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Prediction of Log-Transmissivity 1. Using Specific Capacity

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The log-transmissivity may in many cases be predicted from the log-transformed specific capacity of wells by applying a linear statistical model. The coefficients of the linear equation can be estimated by the least-square method. It is shown that if the estimated slope of the line differs from unity then it may indicate that the specific capacity is correlated with the well efficiency.

The above prediction method is applied to case studies of aquifers in three different formations: the prediction should not be used for a homogeneous fluvioglacial formation because the variance of the well efficiency dominates the variance of the transmissivity; the prediction is fair for a heterogeneous fluvioglacial formation; and the prediction is poor for a homogeneous limnic formation. In the study of the first formation a correlation between specific capacity and well efficiency can be identified directly from the slope of the regression line.

If the predictor values are taken from the drillers log then the standard error of prediction in all the cases is 0.35. This seems to be unacceptable in most practical applications. However, if one needs to predict the mean of a domain and the domain contains a number of observations of the predictor, then the averaging will reduce the prediction error. The averaging procedure ought to take the covariance structure of the variables into consideration.

Introduction

To obtain reliable result from simulations of groundwater flow and transport it is essential that the hydrogeology is described realistically by the simulation model.

As the hydrogeological parameters (*e.g.* transmissivity, conductivity, thickness of a layer, *etc.*) vary in space, the description of the parameter fields ought to be based on actual parameter measurements, with the number and location of measurements adjusted to the variability of the individual parameter. Unfortunately it is expensive to measure most of the hydrogeological parameters and it is therefore attractive to reduce the number of measurements by using parameter predictions instead. Such predictions can be made from parameters that are correlated with the actual parameter and which are attainable in great numbers at low cost.

In predicting the significant hydrogeological parameters it is important to quantify the variance of the prediction. Doing this is helpful when a structured plan for the calibration of a forward groundwater model is to be developed. It also makes it possible to analyze systematically the sensitivity of the calibrated model to the uncertainty of the hydrogeological parameters. Similarly, the parameter uncertainties are an essential input to the inverse groundwater models that are sometimes used for autocalibration, and for quantifying the parameter uncertainties of the calibrated model (*e.g.* Neuman and Yakowitz 1979, Hill 1992).

The aquifer transmissivity (or alternatively the hydraulic conductivity) is a variable that is fundamental to any groundwater flow and transport model. The local transmissivity can be determined quite accurately from pumping tests in wells, but such test are laborious and expensive to carry out in general, and in some wells testing can be very difficult. Therefore good "measures" of transmissivity are often only available at a limited number of wells. This may be insufficient in cases in which the variability in transmissivity is crucial to the simulations and it is therefore tempting to use surrogate parameters such as the specific capacity of wells or the lithological logs of boreholes to predict the transmissivity. In the present paper the application of specific capacity data is discussed and demonstrated, whereas the applicability of lithological data alone and together with the specific capacity will be demonstrated in a subsequent paper, Christensen (1995 – this issue).

Predicting the transmissivity from the specific capacity of wells has been common practice for decades. Theis *et al.* (1963), Walton (1984) and others apply the Theis equation to derive expressions from which the transmissivity can be predicted but these methods suffer from not giving proper prediction variances. Moreover (as argued by Neuman 1980) it is more relevant to predict the log-transmissivity rather than the transmissivity when the predictions are applied as inputs to groundwater models.

Ahmed and De Marsily (1987) have compared different geostatistical methods for estimating log-transmissivity from specific capacity. They conclude that if the residuals of the regression of one variable on the other are spatially uncorrelated or if the correlation coefficient between them is high then one may use a linear model to predict log-transmissivity from log-transformed specific capacity. If none of these requirements are fulfilled one should rather use co-kriging. In the following case studies the first requirement is fulfilled and a linear prediction model is therefore acceptable. Similar models were used by Delhomme (1974), Clifton and Neuman (1982), Ahmed and Marsily (1987) and others.

The present paper summarizes the theoretical background for the linear prediction of log-transmissivity from specific capacity, and derives the traditional linear statistical model. The model parameters are estimated by least-squares estimation and a physical interpretation of the estimated line slope is proposed. The method is applied in three case studies representing different aquifers.

Theoretical Background

Drawdown in a Pumping Well

The unsteady-state drawdown s_w in a well which fully penetrates an infinitely extended, homogeneous, isotropic and confined aquifer, and which has no skin effect, is equal to the aquifer head loss s_a given by the Theis equation

$$s_w = s_a = \frac{Q}{4\pi T}\int_u^{\infty} \frac{e^{-x}}{x}\,dx = \frac{Q}{4\pi T}\,W(u) \quad ; \qquad u = \frac{r_w^2 S}{4Tt} \tag{1}$$

where Q is the pumping rate, T is the transmissivity, S is the storativity, r_w is the radius of the well and t is the time since pumping started. If $u < 0.01$ then Eq. (1) may be approximated by the Jacob equation

$$s_a \approx \frac{Q}{4\pi T}\,(-0.5772 - \ln(u)) \tag{2}$$

It is commonly observed that the well drawdown differs from the aquifer head loss. This will be the case if the well is only partially penetrating the aquifer, or if the flow into the well is disturbed by a skin effect.

If the well penetrates the aquifer only partially, then the pumping induces a vertical flow component close to the well. This increases both the length of the average flow line and the flow velocity in the vicinity of the well and thus causes an extra head loss s_p, which can be estimated according to Eq. (10) of Hantush (1961) using the transformation of Hantush (1964) p. 349

$$s_p = \frac{Q}{4\pi T} f_s \quad ; \qquad t > \frac{BS}{2K_z} \tag{3}$$

$$f_s = \frac{4B^2}{\pi^2 (b-d)^2} \sum_{n=1}^{\infty} \left(\frac{1}{n^2} \mathbf{K}_0\left(\frac{n\pi r_w}{B} \sqrt{K_z/K_r} \right) \{\sin(\frac{n\pi b}{B}) - \sin(\frac{n\pi d}{B})\}^2 \right) \tag{4}$$

where $\mathbf{K_0}$ is the modified Bessel function, B is the aquifer thickness, K_z the vertical hydraulic conductivity, and d and b are respectively the distances from the top of the aquifer to the top and the bottom of the well screen. According to Eqs. (3) and

(4) the head loss due to partial penetration is constant for the given time interval.

The skin effect accounts for the head loss (termed "well loss" in the following) in a thin zone around the well intake. Kruseman and de Ridder (1990) list different procedures for estimating the well loss from a step-drawdown test. When this kind of test is not available the well loss may be estimated with the help of the well efficiency, E, which can be defined as

$$E = \frac{s_a(1 \text{ hour})}{s_w(1 \text{ hour})} \tag{5}$$

If T and S are known the well efficiency is easily calculated from the measured drawdown in the pumping well after 1 hour of pumping and from Eqs. (1) or (2). T can be determined from a pumping test, whereas S, if unknown can be determined from practical experience. According to Eq. (2) s_a varies with the logarithm of $1/S$. Therefore, large relative errors in the estimated S result in small errors in the estimated well efficiency.

By combination of Eqs. (1) and (5) the drawdown after one hour of pumping is

$$s_w(1 \text{ hour}) \equiv \frac{Q}{4\pi T} \frac{W(u_{1 \text{ hour}})}{E} \tag{6}$$

The well efficiency E depends both on the skin effect of the well and on its degree of penetration of the aquifer. An alternative efficiency, ε, which neglects the effect of partial penetration, can be defined and the well drawdown after one hour of pumping is then

$$s_w(1 \text{ hour}) = \frac{Q}{4\pi T} \left(\frac{W(u_{1 \text{hour}})}{\varepsilon} + f_s \right) \tag{7}$$

or

$$s_w(1 \text{ hour}) = \frac{Q}{4\pi T} \left(W(u_{1 \text{ hour}}) + \frac{1-\varepsilon}{\varepsilon} W(u_{1 \text{ hour}}) + f_s \right) \tag{8}$$

where

$$\varepsilon = \frac{W(u_{1 \text{ hour}})\ E}{W(u_{1 \text{ hour}}) - f_s E} \tag{9}$$

According to Eq. (8) the well loss is estimated to be

$$s_w(1 \text{ hour}) = \frac{Q}{4\pi T} \frac{1-\varepsilon}{\varepsilon} W(u_{1 \text{ hour}}) \tag{10}$$

The three bracketed terms on the right side of Eq. (8) are, respectively, the aquifer head loss coefficient, the well loss coefficient and the partial penetration head loss coefficient.

Transmissivity and Specific Capacity

Here we shall define the specific capacity, SC, as the pumping rate relative to the well drawdown after one hour's pumping

$$SC \equiv \frac{Q}{s_w(1\text{ hour})} \tag{11}$$

According to Eq. (7) the well drawdown due to aquifer loss and well loss is

$$s'_w(1\text{ hour}) \equiv s_w(1\text{ hour}) = f_s \frac{Q}{4\pi T} \equiv \frac{Q}{4\pi T}\frac{W(u_{1\text{ hour}})}{\varepsilon} \tag{12}$$

and the specific capacity of a fully penetrating well with a similar well loss is

$$SC' = \frac{Q}{s'_w}(1\text{ hour}) \equiv \frac{4\pi T\varepsilon}{W(u_{1\text{ hour}})} \tag{13}$$

For the reasons mentioned earlier we log-transform Eq. (13) to get an expression for the log-transmissivity. Let $Y = \log_{10}(T)$ and $Z = \log_{10}(SC')$ then

$$Y = \log_{10}(W(u_{1\text{ hour}})) = Z - \log_{10}(\varepsilon) - \log_{10}(4\pi) \tag{14}$$

If the transmisssivity and the storativity of the aquifer vary moderately (*e.g.* three orders of magnitude and one order of magnitude respectively), it is fair to approximate Eq. (14) by

$$Y = Z - \log_{10}(\varepsilon) + \beta_0 \tag{15}$$

where β_0 is a constant. Eq. (15) says that, for fixed ε, pairs of Y and Z will plot on a straight line with a slope of one, but due to errors this is seldom quite true. The errors may for instance be due to inaccurate measurements of the well discharge or the drawdown, or due to inaccurate timing. If the errors are independent with a mean of zero the statistical model is

$$y_i = z_i - \log_{10}(\varepsilon_i) + \beta_0 + e_i \tag{16}$$

where y_i, z_i and ε_i are observation triples of Y, Z and ε respectively and e_i is an error term.

Let us first assume that $\log_{10}(\varepsilon)$ is a random variable independent of Y and Z, which can be written as

$$\log_{10}(\varepsilon_i) = E\{\log_{10}(\varepsilon)\} + e'_i \tag{17}$$

where $E\{\log_{10}(\varepsilon)\}$ is the mean well efficiency and the errors e'_i are independent with a mean of zero. Eqs. (16) and (17) gives

$$y_i \equiv z_i + \beta'_0 + e''_i \tag{18}$$

where $\beta_0' = \beta_0 + E\{\log_{10}(\varepsilon)\}$ and $E\{e_i''\} = 0$.

One the other hand it seems obvious that the specific capacity may depend on ε. If this dependence may be written as

$$\log_{10}(\varepsilon_i) = \alpha z_i + e_i''' \qquad (19)$$

where α is a constant, and if the errors e_i''' are independent with the same mean, then Eq. (16) is transformed to

$$y_i \equiv \beta z_i + \beta_0'''' + e_i'''' \quad ; \quad \beta = 1 = \alpha \qquad (20)$$

where $\beta_0''''=\beta_0'+E(e_i''')$ and $E(e_i'''')=0$.

Normally the parameters are unknown and have to be estimated, for instance by least-squares estimation from pairs of observations of Y and Z. If the errors are independent and joint normally distributed then the least-square estimate is also the maximum-likelihood estimate.

If the estimated ß is close to one (*i.e.* α is close to zero) then the well efficiency may be regarded either as a constant or as a random variable which is uncorrelated with the specific capacity. If ß differs from one then the specific capacity and the well efficiency are correlated variables.

If the specific capacities are not corrected for the effect of partial penetration Eqs. (13) to (20) may still be used on condition that SC' and ε are replaced by SC and E.

Available Data for Present Studies

In the present study pumping tests are available for three different Danish aquifers. A general description of the data is given in the following whereas specific information is given subsequently in the case study section.

Constant rate pumping tests are available for a number of pumping wells within the aquifers. Drawdown as well as recovery tests are available for most of the wells, but for a small number of wells only drawdown or recovery tests are available. The tests have a duration ranging from half an hour to several days. The transmissivity, the well efficiency and the specific capacity can be estimated accurately from these pumping tests.

Specific capacity is obtainable for practically every well in Denmark from the database of the Geological Survey of Denmark (DGU). The capacity measurements are taken by the drilling company immediately after the wells have been developed. The approximately stationary drawdown is typically measured after one to four hours of stationary pumping but in some cases the measurement is taken after several days of pumping. The true duration of the pumping is only known for a small number of the wells.

Lithological logs of the boreholes and the screening interval of the wells are also recorded in the DGU-database.

Data Analysis

Identical methods of data analysis are applied in the three case studies. The methods are briefly described in the following.

Jacob's straight-line method is applied to estimate transmissivities from the drawdown of pumping tests and from the residual drawdown of the recovery tests (Kruseman and de Ridder 1990). The line is fitted to the first linear part of the drawdown curves (typically from 10 minutes and onwards) that are not affected by well bore storage (Krusemand and de Ridder 1990) and partial penetration, see Eq. (3). For all the tested wells the estimated transmissivity represents the local aquifer transmissivity within a distance of at least several hundred metres from the well.

The total well efficiency is estimated by Eq. (5) using storativities estimated either from the observed drawdown in observation wells or from pumping tests of nearby wells. The alternative efficiency, which neglects the effect of partial penetration, is estimated by Eq. (9).

The relative importance of aquifer head loss, well loss and head loss due to partial penetration are estimated by the respective loss coefficients of Eq. (8).

The effect of partial penetration is estimated by Eqs. (3) and (4), where the aquifer thickness and the screening interval are taken from the DGU-database and it is assumed that K_z/K_r is equal to 0.10. In the derivation of Eqs. (3) and (4) it is assumed that the aquifer is homogeneous. Therefore, if the lithological log of a borehole shows that the aquifer at a well site is stratified then only the layer of preferred horizontal groundwater flow is considered. For instance, if the upper half of an aquifer is a layer of fine sand and the lower half is a layer of coarse sand then it is likely that the horizontal flow rate of the coarse layer is at least an order of magnitude larger than in the fine layer. In this case it is assumed that the influence of partial penetration is only a function of the thickness of the coarse sand layer and the depth interval of this layer that is screened by the well.

The regression equations that relate Y and Z are estimated for each of the aquifers. The regression analysis is made for four different data sets of specific capacity: capacity observed during pumping test; the same but modified by Eq. (13) to neglect the influence of partial penetration; capacities recorded in the DGU-database; and the same corrected for partial penetration.

Case Studies

Data are available for three different types of Danish aquifers (Fig. 1): a fluvioglacial, homogeneous aquifer located in the western part of the island Zealand; fluvioglacial but heterogeneous aquifers adjacent to the previous aquifer; and a homogeneous aquifer of Tertiary limnic deposits in the southern part of Jutland.

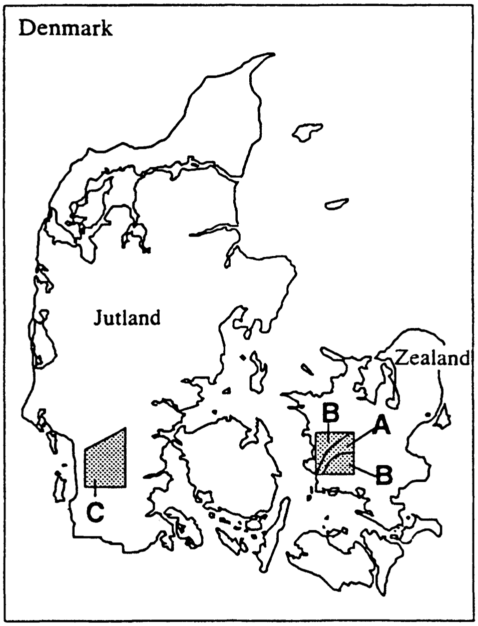

Fig. 1. Location of the studied aquifers.

Homogeneous Fluvioglacial Aquifer

In the western part of Zealand (Fig. 1) a 10 to 30 m thick layer of Quarternary fluvioglacial sand and gravel deposits forms an aquifer which extends over an area of more than 100 km^2 (see Christensen 1994). The aquifer is overlain by glacial till and underlain by Tertiary clay, *i.e.* the aquifer is confined.

More than one hundred wells penetrate the aquifer. The transmissivity, the well efficiency and the specific capacity of 51 wells have been estimated by pumping test analyses. All of the wells are recorded in the DGU-database. In 22 of the wells the aquifer is fully penetrated during the drilling, though only 7 of these screen more than 90% of the aquifer. For the remaining 29 wells the lower boundary of the aquifer and thus the partial penetration coefficient of Eq. (4) have to be estimated from the lithological logs of neighbouring wells.

Statistics for the results of the pumping tests are listed in Table 1 (μ is the sample mean; σ is the sample standard error). Y is relatively large with a moderate standard error, which indicates that the aquifer is homogeneous. Corresponding to this the standard error of the estimated aquifer head loss coefficients is small.

The efficiencies of the wells are varied but low with a mean value of 0.37 and a standard error of 0.29. Corresponding to this the wells loss coefficients are large (on average) and highly variable.

The mean values of the head loss coefficients show that about 20% of the drawdown in a typical pumping well is due to the head loss of horizontal groundwater flow, 15% of the drawndown is due to the partial screening of the aquifer and

Table 1 – Statistics for the pumping test results from the homogeneous fluvioglacial aquifer

	μ	σ
Y ≡ Log_{10} (*T*)	–2.08	0.39
Well efficiency	0.37	0.29
Aquifer loss coefficient	15.94	0.89
Partial penetration loss coeff.	11.64	12.92
Well loss coefficient	55.03	68.71

65% of the drawdown in a typical well is due to well loss. These figures are quite insensitive to the estimated K_z/K_r relationship and to the estimated aquifer thickness at some of the well sites.

Statistics of the regressions between the *Y* and *Z* are listed in Table 2. ß is the slope of the fitted line, $s_ß$ is the standard error of ß, s_y is the standard error of the residuals (which is also termed the "standard error of prediction") and r^2 is the squared sample correlation, which is named the coefficient of determination or the proportion of the variance in *Y* that is explained by the regression.

Table 2 – Statistics for regressions between specific capacity and transmissivity of the homogeneous fluvioglacial aquifer

Case	From	Spec. capacity Part. pen. Correction	No. obs.	ß	$s_ß$	s_y	r^2
a	PT	No	51	0.55	0.11	0.32	0.33
b	PT	Yes	51	0.51	0.11	0.33	0.30
c	DB	No	51	0.53	0.17	0.35	0.17
d	DB	Yes	51	0.64	0.16	0.35	0.15

PT ≡ pumping test; DB ≡ DGU-database

Case a – The estimated regression line between the *Y* and *Z*, where *Z* is obtained from the pumping tests, is shown in Fig. 2. Most of the observation points (*i.e.* 41 out of 51) have abscissas in the range from 0.5 to 1.3 and these points are widely scattered around the line. Seven points have abscissas below 0.5 and three points have abscissas above 1.3. A visual judgement of Fig. 2 leads to the conclusions, that the slope of the line primarily is determined by the few points of high and low abscissas, and that the prediction of *Y* from *Z* is very uncertain. This also appears from the fact that the standard error of prediction, 0.32 (Table 2), is almost as large as the standard error of the sample mean, 0.39 (Table 1), and that the proportion of the total variance that is explained by the regression line is only 0.33.

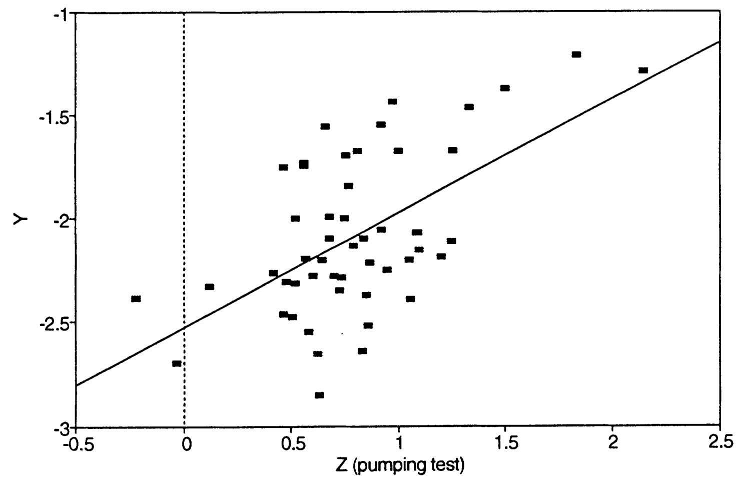

Fig. 2. Regression line of Z and Y from the homogeneous fluvioglacial aquifer.

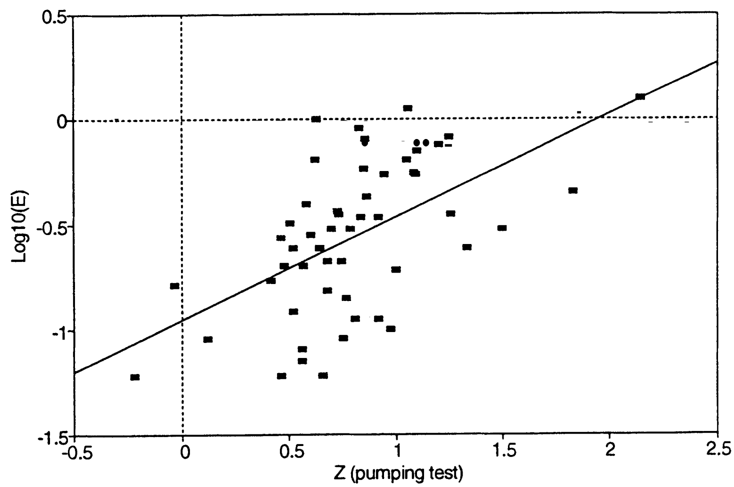

Fig. 3. Regression line of Z and $\mathrm{Log}_{10}(E)$ from the homogeneous fluvioglacial aquifer.

The slope, ß, of the fitted line is 0.55 which indicates that Z may be strongly correlated to the log-transformed well efficiency. This correlation appears directly from Fig. 3 which shows a significant positive correlation between the two variables. The estimated slope, α, of the regression line of Fig. 3 is 0.49 with a standard error of 0.10; the sample correlation is 0.55. According to Eq. (20) the equation $\beta = 1 - \alpha$ must be satisfied. The estimated slopes agrees quite well with this requirement.

The observation points of Fig. 3 are as widely scattered as the points of Fig. 2.

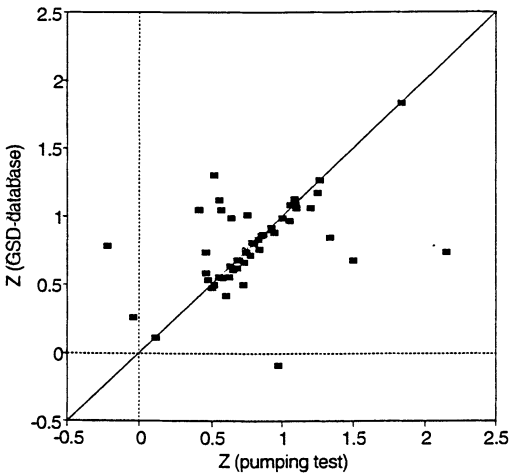

Fig. 4.
Plot of pairs of Z from the homogeneous fluvioglacial aquifer (notice the identity line).

This suggests that the poor fit of the regression line to the observations of Y and Z is mostly due to the large variability of $\log_{10}(E)$. If the regression analysis instead is made between observations of Y and Z-$\log_{10}(E)$, the linear fit is as expected from Eq. (16) practically perfect. Unfortunately $\log_{10}(E)$ is only rarely known.

Case b – In this case the specific capacities of *case a* are corrected for the influence of partial penetration. It is seen from Table 2 that the difference between the regression statistics of *case a* and *case b* is insignificant.

Cases c and d – In these cases Z recorded in the DGU-database is applied instead of Z obtained from the pumping tests. In *case d* Z is corrected for the influence of partial penetration.

In Fig. 4 Z obtained from the pumping tests is plotted against Z recorded in the DGU-database. It is seen that most of the points are plotted along the identity-line, *i.e.* they are almost identical. Some points are plotted significantly above or below the line, *i.e.* the two Z-values differ. This may be due to one or more of the following reasons: the well loss may have changed from the time of well construction to the time of the pumping test (clogging reduces Z, naturally development by pumping increases Z); the time scale for the measurements of the two Z-values are different; or the measurements or recordings are inaccurate.

There is no significant difference between the regression results of *case c* and *case d*. Further, the estimated lines of *cases c* and *d* (DGU data) are comparable to those of *cases a* and *b* (pumping test data), Table 2. On the other hand the standard errors of the estimated line slopes are larger and the coefficients of determinations are smaller in the DGU-cases than in the pumping test cases. This makes it likely that the DGU-data are more erratic than the pumping test data. Possible explanations of this have been mentioned.

Summary – The proposed filtering of the specific capacities to remove the influence of partial penetration does not improve the linear fit between Y and Z. The major reason for this is that the partial penetration only has limited influence on the total drawdown in the pumping wells. Further, the proposed corrections may be unrealistic if the true heterogeneity of the aquifer can not be recognized from the borehole logs. However, in that case it is most likely that the influence of partial penetration is overestimated in the present study.

The estimated linear equations are practically the same whether the Z data are taken from the DGU-database or from the pumping test analysis, although the prediction error is larger when the (more erratic) DGU-data are used as predictors. However, the linear regressions explain only a minor proportion of the variance in the sampled data. This is mostly due to two general facts: firstly, the well efficiency is of major importance to the specific capacity of the wells, whereas the aquifer transmissivity is of less importance; secondly, the variance of the well efficiency is large.

Heterogeneous Fluvioglacial Aquifers

Within the catchments east and west of the previous aquifer (Fig. 1) the layer of fluvioglacial deposits are heterogeneous and embedded in the glacial till. The layer is present within most of the catchments but the thickness varies within short distances. It is likely that the hydraulic conductivity is also highly variable. The fluvioglacial deposits form local confined aquifers from which the groundwater is recoverable in some areas and hardly recoverable in others. An extensive description of the hydrogeology is given by Christensen (1994).

Between one and two hundred wells penetrate these aquifers. The transmissivities of 31 wells were estimated by pumping test analysis. The average Y for these wells is -3.06 with a standard error of 0.71 (Table 3) showing that the transmissivity of the aquifers is generally smaller and more variable than that of the previous aquifer (Table 1). Of the pumping tested wells about half fully screen the aquifer and the majority of the rest screen most of the aquifer.

Table 3 – Statistics for the pumping test results from the heterogeneous fluvioglacial aquifers

	μ	σ
$Y \equiv \mathrm{Log}_{10}(T)$	-3.06	0.71
Well efficiency	0.61	0.34
Aquifer loss coefficient	13.88	1.24
Partial penetration loss coeff.	1.57	2.42
Well loss coefficient	13.63	13.18

The efficiency and the specific capacities of 16 wells were determined by pumping test analysis. The well efficiencies are in general higher (with a mean of 0.61, Table 3) than those of the homogeneous aquifer (with a mean of 0.37, Table 1).

The mean values of the estimated head loss coefficients for these 16 wells show that about half of the drawdown in a typical pumping well is due to the head loss of horizontal groundwater flow, and the other half is due to well loss, Table 3. For most of the wells the drawdown due to partial penetration is insignificant compared to the total drawdown.

The statistics of the regression analysis between Y and Z are shown in Table 4.

Table 4 – Statistics for regressions between specific capacity and transmissivity of the heterogeneous fluvioglacial aquifers

Case	Spec. capacity From	Part. pen. Correction	No. obs.	ß	s_β	s_y	r^2
e	PT	No	16	1.07	0.13	0.23	0.83
f	PT	Yes	16	1.07	0.13	0.23	0.82
g	DB	No	13	0.97	0.23	0.34	0.61
h	DB	Yes	13	0.84	0.24	0.38	0.52

PT = pumping test; DB = DGU-database

Case e – The estimated regression line between Y and Z, where Z is obtained from pumping tests, is shown in Fig. 5. The data points are scattered about the line within the entire range of observations and a visual judgement indicates that the estimated linear model fits the observations quite well. The standard error of

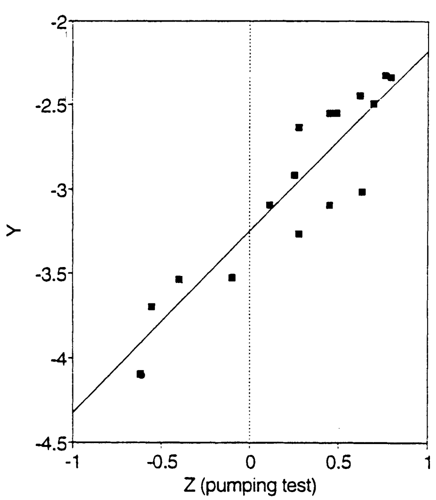

Fig. 5. Regression line of Z and Y from the heterogeneous fluvioglacial aquifer.

prediction is 0.23 and the proportion of the total variance that is explained by the regression line is 0.83.

There is no significant correlation between the specific capacity and the well efficiency (the slope of the line is estimated at 1.07).

Case f = In this case the *Z* values of *case e* are corrected for the influence of partial penetration but according to Table 4 the corrections are so small that they hardly change the results.

Cases g and h – In these cases *Z* recorded in the DGU-database is applied instead of *Z* obtained from the pumping tests. In *case h Z* is corrected for the influence of partial penetration. Table 4 shows that the statistics of *case g* and *case h* are comparable, although it seems as if the correction of *Z* slightly worsens rather than improves the linear fit.

The estimated lines in *cases g* and *h* are comparable to those from *cases e* and *f* but the standard errors of prediction are considerable larger when *Z* is taken from the DGU-database, Table 4. However, both in *case g* and *case h* the standard error of prediction is still only half of the standard error of the sample mean.

In Fig. 6 the *Z* values obtained from the pumping tests are compared to those of the DGU-database. Even though there are considerable scatter, most of the points fall below the identity-line indicating that there may be some systematic error in the *Z*-data of the DGU-database. Possible sources of error may be that the wells have not been completely developed at the time of the capacity measurements or the wells have been pumped longer than one hour.

Summary –For a typical well the drawdown is caused by the head loss of horizontal groundwater flow and the well loss, which are of comparable magnitudes. The

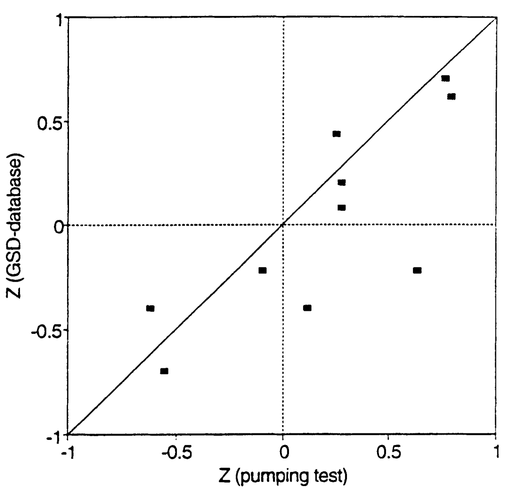

Fig. 6.
Plot of pairs of *Z* from the heterogeneous fluivoglacial aquifer (notice the identity line).

wells generally penetrate most of the aquifer and thus the head loss due to partial penetration is of no practical importance. Therefore correcting the specific capacities does not improve the linear fit to the observations.

It may be concluded that the linear model is fair in predicting *Y* from *Z* regardless of whether the latter is obtained from the DGU-database or from the pumping tests. The prediction uncertainty is mostly due to the variance of the well efficiency. Apparently there is no strong correlation between the specific capacity and the efficiency of the pumping wells.

Homogeneous Tertiary Aquifer

The third case study is from the south-western part of Jutland (Fig. 1) where aquifers are found in the Quaternary and Miocene formations. The present aquifer is in a 15 to 40 m thick Miocene formation of limnic sand that extends over an area of several thousands of square kilometres. This Ribe formation is overlain by 150-200 m of deposits of Miocene clay and sand (containing substantial amounts of mica) and Quartenary sand and till, whereas the underlying deposits are marine clay and sand from the Miocene or Oligocene.

The transmissivity, the efficiency and the specific capacity of 17 wells, which are situated within a 600 km^2 area, have been estimated by pumping test analysis. 11 wells fully penetrate the aquifer, whereas at the remaining 6 wells the lower boundary of the aquifer is estimated from geological profiles. The wells screen most or all of the formation.

The estimated values of *Y* (Table 5) are relatively high with a mean of −1.99 and a moderate standard error of 0.42 which indicates that the aquifer is homogeneous. The well efficiencies are moderate but variable with a mean of 0.50 and a standard error of 0.25.

The mean values of the head loss coefficients (Table 5) show that 60% of the drawdown in a typical well is due to well loss, 35% is due to the head loss of horizontal groundwater flow and only 5% is due to partial screening of the aquifer.

The statistics of the regression analysis between *Y* and *Z* are shown in Table 6. The four analyses are based on the data of the 17 pumping tested wells.

Table 5 – Statistics for the pumping test results from the homogeneous tertiary aquifer

	μ	σ
$Y \equiv \text{Log}_{10}(T)$	–1.99	0.42
Well efficiency	0.50	0.25
Aquifer loss coefficient	16.16	0.98
Partial penetration loss coeff.	2.16	2.81
Well loss coefficient	26.68	31.58

Table 6 – Statistics for regressions between specific capacity and transmissivity of the homogeneous tertiary aquifer

Case	Spec. capacity From	Part. pen. Correction	No. obs.	ß	s_β	s_y	r^2
i	PT	No	17	0.73	0.16	0.28	0.58
j	PT	Yes	17	0.70	0.16	0.29	0.57
k	DB	No	17	0.71	0.26	0.36	0.34
l	DB	Yes	17	0.71	0.25	0.35	0.36

PT ≡ pumping test; DB ≡ DGU-database

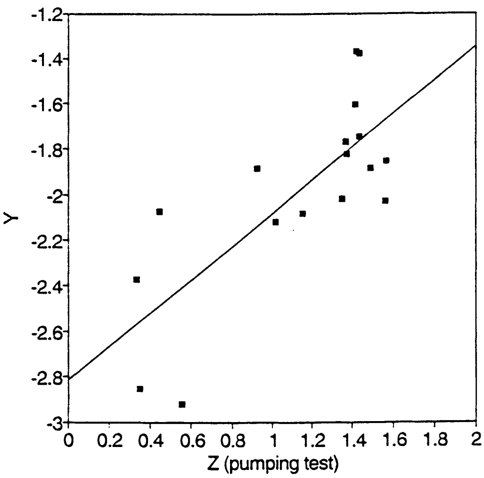

Fig. 7.
Regression line of *Z* and *Y* from the homogeneous tertiary aquifer.

Case i – In this case the *Z* values are obtained by pumping test analysis. Fig. 7 shows that the datapoints are scattered around the estimated regression line. The standard error of prediction is 0.28 which is two thirds of the standard error of the sample mean (0.42, Table 5). The proportion of the total variance that is explained by the linear model is 0.58.

The estimated slope of the regression line is 0.73 with a standard error of 0.16. A hypothesis that ß $\geq$ 1.0 can be rejected at a 7% level of significance and it therefore seems likely that *Z* may be positively correlated to the log-transformed well efficiency. However, if two points of low *Z* value are excluded from the regressions the estimate of ß is close to 1.0. Even though there is no obvious reason for excluding these points, the conclusion that *Z* and well efficiency are correlated remains uncertain.

Case j – As mentioned above the wells screen most of the formation and the correction of data due to partial penetration is therefore so small that the statistics of this regression analysis are similar to those of the previous case.

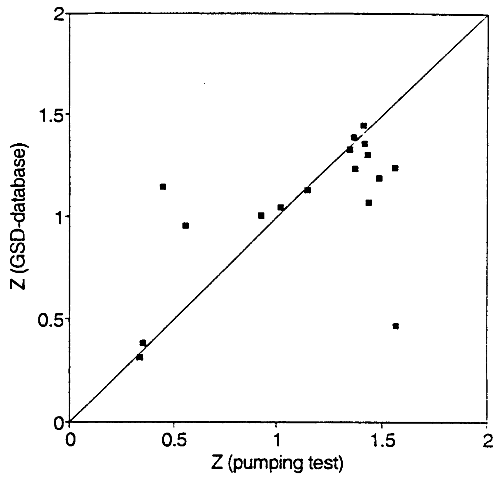

Fig. 8.
Plot of pairs of Z from the homogeneous tertiary aquifer (notice the identity line).

Cases k and l – In these cases the Z values are obtained from the DGU-database. In *case l* the data are corrected for partial penetration but as above the corrections are small and almost irrelevant.

The estimated lines are similar to those of the two previous cases but the standard error of prediction of 0.36 is approximately 25% higher that that of *case i*. The proportion of the total variance that is explained by the regression line is 0.34.

In Fig. 8 the Z values obtained from the pumping tests are compared to those obtained from the DGU-database. The points are scattered about the identity-line with a few points falling a long way off. Possible explanations of such deviations have been given earlier and the best conclusion is that the DGU-data are more erratic that the pumping test data.

Summary – The drawdown in the pumping wells are mainly caused by well loss and head loss of groundwater flow. The linear fit to the observations is therefore not improved by the proposed correction of the specific capacities to eliminate the influence of partial penetration.

The estimated linear equations are identical whether the Z-data are obtained from the pumping tests or from the DGU-database. However, the standard error of prediction is considerably larger when the latter data are applied as predictors. In this case the proportion of variance explained by the estimated model is 0.35. As in the first case study this is mainly due to the fact that the well efficiency varies greatly and plays a major role in determing the specific capacity.

Conclusion

The specific capacity is obtainable for pratically every well from the drillers log (in Denmark these logs are stored in the database of the Geological Survey of Denmark, DGU). One of the main objectives of this paper is to study the applicability of such data for the prediciton of log-transmissivity for different types of aquifers. Pumping test data and DGU-data are available for aquifers in three formations of different origins: a homogeneous formation of Quarternary fluvioglacial deposits; heterogeneous formations of Quarternary fluvioglacial deposits; and a homogeneous formation of Miocene limnic sand. The main conclusions of these studies are:

The total drawdown in the pumping wells are mainly due to well loss and to the head loss of horizontal groundwater flow. A number of the wells do only partially screen the aquifers but the equation of Hantush (1961 and 1964) shows that the head loss due to partial penetration plays a minor role in the total well drawdown. Therefore, the proposed correction of the data to remove the influence of partial penetration does not improve the predictions in any of the case studies.

Following the conclusions of Ahmed and De Marsily (1987) it is acceptable to use a linear model for the prediction of Y ($\log_{10}$-transmissivity) from Z ($\log_{10}$-transformed specific capacity) in the studied cases. The estimated linear equations are practically the same whether the Z data are taken from the DGU-database or from the pumping test analyses, but due to the fact that the former data are more erratic than the latter, the prediction variance is larger when the DGU-data are used as predictors.

For the homogeneous fluvioglacial aquifer the slope of the fitted line is primarily determined by a few points with high and low values of Z. The poor fit of the regression line is mostly due to the large variability of the well efficiency and it is therefore questionable to use the linear model to predict Y from Z. The estimated line slope is 0.5 which indicates that Z and the well efficiency are positively correlated.

For the heterogeneous fluvioglacial aquifer the linear model is fair in predicting a specific Y from Z. The estimated line slope is close to 1.0 and there is no correlation between Z and well efficiency.

For the homogeneous Tertiary aquifer the estimated line slope is 0.7. However, if two out of 17 sets of well data are excluded from the regressions the estimated slope is 1.0. It therefore remains uncertain whether Z is correlated with the well efficiency. In any case, the estimated model is a rather poor prediction model.

If Z-values of the DGU-database are applied as predictors for Y and if the regression lines are applied as prediction models then the standard error of prediction is in all the cases together approximately 0.35. The magnitude of the standard error is mainly due to the variance of the well efficiency and secondly due to random errors in the DGU-data.

If the transmissivity of an aquifer is expected to vary within one or two orders of magnitude (which is the case for the homogeneous fluvioglacial aquifer and the tertiary aquifer studied in the present paper) then an individual prediction of Y is of no practical use because the prediction variance is comparable to the natural variance of Y. However, if one needs to estimate the mean Y within a domain and if the domain holds a number of wells with known Z from which Y can be predicted, then the individual predictions may be averaged and the prediction error thus reduced. It is likely that there is a spatial covariance between Y (or Z) of the wells and therefore the procedure of averaging ought to take the covariance structure into consideration in the weighting of observations. Kriging is one such procedure, Journel and Huijbregts (1978).

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