{2}} \tag{9}$$

the partial differential Eq. (6) is reduced to ordinary one

$$\frac{d^2\overline{K}}{d\eta^2} - \frac{p}{2D_x}\overline{K} = 0 \tag{10}$$

and the conditions Eq. (7) and Eq. (8) convert to the form

$$\overline{K} = \frac{C_0}{p-\alpha}\exp-(h_1L+h_2H) \quad \text{at} \quad \eta = L + H\left(\frac{D_x}{D_y}\right)^{\frac{1}{2}} \tag{11}$$

$$\overline{K} \equiv 0 \quad \text{at} \quad \eta = 0 \tag{12}$$

The solution of above boundary value problem can be obtained as

$$\overline{K}(x,y,p) \equiv \frac{C_0}{p-\alpha}\exp-(h_1L+h_2H)\,\frac{\sinh M\eta}{\sinh M\xi} \tag{13}$$

where $M = (p/2D_x)^{1/2}$ and $\xi = L+H(D_x/D_y)^{1/2}$, and is obtained by substituting $x = L$ and $y = H$ in Eq. (9).

To complete the solution for concentration $C(x,y,t)$, the inverse Laplace transformation of Eq. (13) is found and using back the transformation Eq. (5), we get

$$\frac{C}{C_0} \equiv \exp[h_1(x-L) + h_2(y-H)]\left[\frac{\sinh \eta(\alpha/2D_x)^{\frac{1}{2}}}{\sinh \xi(\alpha/2D_x)^{\frac{1}{2}}} + 2\pi\sum_{n=1}^{\infty}\frac{n(-1)^n \sin(n\pi\eta/\xi)}{n^2\pi^2+\alpha\,\xi^2/2D_x}\exp-\left(\alpha+2D_x\frac{n^2\pi^2}{\xi^2}\right)t\right] \tag{14}$$

where h_1,h_2 and α are defined in the transformation Eq. (5).

The solution Eq. (14) converges rapidly for large values of time. The solution converging rapidly for small values of time can be obtained by rewriting the solution Eq. (13) in the exponential form (Carslaw and Jaeger 1986, Ch. XII) as

$$\overline{K}(x,y,p) = \frac{C_0}{p-\alpha}\exp-(h_1L+h_2H)\sum_{n=0}^{\infty}[\exp-N_1M - \exp-N_2M] \tag{15}$$

where $N_1 = (2n+1)\xi-\eta$ and $N_2 = (2n+1)\xi+\eta$

Taking the inverse Laplace transform of Eq. (15) and using the transformation Eq. (5) we get the required solution for $C(x,y,t)$ as

$$\frac{C}{C_0} = \frac{1}{2}\exp[h_1(x-L) + h_2(y-H)] \times$$
$$\sum_{n=0}^{\infty}[\exp-N_1\left(\frac{\alpha}{2D_x}\right)^{\frac{1}{2}} \operatorname{erfc}\left\{\frac{N_1}{2(2D_xt)^{\frac{1}{2}}} - (\alpha t)^{\frac{1}{2}}\right\} +$$
$$\exp N_1\left(\frac{\alpha}{2D_x}\right)^{\frac{1}{2}} \operatorname{erfc}\left\{\frac{N_1}{2(2D_xt)^{\frac{1}{2}}} + (\alpha t)^{\frac{1}{2}}\right\} -$$
$$\exp-N_2\left(\frac{\alpha}{2D_x}\right)^{\frac{1}{2}} \operatorname{erfc}\left\{\frac{N_2}{2(2D_xt)^{\frac{1}{2}}} - (\alpha t)^{\frac{1}{2}}\right\} -$$
$$\exp N_2\left(\frac{\alpha}{2D_x}\right)^{\frac{1}{2}} \operatorname{erfc}\left\{\frac{N_2}{2(2D_xt)^{\frac{1}{2}}} + (\alpha t)^{\frac{1}{2}}\right\}] \qquad (16)$$

Case 2: Changes in concentration at the boundaries $x = 0$ and $y = 0$ being proportional to the flow components u and v respectively

The boundary condition that concentration at the origin $x = 0$, $y = 0$, of the porous medium is held at zero may not occur at all times in practice. Alternative conditions which allow some concentration to be transported after long times at the origin are given by two conditions

$$\frac{\partial C}{\partial x} = \frac{u}{2D_x}C; \quad x = 0, \quad y > 0, \quad t \geq 0 \qquad (17)$$

$$\frac{\partial C}{\partial y} \equiv \frac{v}{2D_y}C; \quad x > 0, \quad y = 0, \quad t \geq 0 \qquad (18)$$

Thus for the second type of dispersion problem using the above two conditions in place of Eq. (4), we can obtain the solution

$$\overline{K}(x,y,p) = \frac{C_0}{p-\alpha}\exp-(h_1L+h_2H)\frac{\cosh M\eta}{\cosh M\xi} \qquad (19)$$

where M and ξ are defined similarly as in Eq. (13). Applying inverse Laplace transformation on Eq. (19) and simplifying after required substitutions, the concentration distribution $C(x,y,t)$ converging rapidly for large values of time, can be obtained as

$$\frac{C}{C_0} = \exp[h_1(x-L)+\tilde{h}_2(y-H)]\left[\frac{\cosh\eta(\alpha/2D_x)^{\frac{1}{2}}}{\cosh\xi(\alpha/2D_x)^{\frac{1}{2}}} -\right.$$
$$2\pi\sum_{n\equiv 0}^{\infty}\frac{(n+\frac{1}{2})(-1)^n\cos\{(n+\frac{1}{2})\pi\eta/\xi\}}{(n+\frac{1}{2})^2\pi^2+\alpha\,\xi^2/2D_x} \times$$
$$\left.\exp-\{\alpha+2D_x(n+\tfrac{1}{2})^2\pi^2/\xi^2\}t\right] \qquad (20)$$

To ensure the solution to converge rapidly for small values of time Eq. (19) can be rewritten as

$$\bar{K}(x,y,p) = \frac{C_0}{p-\alpha}\exp-(h_1L+\tilde{h}_2H)\sum_{n=0}^{\infty}(-1)^N[\exp-MN_1+\exp-MN_2] \quad (21)$$

where N_1 and N_2 are defined in the solution Eq. (15). The required solution can be obtained as

$$\frac{C}{C_0} = \frac{1}{2}\exp[h_1(x-L)+h_2(y-H)]\sum_{n=0}^{\infty}(-1)^n \times$$
$$[\exp-N_1\left(\frac{\alpha}{2D_x}\right)^{\frac{1}{2}}\operatorname{erfc}\left\{\frac{N_1}{2(2D_xt)^{\frac{1}{2}}}-(\alpha t)^{\frac{1}{2}}\right\}+$$
$$\exp N_1\left(\frac{\alpha}{2D_x}\right)^{\frac{1}{2}}\operatorname{erfc}\left\{\frac{N_1}{2(2D_xt)^{\frac{1}{2}}}+(\alpha t)^{\frac{1}{2}}\right\}-$$
$$\exp-N_2\left(\frac{\alpha}{2D_x}\right)^{\frac{1}{2}}\operatorname{erfc}\left\{\frac{N_2}{2(2D_xt)^{\frac{1}{2}}}-(\alpha t)^{\frac{1}{2}}\right\}-$$
$$\exp N_2\left(\frac{\alpha}{2D_x}\right)^{\frac{1}{2}}\operatorname{erfc}\left\{\frac{N_2}{2(2D_xt)^{\frac{1}{2}}}+(\alpha t)^{\frac{1}{2}}\right\}] \quad (22)$$

Numerical Example and Discussion

To explain the behaviour of the concentration distribution of the non-reactive pollutant against the uniform horizontal flow, a numerical example has been chosen. The velocity and dispersion coefficient components have been assigned values as u = 0.01 cm/min, v = 0.001 cm/min, D_x = 1.04 cm^2/min and D_y = 0.104 cm^2/min. The horizontal dimensions of the porous medium have been taken 100 cm and 50 cm along x and y axes respectively. The computation efforts reveal that the solutions for large values of time given by Eq. (14) (for Case-1) and Eq. (20) (for Case-2) converge rapidly from times t = 4,000 min and t = 10,000 min, respectively. The solution Eq. (14) has been computed at t = 4,000 min and 12,000 min while solution Eq. (20) at t = 10,000 min and 30,000 min and the numerical values of C/C_0 in both the solutions have been plotted in the Figs. 2 and 3, respectively. From Fig. 2, it can be observed that the solution Eq. (14) satisfies the boundary condition Eq. (4), *i.e.* (C/C_0) at the origin is always zero. While Fig. 3, shows that (C/C_0) at the origin at the two values of time are 0.20 and 0.35 respectively which confirms that the boundary conditions Eqs. (17) and (18) allow some concentrations to reach at the origin after some time.

The solution for small values of time given by Eq. (16) for Case-1 has been computed between t = 1,000 min, and 3,000 min, and the similar solution in Case-2 given by Eq. (22) computed between t = 2,000 min and 8,000 min. These two solutions also satisfy the respective boundary conditions. As the profiles showing the concentration distribution are similar to those at large values of time, graphs

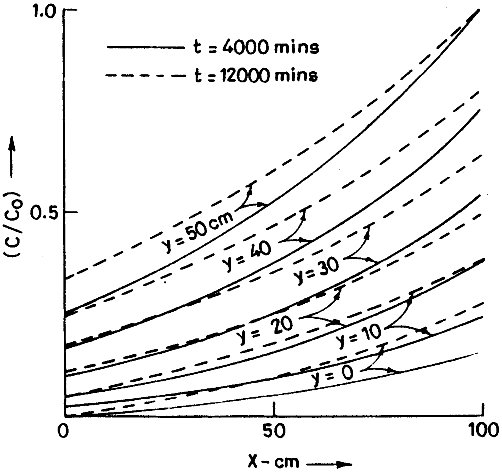

Fig. 2. Concentration distribution with Case-1 boundary conditions representing Eq. (14).

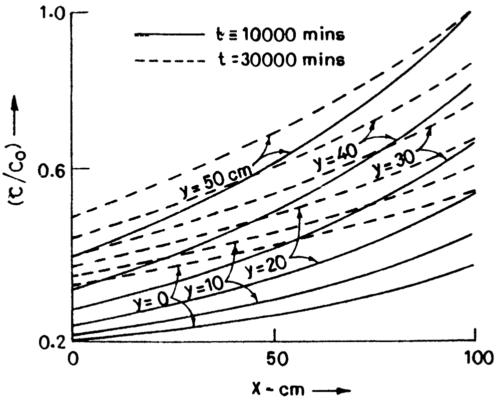

Fig. 3. Concentration distribution with Case-2 boundary conditions representing Eq. (20).

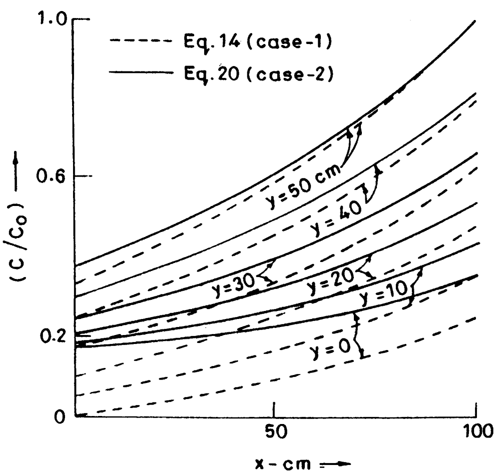

Fig. 4. Comparison of concentration distribution in the two dispersion models at $t = 10{,}000$

are not presented for them. The number of terms in the expansions in all the four solutions has been taken twenty, and beyond this value the summation causes absolutely no difference. Fig. 4 shows the comparison of concentration values (C/C_o) for the two dispersion models Eq. (14) Case-1, and Eq. (20) Case-2, at the common time value t = 10,000 min.

The effects of horizontal flow components as well as dispersion coefficients on the concentration distribution have also been studied. Two more sets of data have been chosen for the purpose as a) u = 0.005 cm/min, v = 0.0005 cm/min, D_x = 1.04 cm^2/min, and D_y = 0.104 cm^2/min and b) u = 0.005 cm/min, v = 0.0005 cm/min, D_x = 0.52 cm^2/min and D_y = 0.052 cm^2/min. The comparison of numerical values of C/C_o for each of the four analytical solutions Eqs. (14), (16), (20) and (22) at particular times obtained for first and second sets of data shows that reduction in the velocity component values causes increase in the concentration values at different positions of the horizontal cross-section of the porous medium. The increase (in percentage) in the concentration values are put in Table 1. But reduction in the dispersion coefficient values (comparison of the results of any one solution solved for second and third set of data) attenuates the concentration values at any position. The attenuation (in percentage) in the concentration values are put in Table 2. The two tables also reveal that the effects of the velocity and dispersion coefficient increase as the pollutants are transported against the flow nearer to the boundaries $x = 0$ and $y = 0$ respectively.

The normal values of groundwater velocities vary widely from 2 metre/year (0.0003 cm/min) to 2 metre/day (0.14 cm/min) depending upon local hydrogeological conditions (Todd 1980). Where the groundwater is moving relatively slow and the aquifer is of shallow depth, the lateral velocity component can not be neglected in comparison to the longitudinal one. The pollutants or poor quality water tend to spread more laterally to form wider plumes against the slowly moving groundwater horizontally. The different values to the velocity and dispersion coefficients have been assigned keeping in view the above facts. The lateral components of both, have been taken one-tenth of the respective longitudinal components.

Conclusion

Concentration distribution behaviour of a non-reactive pollutant from a point source against the horizontal uniform flow in a horizontal cross-section of a shallow finite saturated aquifer are explained with the help of analytical solutions of two types of dispersion models through porous media. Using Laplace transformation time variable in the partial differential equation is eliminated and then introduction of a new space variable reduces that into an ordinary differential equation. The solution of which is put in two forms, one for large values of time and the other for small values of time. The inverse Laplace transformation of those

Table 1 – Increase (%) in (C/C_o) due to reduction in the values of u and v by half, obtained by comparing the two results from the data:
(i) $u \equiv 0.01$ cm/min, $v \equiv 0.001$ cm/min, $D_x = 1.04$ cm^2/min, $D_y \equiv 0.104$ cm^2/min, and
(ii) $u \equiv 0.005$ cm/min, $v \equiv 0.0005$ cm/min, $D_x = 1.04$ cm^2/min, $D_y \equiv 0.104$ cm^2/min, of the solutions
Case-1: a) Eq. (14) at $t = 8{,}000$ min, b) Eq. (16) at $t \equiv 2{,}000$ min.
Case-2: a) Eq. (20) at $t = 20{,}000$ min, b) Eq. (22) at $t \equiv 5{,}000$ min.

y(cm)→	50		40		30		20		10		0	
x(cm) ↓	Case-1	Case-2	1	2	1	2	1	2	1	2	1	2
100 (a)	-	-	4.21	5.90	7.99	11.18	11.39	15.90	14.45	20.07	17.22	23.71
(b)	-	-	3.28	3.87	6.22	7.35	8.91	10.51	11.41	13.42	13.76	16.13
80 (a)	5.86	6.91	9.66	12.24	13.08	17.00	16.17	21.23	18.93	24.94	21.44	28.13
(b)	5.28	5.65	8.24	9.14	10.92	12.31	13.40	15.21	15.72	17.90	17.92	20.41
60 (a)	11.28	13.25	14.72	18.05	17.82	22.33	20.61	26.11	23.12	29.38	25.38	32.14
(b)	10.19	10.88	12.88	14.05	15.34	16.94	17.64	19.62	19.81	22.11	21.86	24.45
40 (a)	16.29	19.06	19.41	23.38	22.21	27.21	24.75	30.56	27.02	33.41	29.06	35.76
(b)	14.77	15.74	17.22	18.63	19.50	21.29	21.64	23.76	23.67	26.08	25.60	28.23
20 (a)	20.94	24.38	23.76	28.26	26.31	31.67	28.60	34.60	30.65	37.05	32.49	39.03
(b)	19.05	20.26	21.31	22.91	23.43	25.36	25.43	27.66	27.33	29.81	29.13	31.75
0 (a)	25.26	29.25	27.82	32.72	30.12	35.73	32.19	38.28	34.04	40.36	-	41.98
(b)	23.07	24.49	25.17	26.92	27.14	29.20	29.01	31.33	30.79	33.30	-	35.01

Table 2 – Attenuation (%) in (C/C_o) due to reduction in the values of D_x and D_y, obtained by comparing the two results from the data:
(i) $u \equiv 0.005$ cm/min, $v \equiv 0.0005$ cm/min, $D_x \equiv 1.04$ cm^2/min, $D_y \equiv 0.104$ cm^2/min, and
(ii) $u \equiv 0.005$ cm/min, $v \equiv 0.0005$ cm/min, $D_x \equiv 0.52$ cm^2/min, $D_y \equiv 0.052$ cm^2/min, of the solutions
Case-1: a) Eq. (14) at $t \equiv 8{,}000$ min, b) Eq. (16) at $t \equiv 2{,}000$ min.
Case-2: a) Eq. (20) at $t \equiv 20{,}000$ min, b) Eq. (22) at $t \equiv 5{,}000$ min.

y(cm)→	50		40		30		20		10		0	
x(cm) ↓	Case-1	Case-2	1	2	1	2	1	2	1	2	1	2
100 (a)	-	-	9.25	10.25	18.87	20.09	28.47	29.36	37.54	37.82	45.55	45.16
(b)	-	-	16.85	11.49	36.71	24.19	56.36	37.50	72.93	50.74	84.96	63.27
80 (a)	8.83	9.55	18.06	19.23	27.40	28.43	36.42	36.93	44.61	44.47	51.48	50.73
(b)	13.07	10.11	31.54	22.03	51.00	34.76	68.39	47.67	81.71	60.11	90.51	71.52
60 (a)	17.39	18.44	26.44	27.54	35.34	36.04	43.62	43.71	50.78	50.26	56.33	55.38
(b)	26.91	20.19	45.75	32.29	63.61	44.79	78.08	57.07	88.17	68.54	94.23	78.56
40 (a)	25.59	26.70	34.31	35.16	42.59	42.91	49.98	49.69	56.01	55.18	60.32	59.13
(b)	40.76	30.10	58.72	42.13	74.11	54.15	85.45	65.60	92.66	75.91	96.54	84.16
20 (a)	33.36	34.32	41.58	42.10	49.10	49.03	55.46	54.84	60.28	59.24	63.28	62.03
(b)	53.84	39.69	69.89	51.39	82.38	62.74	90.76	73.19	95.58	82.06	97.78	87.92
0 (a)	40.60	41.29	48.18	48.33	54.80	54.38	60.04	59.16	63.61	62.45	-	64.12
(b)	65.49	48.81	78.97	59.98	88.52	70.47	94.35	79.73	97.31	86.69	-	69.52

completes the required solutions. The effects due to change in the velocity and dispersion coefficient components on the concentration distribution have also been explained. The concentration distribution trend in the horizontal cross-section will describe the trend of pollution transport against the flow in the whole aquifer possessing uniform horizontal groundwater flow, away from the source boundary (vertical along the depth).

Notations

C – concentration of the dispersing pollutant at any position of the horizontal cross-section of the porous medium, in liquid phase
C_o – concentration of entrained pollutant into the cross-section from the point source.
D_x, D_y – Dispersion coefficient along longitudinal and lateral direction respectively.
H – length of the porous medium along lateral direction
h_1 – $u/2D_x$
h_2 – $v/2D_y$
L – length of the porous medium along longitudinal direction
n – an integer
N_1 – $(2n + 1)\,\xi - \eta$
N_2 – $(2n + 1)\,\xi + \eta$
p – Laplace tranformation parameter
t – time variable
u, v – longitudinal and lateral seepage velocity components respectively.
x, y – space variable along longitudinal and lateral directions
α – $D_x h_1^2 + D_y h_2^2$
η – $x + y(D_x/D_y)^{1/2}$, a new space variable
ξ – $L + H(D_x/D_y)^{1/2}$

References

Al-Niami, A. N. S., and Rushton, K. R. (1977) Analysis of flow against dispersion in porous media, *J. Hydrol, Vol. 33,* pp. 87-97.

Banks, R. B., and Ali, J. (1964) Dispersion and adsorption in porous media flow, *J. Hydraul. Div., Vol. 90,* pp. 13-31.

Banks, R. B., and Jerasate, S. (1962) Dispersion in unsteady porous media flow, *J. Hydraul. Div., Vol. 88,* pp. 1-21.

Bastian, W. C., and Lapidus, L. (1956) Longitudinal diffusions in ion-exchange and chromatographic columns, *J. Phys. Chem., Vol. 60,* pp. 816-817.

Bruce, J. C., and Street, R. L. (1966) Studies of free surface flow and two dimensional dispersion in porous media. Report No. 63, Civil Eng. Dept. Stanford University, Stanford, California.

Carnahan, C. L., and Remer, J. S. (1984) Non-uniform and equilibrium sorption with a linear sorption isotherm during mass transport through an infinite porous medium: Some analytical solutions, *J. Hydrol., Vol. 73,* pp. 227-258.

Carslaw, H. S., and Jaeger, J. C. (1986) *Conduction of Heat in solids,* Clarendon press, Oxford, 2nd Ed.

Harleman, D. R. F., and Rumer, R. R. (1963) Longitudinal and lateral dispersion in an isotropic porous media, *J. Fluid Mech., Vol. 16,* pp. 385-394.

Hunt, B. (1978) Dispersion sources in uniform groundwater flow, *J. Hydraul. Div., Vol. 104,* pp. 75-85.

Kumar, N. (1983) Dispersion of pollutants in semi-infinite porous media with unsteady velocity distribution, *Nordic Hydrology, Vol. 14,* pp. 167-178.

Kumar, N. (1983) Unsteady flow against dispersion in finite porous media, *J. Hydrol., Vol. 63,* pp. 345-358.

Latinopoulos, P., Tolikas, D., and Mylopoulos, Y. (1988) Analytical solutions for two dimensional chemical transport in aquifer, *J. Hydrol., Vol. 96,* pp. 11-19.

Mariño, M. A. (1974) Model of dispersion in granular medium, *J. Hydrol., Vol. 23,* pp. 313-318.

Mariño, M. A. (1978) Flow against dispersion in non-adsorbing porous media, *J. Hydrol., Vol. 37,* pp. 149-158.

Ogata, A. (1970) Theory of dispersion in granular medium, U. S. Geol. Surv., Prof. Pap., 411-I, 34.

Rumer, R. R. (1962) Longitudinal dispersion in steady and unsteady flow, *J. Hydraul. Div., Vol. 88,* pp. 147-172.

Shen, H. T. (1976) Transient dispersion in uniform porous media flow, *J. Hydraul. Div., Vol. 102,* pp. 707-715.

Todd, D. K. (1980) *Groundwater Hydrology,* John Wiley, N. Y., 2nd Ed.

Van Genuchten, N. Th. (1981) Analytical solutions for chemical transport with simultaneous adsorption, zero-order production and finite order decay, *J. Hydrol., Vol. 49,* pp. 213-233.

Yadava, R. R., Vinda, R. R., and Kumar, N. (1990) One-dimensional dispersion in unsteady flow in an adsorbing porous medium: An analytical solution, *Hydrological Processes, Vol. 4,* pp. 189-196.

First received: 9 September, 1991
Revised version received: 2 December, 1991
Accepted: 12 December, 1991

Address:
Dept. of Mathematics,
Faculty of Science,
Banaras Hindu University,
Varanasi-221005, India.