Mathematical modeling of aeration efficiency and dissolved oxygen provided by stepped cascade aeration

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ABSTRACT

Mathematical modeling has been a vital tool in the field of environmental engineering. Various models have been developed to simulate the level of aeration efficiency (AE) provided by different aerating structures to raise levels of dissolved oxygen (DO) in streams; one of which is the stepped cascade structure. Three models developed by Gameson et al. WRL, and Nakasone, in addition to Qual2k, a computer program for stream modeling, have been used in this research; values of AEs obtained have been compared to those computed using DO measured from a built model at a WWTP. A stepped cascade structure was installed with different heights to aerate five flowrates with different levels of COD. An adjustment has been made to the Nakasone model to test the effect of pollutant load on the amount of aeration that could be reached. Values of AEs computed using the Gameson model were 30%, 39.5%, and 40% for cascade heights (Hd) 45, 60, and 75 cm respectively for the five flowrates (q) that ranged from 21–66 m3/hr. Values of AEs from WRL model were 32.8%, 42%, and 43% consequently. Values of AEs from Nakasone model ranged from 4.6–7.5%, 6–10%, and 7.6–12% respectively. For the adjusted Nakasone model, values of AEs ranged from 3.2–4.9%, 3.3–5.3%, and 4.1–6.7% respectively. Finally, the AEs computed using the values of downstream DO generated by Qual2k ranged from 4–18%, 2–15%, and 2.5–5.1% correspondingly. Around 80% of the downstream DO values computed using the Nakasone and adjusted Nakasone model were closer to those measured in the field, thus more reliable in cascade design.

Key words | aeration efficiency, cascade aeration, cascade height, COD, flowrate, mathematical modeling

INTRODUCTION

Streams provide water for uses such as agriculture, industry, power supply, and fishing yet, they receive lots of point source pollution and residual waste daily produced by the community leading to dramatic deterioration in steam quality specially the levels of DO. Different hydraulic structures have been designed to enhance the level of aeration in streams, one of which is the use of stepped cascade which has shown to be one of the cost effective promising techniques (Koduri & Barkdoll 2003) in replenishing dissolved oxygen (Metcalf & Eddy 2003) in stream water and wastewater from wastewater treatment plants (WWTPs). They have been used in various hydraulic structures for more than 3500 years (Chanson 2000).

Various equations and models have been developed for oxygen transfer over cascade stepped aerators; however it should be considered that most of them are empirical and are limited to certain situations and boundary conditions (Koduri & Barkdoll 2003). Three mathematical models have been used in this research to compute the expected amount of aeration efficiency and hence the resulting increase in downstream DO due to cascade aeration. The models used were developed by Gameson et al. WRL and Nakasone. An adjustment was done to the Nakasone model in which the value of chemical oxygen demand (COD) has been included in the equation to consider the effect of pollutant load and is
called the adjusted Nakasone equation in this research. The mathematical equations used are presented below.

**Theoretical downstream DO (C_{d1})**

This is computed using the Gameson et al. (1958) model in computing the oxygen deficit ratio \( r_1 \) such that:

(a) \( r_1 = 1 + 0.34 \times a_w b_w H_d (1 + 0.046 T) \)

where \( a_w = 0.85 \) for sewage water and \( b_w = 1.3 \) for stepped weirs (Mueller et al. 2002); \( T = \) temperature (°C);

(b) aeration efficiency \( (E_1) = E_1 = 1 - 1/r_1 \)

(c) the overall deficit ratio (\( r_{tot} \)) and efficiency (\( E_{tot} \)) for a number of cascade steps \( N \) is computed using the following equations developed by Avery and Novak (1978):

\[
\frac{r_{tot}}{E_{tot}} = r_1^n \quad E_{tot} = 1 - (1 - E_1)^n
\]

Theoretical downstream DO: \( C_{d1} = (C_{o}.(r_{tot} - 1) + C_{u})/r_{tot} \)

**Theoretical downstream DO (C_{d2})**

This is computed using the WRL (Water Research Laboratory, 1973) model in computing the aeration efficiency such that:

(a) aeration efficiency \( (E_2) \): \( E_2 = 1 - \left[ 1 + 0.38 a_w b_w H_d (1 + 0.046 T) \right]^{-1} \)

(b) oxygen deficit ratio (\( r_2 \)): \( r_2 = 1/(1 - E_2) \)

(c) theoretical downstream DO: \( C_{d2} = (C_{o}.(r_2 - 1) + C_{u})/r_2 \)

where \( r_2 \) is the overall deficit ratio of the system, \( E_2 \) is the overall efficiency of the system, \( C_{o} \) is concentration of upstream DO; \( C_{o} \) is concentration of DO at level of saturation.

**Theoretical downstream DO (C_{d3})**

This is computed using the Nakasone (1986) model in computing the aeration efficiency such that:

(a) aeration efficiency \( (E_3) \): \( E_3 = 1 - e^{-aH_{o}(q_w/d_i)} \)

where \( E_3 \) is the overall efficiency of the system

(b) oxygen deficit ratio (\( r_3 \)): \( r_3 = 1/(1 - E_3) \)

where \( r_3 \) is the overall deficit ratio of the system

(c) theoretical downstream DO: \( C_{d3} = (C_{o}.(r_3 - 1) + C_{u})/r_3 \)

**Adjusted theoretical downstream DO (C_{d3})**

This is computed after introducing the concentration of COD as a measure of level of the pollutant load to the Nakasone (1986) model in computing the aeration efficiency such that:

(a) aeration efficiency \( (E_3) \): \( E_3 = 1 - e^{-aH_{o}.(q_w/d_i).C_{cod}/d_i} \)

where: the unit of COD is in kg/l

(b) adjusted oxygen deficit ratio (\( r_3 \)): \( r_3 = 1/(1 - E_3) \)

(c) adjusted theoretical downstream DO: \( C_{d3} = (C_{o}.(r_3 - 1) + C_{u})/r_3 \)

where \( H_{o} \) is the weir drop height above the downstream water level; \( H_c \) is critical water depth on the weir. Values of \( a, b, c, \) and \( d \) are factors that depend on \( q_w \) and \( H_{o} \).

**Qwal2k**

The equation used by Qwal2k to compute aeration efficiency is that presented by Butts & Evans (1983) which was initially developed by WRL (1973) as presented above.

**Objectives**

The objectives behind this research were to investigate the effect of the following parameters on computing aeration efficiency using the above mentioned mathematical models and find the most reliable model to be used in designing a stepped cascade structure:

1. Pollutant load (COD)
2. Height of stepped cascade (\( H_{o} \))
3. Amount of flow being discharged over the cascade (\( Q \))

Those factors have been studied for the purpose of reaching the best possible means for enhancing the levels of DO in streams and/or the effluent water from WWTPs that eventually discharge into streams and lakes.

**Site description and work plan**

The model built consisted of an inlet chamber that discharges the pumped wastewater from the main channel into two long concrete channels constructed at Gabal El Asfar WWTP as shown in Figure 1a and b. The inlet chamber had dimensions of 1.5 m length × 1.5 m wide × 1.25 m deep. Each channel was 20.0 m long, 0.5 m wide and 0.9 m deep. The pumped wastewater over flew over two identical weirs to be divided equally into the channels. The pumps used had a 20.0 m head and a 25 l/s discharge rate. The steps used to form cascade were made of iron and wire mesh and filled with plastic star shape media. Variation in flowrate and height of cascade steps were tested to find their effects on the level of aeration. The tested cascade heights were 45 cm, 60 cm, and 75 cm each for a set of flowrates of \( 0.018 \text{ m}^3/\text{s} \) (66 m3/hr), 0.015 m3/s (55 m3/hr), 0.011 m3/s (41 m3/hr), 0.0086 m3/s (31 m3/hr), and 0.0058 m3/s (21 m3/hr) along with different concentrations of COD that ranged from 150 mg/l to 400 mg/l.
The concentration of dissolved oxygen, wastewater temperature and water height over the concrete weir and steps were measured at the site. The height of water over the weir and steps were measured to compute the flowrates. The steps were placed at a distance of 0.75 m from the weir of the inlet chamber. Samples were taken from different locations: inlet chamber (sample 1), just before the stepped cascade (sample 2), just after the stepped cascade (sample 3a), and at the end of the channel (sample 4) as illustrated in Figure 1a. The parameters examined included different cascade heights, flowrates, and pollutant loads, mainly COD. The samples taken during the field work were analyzed in the lab and the results are presented in Table 1.

RESULTS AND DISCUSSION

The following lines show the different results of aeration efficiency computed using the three mathematical models mentioned and are compared to those computed from the field DO readings recorded in situ. This shall be followed by a comparison between the values of DO for the different flowrates and cascade heights computed downstream the stepped cascade using the values of aeration efficiencies computed and Qual2k and are compared to the values of DO measured using the DO meter in the field.

Comparison of computed aeration efficiency for the different flowrates and cascade heights

Figures 2–5 compare the percentages of aeration efficiency for the overall system computed using the three mathematical models developed by Gameson, WRL, Nakasone, and the adjusted Nakasone model for the 45 cm, 60 cm and 75 cm cascade heights and for the five different flowrates.

Figure 2 illustrates a comparison for the aeration efficiency computed using the Gameson model for the three cascade heights and five flowrates. The results showed that as the cascade height increased from 45 cm to 60 cm to 75 cm, the cumulative aeration efficiency increased from 30% to 40% to 48% consecutively. This could be attributed to the mechanism of aeration which could have occurred over three steps: during the transfer of the water from above the weir to the level of destination, then aeration that occurs at the surface due to surface agitation and finally the entrance of the air bubbles into the water (Mueller et al. 2002). However, the percentage efficiency for every cascade height does not vary with the variation in flowrate. This is attributed to the fact that the Gameson model does not consider the amount of discharge in its equation and thus could make the model less reliable in reaching the optimal dimensions when designing cascade structures to achieve the best possible level of aeration efficiency.

Similar to the above, Figure 3 illustrates a comparison for the aeration efficiency computed using the WRL model for the three cascade heights and five flowrates. The results show that as the cascade height increased from 45 cm to 60 cm to 75 cm, the overall aeration efficiency increased from 32% to 41% to nearly 50% successively. However, again the percentage efficiency for every cascade height does not vary with the variation in flowrate. This is ascribed to the fact that the WRL model does not consider the amount of discharge in its equation and thus could make the model less reliable in reaching the optimal dimensions when designing cascade structures to achieve the best possible level of aeration efficiency.

Comparable to the above, Figure 4 illustrates a comparison for the aeration efficiency computed using the...
Nakasone model for the three cascade heights and five flowrates. The results show that as the cascade height increased, the cumulative aeration efficiency of the system increased for cascades located at the beginning of the channel. However, it is noticed that as the flowrate decreased the percentage aeration also decreased which complies with the findings of Toombes (2002) where he stated that, ‘’the increase in aeration efficiency is greatest for the higher flowrates’’. This also complies with the findings of Chapman (1996) as he found that as the flow increase, higher turbulence would occur at the water surface and thus increase the surface area of the water through which gas could be transferred. It is worth mentioning here that the Nakasone model does consider the amount of discharge in its equation and thus the model would be more reliable in reaching the optimal dimensions when designing cascade structures to achieve the best possible level of aeration efficiency compared to the previous two models.

### Table 1

Characteristics of wastewater for different cascade heights and flowrates.

<table>
<thead>
<tr>
<th>Flowrate (m³/hr)</th>
<th>Cascade Heights – 45 cm</th>
<th>Cascade Heights – 60 cm</th>
<th>Cascade Heights – 75 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inlet chamber</td>
<td>Before steps</td>
<td>After steps</td>
</tr>
<tr>
<td>21</td>
<td>SS (mg/l)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>COD (mg/l)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>BOD₅ (mg/l)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>31</td>
<td>SS (mg/l)</td>
<td>148</td>
<td>138</td>
</tr>
<tr>
<td></td>
<td>COD (mg/l)</td>
<td>288</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>BOD₅ (mg/l)</td>
<td>–</td>
<td>173</td>
</tr>
<tr>
<td>41</td>
<td>SS (mg/l)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>COD (mg/l)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>BOD₅ (mg/l)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>53</td>
<td>SS (mg/l)</td>
<td>116</td>
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</tr>
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<td>COD (mg/l)</td>
<td>240</td>
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<tr>
<td></td>
<td>BOD₅ (mg/l)</td>
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<td>66</td>
<td>SS (mg/l)</td>
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<td></td>
<td>COD (mg/l)</td>
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</tr>
<tr>
<td></td>
<td>BOD₅ (mg/l)</td>
<td>188</td>
<td>190</td>
</tr>
</tbody>
</table>

*Figure 2* | Comparison of $E_1$ corresponding to different $Q$ & $H_d$. Subscribers to the online version of Water Science and Technology can access the colour version of this figure from [http://www.iwaponline.com/wst](http://www.iwaponline.com/wst).

*Figure 3* | Comparison of $E_2$ corresponding to different $Q$ & $H_d$. Subscribers to the online version of Water Science and Technology can access the colour version of this figure from [http://www.iwaponline.com/wst](http://www.iwaponline.com/wst).

$$r = 1 + 0.34a_b H_d (1 + 0.046 I) \quad E = 1 - \frac{1}{r}$$

$$E_f = 1 - \frac{1}{1 + 0.38a_b H_d (1 + 0.046 I) (1 - 0.11 H_d)}$$

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*Figure 2* | Comparison of $E_1$ corresponding to different $Q$ & $H_d$. Subscribers to the online version of Water Science and Technology can access the colour version of this figure from [http://www.iwaponline.com/wst](http://www.iwaponline.com/wst).

*Figure 3* | Comparison of $E_2$ corresponding to different $Q$ & $H_d$. Subscribers to the online version of Water Science and Technology can access the colour version of this figure from [http://www.iwaponline.com/wst](http://www.iwaponline.com/wst).
Comparable to the above, Figure 5 illustrates a comparison for the aeration efficiency computed using the adjusted Nakasone model for the 3 cascade height and 5 flowrates after introducing the concentration of COD into the model. The results show that as the cascade height increased, the aeration efficiency increased for cascade located at the beginning of the channel. Similar to the above model, it is observed that as the flowrate decreases the percentage of aeration also decreases. However, it is noticed that the percentages of aeration efficiency computed here less than those computed using the original Nakasone equation. This could be contributed to the effect of considering the level of pollutant load in the wastewater and which show how it could directly have an attainable effect on the amount of aeration.

Comparison of computed and measured DO for the different flowrates and cascade heights

For the 45 cm cascade height

Figures 6–8 represent a comparison between the values of DO for the different flowrates recorded and computed upstream and downstream the stepped cascade using the above mathematical equations and the computer model and are compared to the values of DO measured using the DO meter in the field and the following could be concluded.
Values of downstream DO were always greater than the upstream concentration which proves the constructive and practical effect of using stepped cascade in ameliorating the level of DO in water and hence would aid in the rehabilitation process of streams.

Values of downstream DO computed using the Gameson model \( (C_{d1}) \) and WRL model \( (C_{d2}) \) are nearly the same and are higher than the field measured downstream DO \( (C_d) \). This could be attributed to the fact that these models consider the height of the cascade and the water temperature; however they neither consider the flowrate of the water nor the pollutant load present although both parameters do affect the level of aeration according to Chapman (1996).

Values of downstream DO computed using the Nakasone model appeared to be much closer to the values of DO recorded at the field. The Nakasone model considered the flowrate in its calculations and thus would provide more reliable results as shown in the graphs compared to the other two models.

Values of downstream DO computed using the Nakasone model after introducing the pollutant load factor into the equation were very close to the prime state of the equation without the added factor and were also closer to the values of DO measured directly in the field compared to the Gameson and the WRL models. Thus the adjusted equation considers the effects of cascade height, the amount of discharged flow and the concentration of COD as a pollutant factor that signifies the level of pollution in the water; in other words the equation combines all the important parameters that affect the level of aeration and hence could be more dependable in designing cascade structures needed to be installed in streams that are in need for rehabilitation. It is worth mentioning that COD is one of the easiest and quickest parameters that could be measured in the lab as it takes 2 hours to measure COD; other COD reactors are designed to provide instant readings of COD such as the visible light spectrometer spectrophotometer (Jikra & Carter 1975) compared to BOD which requires 5 days to be measured.

The value of downstream DO resulting from the computer model differed from that obtained from the field by 5% for the 66 m³/hr flowrate, 17% for the 53 m³/hr flowrate and by 20% for the 31 m³/hr flowrate. The output DO values from the computer model were very close to those computed using the Nakasone and adjusted Nakasone model for the three flowrates as illustrated in the figures below. This could be attributed to the fact that Qual2k considers the amount of flow and the concentration of pollutant load (COD in this case) in calculating the downstream concentration of DO in the stream. Thus the results show the importance of considering pollutant loads in streams which would support the adjusted Nakasone equation in considering the concentration of COD while computing the aeration efficiency.

For the 60 cm cascade height

Figures 9–12 represent a comparison between the values of DO for the different flowrates recorded and computed upstream and downstream the stepped cascade using the above mathematical equations and the computer model and are compared to the values of DO measured using the DO meter in the field. Similar to the 45 cm cascade height, values of downstream DO measured using the Nakasone and adjusted Nakasone equations are much closer to the values of DO measured directly in the field compared to the Gameson and the WRL models.

The value of downstream DO resulting from the computer model differed from that obtained from the field by 16%...
For the 66 m³/hr flowrate, 25% for the 53 m³/hr flowrate, 28% for the 31 m³/hr flowrate, and by 5% for the 21 m³/hr flowrate.

The output DO values from Qual2k were quite equivalent to those computed using the Nakasone and adjusted Nakasone model for the four flowrates as presented in the figures below. Again, this could be attributed to the fact that Qual2k considers the amount of flow and the concentration of pollutant load (COD in this case) in calculating the downstream concentration of DO in the stream. Thus, the results show the significance of considering pollutant loads in streams which would support the adjusted Nakasone equation in considering the concentration of COD while computing the aeration efficiency.

For the 75 cm cascade height

Figures 13–16 represent a comparison between the values of DO for the different flowrates recorded and computed upstream and downstream the stepped cascade using the mathematical equations presented above and Qual2k and are compared to the values of DO measured using the DO meter in the field. Most of the DO values computed using the theoretical models appear to be quite close to results of DO measured at the field. However, it is noticed that the theoretical DO values computed for the highest flowrate, 66 m³/hr using the adjusted Nakasone model is
the closest to the measure DO in the field and thus would be more reliable compared to the other models. Consequently, it could be more helpful in selecting the optimum cascade dimensions to provide the optimal level of aeration efficiency.

The value of downstream DO resulting from the computer model differed from that obtained from the field by 21% for the 66 m³/hr flowrate, 25% for the 53 m³/hr flowrate, 27% for the 41 m³/hr flowrate, and by 42% for the 21 m³/hr flowrate. The last output result could be discarded from the comparison as 42% difference is quite high. This could be due to a false DO measurement in the field. The output DO values from the computer model appear to comply with those computed using the Nakasone and adjusted Nakasone model for the four flowrates as shown in the figures. Again, this could be attributed to the fact that Qual2k considers the amount of flow and the concentration of pollutant load (COD in this case) in calculating the downstream concentration of DO in the stream. Thus the results show the value of considering pollutant loads in streams which would support the adjusted Nakasone equation in considering the concentration of COD while computing the aeration efficiency.

**CONCLUSIONS**

Outcomes from this research show the promising effect of cascade aeration in enhancing the levels of DO and would be recommended towards using them more repeatedly in stream restoration and for effluent wastewater from primary and secondary units in WWTPs. They also demonstrate the significance of considering the prevailing water and wastewater quality in terms of pollutant load (such as COD) and flowrate to be able to reach an optimum cascade design with a maximum possible performance level at an economic cost. Specific conclusions withdrawn from this work include the following.

- Values of aeration efficiency computed by Gameson and WRL did not change for the different flowrates and concentrations of COD whereas those calculated by the other two models showed a variation in the values of the aeration efficiency.
- For the 45 cm cascade height: the percentage of aeration efficiencies computed using Gameson et al. model was 30%, using the WRL model was 32.8%, using the Nakasone model, the values ranged from 4.6% to 7.5% and for the adjusted Nakasone model, values of aeration efficiency ranged from 3.2% to 4.9%.
- For the 60 cm cascade height: the percentage of aeration efficiencies computed using Gameson et al. model was 39.6%, using the WRL model was 42.3%, using the Nakasone model, the values ranged from 6.0% to 10% and for the adjusted Nakasone model, values of aeration efficiency ranged from 3.3% to 5.3%.
- For the 75 cm cascade height: the percentage of aeration efficiencies computed using Gameson et al. model was 40%, using the WRL model was 43%, using the Nakasone model, the values ranged from 7.6% to 12.1% and for the adjusted Nakasone model, values of aeration efficiency ranged from 4.1% to 6.7%.
- Around 80% of the downstream dissolved oxygen concentrations and the aeration efficiencies computed for the different cascade heights and flowrates using the Nakasone and adjusted Nakasone model have shown to be in compliance with the downstream DO measured at the field and the corresponding level of aeration efficiency. Consequently, they would be more reliable in using them to design an optimum stepped cascade structure that would provide the highest possible level of aeration that would result in higher levels of dissolved oxygen compared to the other two mathematical models used.
- Qual2k utilized the equation developed by WRL as previously presented; however, it should be considered that the computer model has the option of considering the concentration of various pollutant loads, one which is COD which has been considered in computing the levels of dissolved oxygen.
- The model was validated and its outcomes have shown to comply with the measured values of downstream dissolved oxygen in the field in a range that varied from 5%–27%.
This shows that the program could be a promising and supportive tool in simulating stepped cascade to aerate stream water as it would facilitate computation and would provide designers with a preliminary assumption about the dimensions of the cascade that would need to be installed depending on the available boundary conditions.

Further investigation in this area could include the use of the mathematical models in studying the effect of cascade aeration for different pollutant loads, different temperatures (as in hot and cold climates), higher flowrates, and steps of longer width.

REFERENCES


