If \( f \) is replaced by \( x \), and derivatives of \( x \) replaced by

\[
x^{(j)} = A^j x + \sum_{k=0}^{j-1} A^{j-1-k} B \{x^{(k)}\},
\]

which can be derived from equation (1), equation (2) follows.

It is interesting to compare equation (2) with the formally exact solution

\[
x(h) = \exp(A h) x(0) + \int_0^h \exp(A(h - t)) B \text{d}t.
\]

It will be seen that \( \exp(A h) \) is approximated by a rational function; the expression is, in fact, the Padé approximation, which is best in the sense that, given the degrees of numerator and denominator, the coefficients are chosen so that the Taylor series for the approximation agrees with that of \( \exp(A h) \) for as many terms as possible, up to the term in \( (Ah)^{2n} \) in the present case.

Yours faithfully,

W. E. THOMSON

P.O. Research Station,
London, N.W.2.
August 1967

References
