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Unsteady Flow to a Non-Penetrating Large Diameter Well in Extensive Aquifers

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Non-equilibrium groundwater movement equations are written explicitly for the continuity and flow laws in the case of a non-penetrating large diameter well discharging from extensively thick confined porous medium aquifers. The equations representing these laws are combined together with the Boltzmann transformation which leads to the specific discharge equation within the aquifer. Type curve expressions are obtained with convenient definitions of the well function and dimensionless time factor for a non-penetrating well. The forms of these curves on double logarithmic paper appear identical to the fully penetrating large diameter well type curves but with differences in the variables as well as the aquifer parameters. In the non-penetrating well case the significant aquifer parameters are the hydraulic conductivity and specific storage coefficient which are intact of the aquifer saturated thickness. The methodology developed herein is applied to the time-drawdown data from a non-penetrating well in Saudi Arabia. Aquifer parameter determinations can be obtained through a similar procedure to the Theis type curve usage.

Introduction

Groundwater potential assessment of an aquifer requires essentially determination of aquifer parameters, namely, the storativity or specific storage coefficient and transmissivity or hydraulic conductivity. Their determination depends mainly on the field data which are collected during an unsteady flow condition with withdrawal of water through the wells. The mathematical models developed by Theis (1935), Cooper and Jacob (1946), Hantush (1964), Papadopulos and Cooper (1967) Cooper *et al.* (1967), *etc.,* are useful for aquifer parameter evalutation by conducting tests in small or large diameter wells existing in that aquifer with full penetration only. Due to economical reasons and especially for preliminary groundwater investigations short wells have been preferred in the practical activities. Therefore, non-penetrating wells have been used commonly in the Middle Eastern countries and India because of economy and simple construction. These wells do not require screens and take in water from a cavity formed at the bottom of the well. In confined aquifers, they are constructed by puncturing the upper impervious layer. Similar considerations are valid in the form of pits and swamps for unconfined aquifers where the groundwater table is close to earth surface. In the Kingdom of Saudi Arabia, in regions where the piezometric levels are not far from the earth's surface even the bulldozers have been used in Quaternary alluvium deposits to form such non-penetrating wells. On the other hand, in the limestone regions of the world some oasis have developed which tap water from the underlying sandstone aquifers through vertical cavities. These cavities do not penetrate the sandstone and play exactly the same role as the non-penetrating wells. For instance, in the eastern provinces of the Kingdom of Saudi Arabia, the Umm Er Radhuma limestone includes many small and large scale oases which withdraw water from the underlying Saq, Tabuk or Wajid sandstones.

Steady state solutions of non-penetrating wells have been presented by Muskat (1937), Aravin and Numerov (1965) and Basak (1977). However, the first treatment of these wells with unsteady flow conditions has been proposed by Kanwar (1974). Later, Kanwar and Chauhan (1974, 1978), Kanwar *et al.* (1974, 1979), Abdul-Khader and Ramadurgaiah (1976), Jaiswal *et al.* (1977), Jaiswal and Chauhan (1978) have derived approximate solutions with variable discharges from non-penetrating wells. On the other hand, Şen (1985) has obtained solutions for constant discharge groundwater flow case in infinitesimally small diameter non-penetrating wells.

In this paper, however, a new analytical solution for unsteady flow to a large non-penetrating well in a confined aquifer has been obtained with Boltzmann transformation of relevant differential equations of groundwater movement. The general solutions of the drawdown distribution in and around a large diameter non-penetrating well have been presented in forms of type curves.

Problem Presentation and Analytical Solution

General description of a non-penetrating well in an extensively thick confined aquifer is given in Fig. 1. The well radius is r_w and the initial piezometric level is h_0 above the impervious layer. This well is blind in the sense that no water enters through the wall of the well. The aquifer parameters are the hydraulic conductivity,

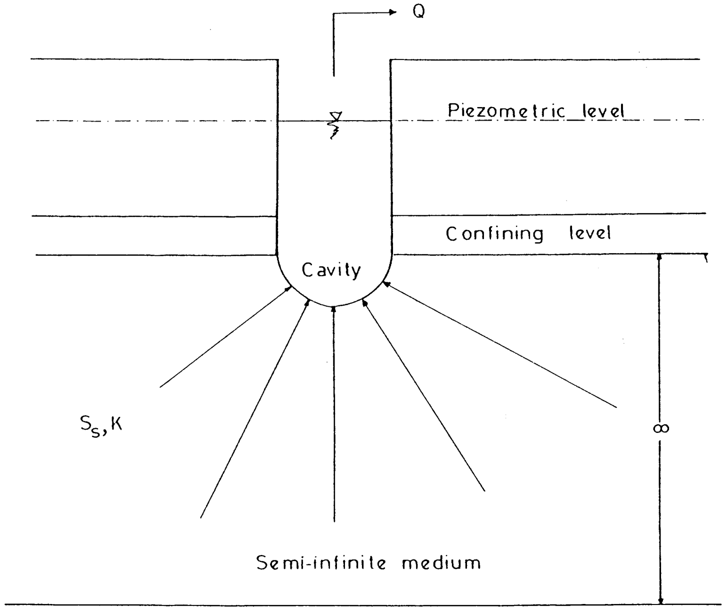

Fig. 1. Large diameter non-penetrating well.

K, and the specific storage coefficient, S_s. The well is pumped at a constant discharge, Q. The aquifer is assumed to be uniform, horizontal, homogeneous and isotropic. The groundwater flow obeys the Darcy law everywhere within the aquifer.

During a small time interval, Δt, the continuity equation for the aquifer control domain between two concentric hemispheric surfaces of radii r and $r+\Delta r$ can be written as

$$\Delta t[Q(r,t) - Q(r+\Delta r,t)] = S_s 2\pi r^2 \Delta r[h(r,t) - h(r,t+\Delta t)] \quad (1)$$

in which t and r are independent time and radial distance variables and $h(r,t)$ is the piezometric level. The left hand side of Eq. (1) shows the volumetric difference between the input and output whereas the right hand side is the volumetric change of storage within the aforementioned control domain. For infinitesimally small time and distance intervals (*i.e.* as Δt and simultaneously Δr go to zero), Eq. (1) leads to the following partial differential equation

$$\frac{\delta Q(r,t)}{\delta r} \equiv S_s 2\pi r^2 \frac{\delta h(r,t)}{\delta t} \quad (2)$$

On the other hand the general expression of the discharge to the well can be written

$$Q(r,t) = 2\pi r^2 v(r,t) \quad (3)$$

where $v(r,t)$ is the specific discharge at time t and distance r. The substitution of Eq. (3) into Eq. (2) gives after some simple differentiation and algebraic manipulation

$$\frac{\delta v(r,t)}{\delta r} + \frac{2}{r} v(r,t) \equiv S_s \frac{\delta h(r,t)}{\delta t} \tag{4}$$

All of the aforementioned equations are different versions of the principle of continuity. The specific discharge of the groundwater movement in a porous medium can be expressed by Darcy's law as

$$v(r,t) \equiv K \frac{\delta h(r,t)}{\delta r} \tag{5}$$

The substitution of Eq. (5) into Eq. (4) results in the general groundwater movement equation of flow towards a non-penetrating well as

$$\frac{\delta^2 h(r,t)}{\delta r} + \frac{2}{r}\frac{\delta h(r,t)}{\delta r} = \frac{S_s}{K}\frac{\delta h(r,t)}{\delta t} \tag{6}$$

Kanwar (1974) and Kanwar *et al.* (1979) have based their results on direct solution of Eq. (6) by use of the Laplace transformation. The solution of the groundwater movement equation in this paper is sought by the use of Boltzmann transformation which, first of all, renders the two partial equations, namely, Eqs. (4) and (5) into ordinary differential forms. The Boltzmann variable, η, is given as

$$\eta = \frac{r}{2\sqrt{t}} \tag{7}$$

After using the chain rule for the differentiation of composite functions one can obtain, corresponding versions of Eqs. (4) and (5) as

$$\frac{dv(\eta)}{d\eta} + \frac{2}{\eta} v(\eta) = -\frac{\eta}{\sqrt{t}}\frac{dh(\eta)}{d\eta} \tag{8}$$

and

$$v(\eta) = \frac{K}{2\sqrt{t}}\frac{dh(\eta)}{d\eta} \tag{9}$$

Elimination of $dh(\eta)/d\eta$ between these two equations yields

$$\frac{dv(\eta)}{d\eta} + 2\left(\frac{1}{\eta} + \frac{\eta}{K} S_s\right) v(\eta) \equiv 0 \tag{10}$$

The general solution of which is straightforward to obtain as

$$v(\eta) = C\frac{\exp(-S_s\eta^2/K)}{\eta^2} \tag{11}$$

or substitution of Eq. (7) into Eq. (11) leads to

$$v(r,t) \equiv 4tC(t)\frac{\exp(-S_s r^2/4Kt)}{r^2} \tag{12}$$

This is the general expression of the groundwater specific discharge towards a non-penetration well whatever the diameter of the well is. However, its specific solution depends on the initial and boundary conditions of the problem at hand which are for the case considered herein as follows

$$h(r,0) = h_0 \tag{13}$$

$$h(\infty,t) = h_0 \tag{14}$$

and

$$\lim_{r\to r_w}[2\pi r^2 v(r,t)] \equiv Q - \pi r_w^2 \frac{ds_w(t)}{dt} \tag{15}$$

where $s_w(t)$ is the drawdown within the large diameter well. By substituting Eq. (12) into Eq. (15) it is possible to obtain the integration constant as

$$C(t) \equiv \frac{1}{8\pi t}\left[Q-\pi r_w^2 \frac{ds_w(t)}{dt}\right] \exp \frac{r_w^2 S_s}{4Kt} \tag{16}$$

Hence, Eq. (12) becomes with the substitution of this value as

$$v(r,t) = \frac{1}{2\pi}\left[Q-\pi r_w^2 \frac{ds_w(t)}{dt}\right]\exp\left(\frac{r_w^2 S_s}{4Kt}\right) \frac{\exp(-r^2 S_s/4Kt)}{r^2} \tag{17}$$

which gives the specific discharge variations with time and space towards the large diameter non-penetrating well. For any large time Eq. (17) reduces to

$$v(r,t) = \frac{Q}{2\pi r^2} \tag{18}$$

In fact, this is the steady state flow specific discharge expression. By considering Eq. (5) and the initial conditions in Eqs. (13) and (14) the head distribution can be found from Eq. (17) and through taking integration as

$$h_0 - h(r,t) = \frac{1}{2\pi K}\left[Q-\pi r_w^2 \frac{ds_w(t)}{dt}\right]\exp\left(\frac{r_w^2 S_s}{4Kt}\right) \int_r^\infty \frac{\exp(-r^2 S_s/4Kt)}{r^2}\, dr \tag{19}$$

The left hand side is equal to drawdown $s(r,t) = h_0 - h(r,t)$. In addition, definition of new dimensionless variable as

$$u \equiv \frac{r^2 S_s}{4Kt} \tag{20}$$

$$u_w \equiv \frac{r_w^2 S_s}{4Kt} \tag{21}$$

$$W(u) \equiv \frac{4\pi K r}{Q} s(r,t) \tag{22}$$

and

$$W(u_w) \equiv \frac{4\pi K r_w}{Q} s_w(t) \tag{23}$$

and the necessary derivatives from these definitions

$$dt = -\frac{r_w^2 S_s}{4 K u_w^2} du_w \tag{24}$$

and

$$ds_w(t) = \frac{Q}{4\pi K r_w} dW(u_w) \tag{25}$$

render Eq. (19) after necessary substitutions and some algebra into

$$s(r,t) \equiv \frac{Q}{4\pi K} [1 + \frac{u_w^2}{r_w S_S} \frac{dW(u_w)}{du_w}] \exp(u_w) \int_u^\infty \frac{e^{-x}}{x} dx \tag{26}$$

where x is a dimensionless dummy variable and u_w is the dimensionless time factor for the large diameter well. Finally, consideration of Eq. (22) leads to

$$W(u) \equiv [1 + \frac{u_w^2}{r_w S_s} \frac{dW(u_w)}{du_w}] \exp(u_w) \int_u^\infty \frac{\exp(-x)}{x} dx \tag{27}$$

For small diameter wells, *i.e.*, when $u_w \rightarrow 0$ this expression reduces to the form of Theis equation as presented by Şen (1985) with non-penetrating well dimensionless time factor and the well function which are different from their counterparts in the case of Theis solution.

Type Curves and Applications

Eq. (27) is general in the sense that it relates the well function, $W(u)$, at any time and distance within the aquifer to the derivative of the large diameter well function, $dW(u_w)/du_w$. In fact, $W(u)$ is the well function for the observation wells. By considering the non-penetrating large diameter well as an observation well, *i.e.*, if the time-drawdown data are available from this large diameter well then with the assumption that there is no well loss one can write that $u = u_w$. By substitution into Eq. (27) this leads to the relationship between the non-penetrating well function and the dimensionless time factor as

$$W(u_w) = [1 + \frac{u_w^2}{r_w S_s} \frac{dW(u_w)}{du_w}] \exp(u_w) \int_{u_w}^\infty \frac{\exp(-x)}{x} dx \tag{28}$$

The solution of this equation is possible by a simple finite element method when the derivative in the big brackets is written as

$$\frac{dW(u_w)}{du_w} = \frac{W_i(u_w) - W_{i-1}(u_w)}{\Delta u_{wi}} \tag{29}$$

in which i denotes the node number and $\Delta u_{wi} = u_{wi} - u_{w(i-1)}$ The substitution of Eq. (29) into Eq. (28) after the necessary algebraic arrangements leads to

$$W_i(u_w) = [1 + \frac{u_{wi} . W_{i-1}(u_w)}{S_s \Delta u_{wi}}] / [\frac{e^{-u_{wi}}}{\int_{u_{wi}}^{\infty} \frac{e^{-x}}{x} dx} + \frac{u_{wi}^2}{S_s \Delta u_{wi}}] \tag{30}$$

in which the integration term can be obtain from the Theis (1935) table. The numerical solution of Eq. (30) on digital computers with the initial conditions from the Papadopulos and Cooper (1967) tables for $u_{wi} = 10$ leads to a set of type curves for different specific storage values as in Fig. 2.

Simple way of finding the type curve expressions for observation wells can be obtained by taking the ratio of Eq. (27) to Eq. (28)

$$W(u) = W(u_w) \frac{\int_{u}^{\infty} \frac{e^{-x}}{x} dx}{\int_{u_w}^{\infty} \frac{e^{-x}}{x} dx} \tag{31}$$

Furthermore, the ratio of dimensionless time factors gives

$$u = (\frac{r}{r_w})^2 u_w \tag{32}$$

where always $r/r_w > 1$. Eqs. (31) and (32) provide a basis in obtaining type curves for any observation well as follows:

(i) Obtain type curves for drawdowns in the main well by using Eq. (28)

(ii) By knowing the well diameter and the radial distance between the main and observation wells from the field measurements, it is possible to calculate the ratio, r/r_w', hence, u from Eq. (32) corresponding to any u_w in step (i).

(iii) Corresponding, $W(u)$ values can be calculated from Eq. (31).

(iv) Plot of u versus $W(u)$ for a set of S_s values leads to the type curves for observation wells as shown in Figs. 3-5. It is clear that as the distance ratio, (r/r_w), approaches 1, the type curves approach the main well curves. On the contrary, with extensive increases in this ratio all the curves come closer to the Theis type curve.

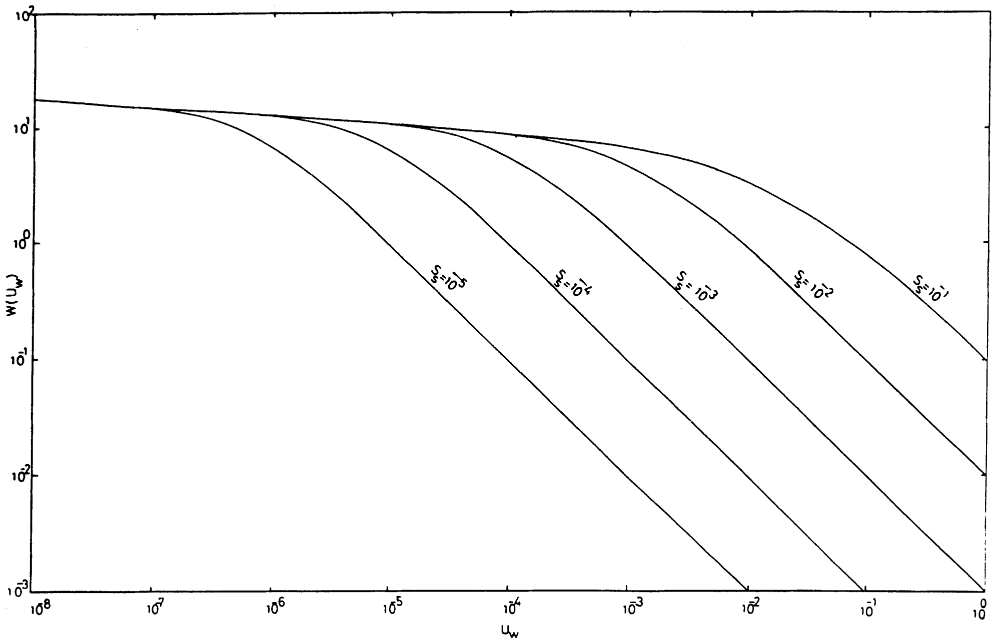

Fig. 2. Type curves for non-penetrating main well.

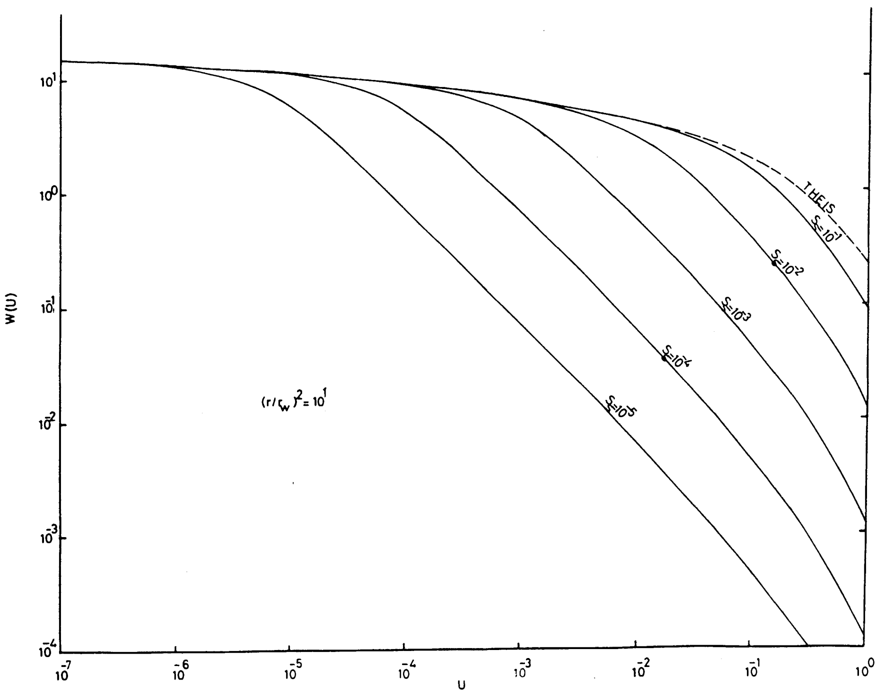

Fig. 3. Type curves for observation well, ($r/r_W \equiv 10$).

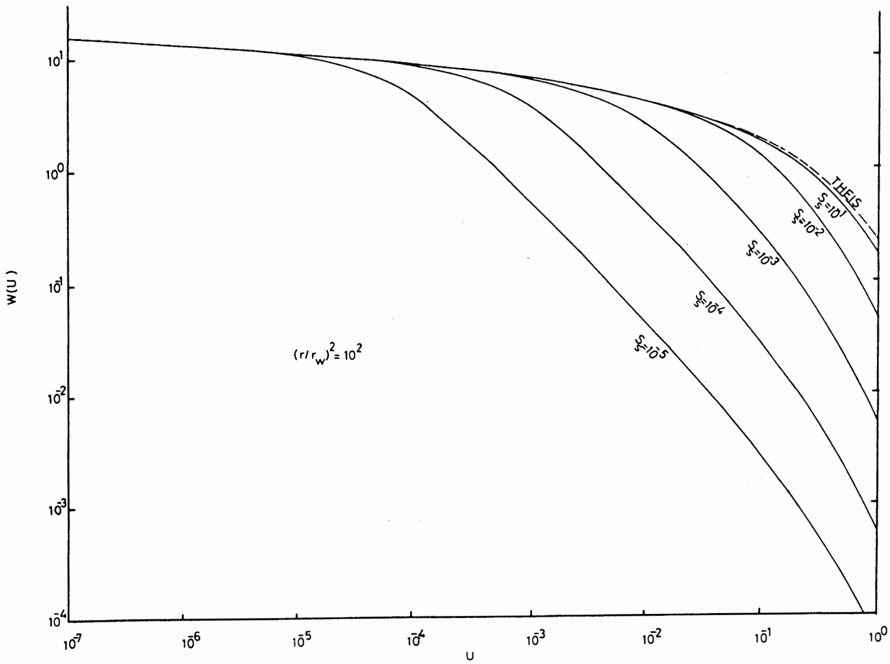

Fig. 4. Type curves for observation well, ($r/r_w \equiv 100$).

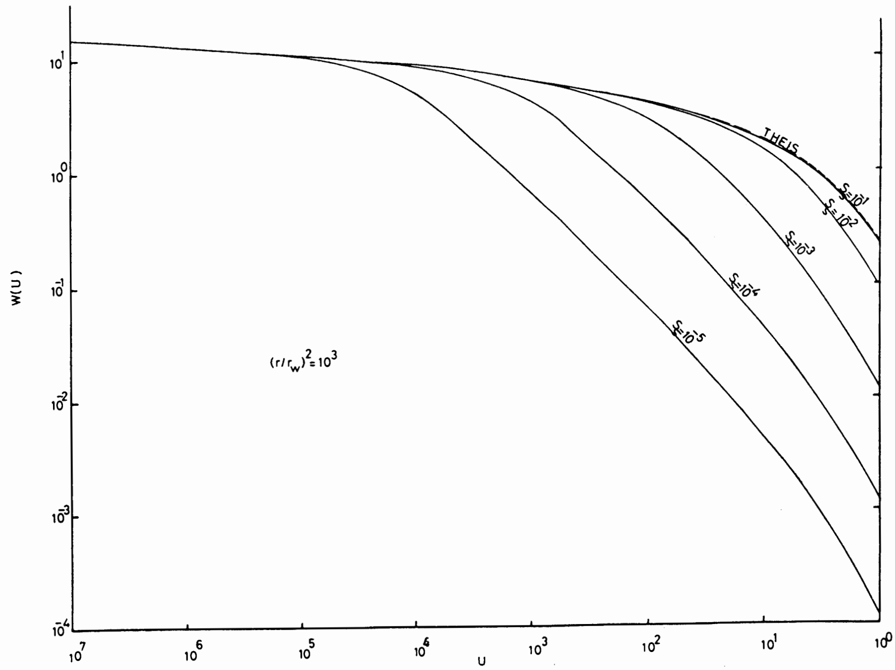

Fig. 5. Type curves for observation well, ($r/r_w = 1000$).

Application

The field data with non-penetrating well have been obtained from the Quaternary wadi deposits within wadi Qudaid in the western part of the Kingdom of Saudi Arabia. The wadi is bound on two sides by impervious crystalline bedrock. Since, these barriers are far away from the pumping well their effect is not felt in the time-drawdown data. In the area there exists a clay layer which gives rise to a confined aquifer behavior to the wadi medium. The large diameter well penetrates this clay layer only and the groundwater is drawn from the underlying sand and gravel mixture layer which has a thickness of 30 m. The well diameter is 2.2 m and the discharge is kept constant at 0.33 m^3/min throughout the pumping period.

Classical type curve matching procedure is applied to the observed data with the set of curves developed herein as in Fig. 6. It is obvious that the best type curve that matches the field data has $S_s = 10^{-5}$. The selected match point coordinates both on the field and type curve sheets are $s_{wM} = 0.26$ m, $t_M = 21$ min, $W_w(u_w) = 10^0$ and $u_{wM} = 10^{-6}$. With these values at hand, hydraulic conductivity can be calculated from Eq. (23) as

$$K = \frac{QW_M(u_w)}{4\pi r_w s_w(t)} = \frac{0.33\times10^0}{4\times3.14\times1.1\times0.26} \equiv 0.092 \text{ m/min}$$

However, the specific storage coefficient can be calculated from Eq. (21) as

$$S_s = \frac{Kt_M}{r_w^2} u_{wM} = \frac{4\times0.092\times21}{1.1^2} 10^{-6} = 6.5\times10^{-7}$$

The difference of this value from the specific storage read off the matching procedure as 10^{-5} indicates that the type curve matching does not yield too reliable results so far as this parameter is concerned. Therefore, the calculation of S_s should be supported by some other technique such as the one proposed by Şen (1987) based on the quasi-steady state flow.

Conclusions

Analytical solutions of groundwater movement problem toward a non-penetrating large diameter well discharging from an extensive thick confined aquifer have been presented. The drawdown variations with time within the well itself are exhibited in terms of dimensionless type curves by taking into consideration the well storage. These curves are useful in estimating the aquifer parameters, namely, the hydraulic conductivity and the specific storage coefficient from the field measurements of time-drawdown data. The following significant points can be drawn from the study in this paper:

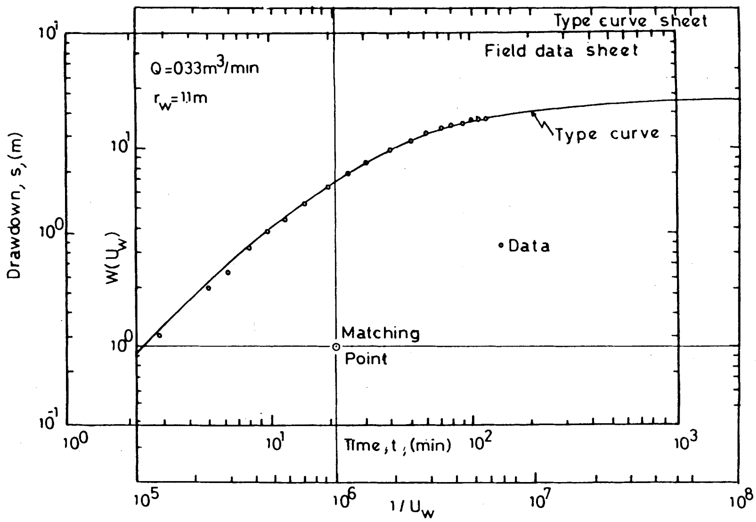

Fig. 6. Type curve matching to field data.

(i) The non-penetrating large diameter well type curves are identical with the large diameter fully penetrating well curves in their forms but different in the dimensionless well function and time factor definitions. In a non-penetrating well instead of transmissivity and storativity, the hydraulic conductivity and the specific storativity become important.

(ii) Each type curve for different specific storage value are slightly different from each other and therefore their use in the specific storage determinations is questionable since a slight change in the best matching type curve to the field data may lead to differences in the magnitude of specific storage. Therefore, some other technique should be used to support the specific storage calculations. However, the hydraulic conductivity determinations, from the methodology presented herein, are reliable.

(iii) Type curves are all straight lines initially which indicates the active effect of the non-penetrating well storage.

(iv) Final portions of type curves, that is for large times, merge with a single curve identical to the Theis type curve but different in the dimensionless variables only.

References

Abdul-Khader, M.H., and Ramadurgaiah, D. (1976) Unsteady flow to a non-penetrating artesian well, *Ground Water, Vol. 14,* No. 4, pp. 200-204.

Aravin, V.I., and Numerov, S.N. (1965) Theory of fluid flow in undeformable porous media, Israel Program for Scientific Translation, 511 pp.

Basak, P. (1977) Non-penetrating well in a semi-infinite medium with non-linear flow, *Journal of Hydrology, Vol. 133,* pp. 375-382.

Cooper, Jr., H.H., Bredehoft, J.D., and Papadopulos, I.S. (1967) Responce of a finite diameter well to an instantaneous charge of water, *Water Resources Research, Vol. 3,* No. 1, pp. 263-269.

Cooper, Jr., H.H., and Jacob, C.E. (1946) A generalized graphical method for evaluating formation constants and summarizing well field history, *Trans. Amer. Geophys. Union, Vol. 27,* pp. 526-534.

Hantush, M.S. (1964) Hydraulics of Wells. In *Advances in Hydrosciences, 1.* Edited by V.T. Chow, Academic Press, New York, pp. 281-432.

Jaiswal, C.S., Chauhan, H.S., and Ghildyal, B.P. (1977) Transient spherical flow to a cavity well of finite diameter, *Ground Water, Vol. 15,* No. 5, pp. 372-378.

Jaiswal, C.S., and Chauhan, H.S. (1978) Flow to a non-penetrating well in a leaky artesian aquifer with variable discharge, *Ground Water, Vol. 16,* No. 2, pp. 119-125.

Kanwar, R.S., (1974) Studies on nonsteady flow towards a non-penetrating well. Thesis presented to G.B. Pant University, India.

Kanwar, R.S., and Chauhan, H.S. (1974) Unsteady spherical flow to a non-penetrating well in an artesian aquifer, *Journal of Agricultural Engineering,* Indian Society of Agricultural Engineering, *Vol. 11,* No. 3, pp. 14-21.

Kanwar, R.S., and Chauhan, H.S. (1978) Non-steady spherical flow to a cavity well in an infinite nonleaky aquifer, Water Resources Bulletin, American Water Resources Association, Vol. 14, No. 3, pp. 605-612.

Kanwar, R.S., Johnson, H.P., and Chauhan, H.S. (1979) Nonsteady flow to non-penetrating well, *Journal of the Irrigation and Drainage Division, ASCE, Vol. 105,* No. IR1, pp. 45-55.

Muskat, M. (1937) *The flow of homogeneous fluids through porous media,* McGraw-Hill Book Company, Inc., New York, 763pp.

Papadopulos, I.S., and Cooper, H.H., Jr. (1967) Drawdown in a well of large diameter, *Water Resources Research, Vol. 3,* No. 1, pp. 241-247.

Şen, Z., (1985) Unsteady flow to non-penetrating well in extensive aquifers, *Nordic Hydrology, Vol. 16,* pp. 213-220.

Şen, Z. (1987) Storage coefficient determination from quasi-steady state flow, *Nordic Hydrology, Vol. 18,* pp. 101-110.

Theis, C.V., (1935) The relationship between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage, *Trans. Amer. Geophys. Union, Vol. 16.* pp. 519-524.

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