Pseudoscalar Interaction in the Theory of Beta-decay and RaE

Hisao TAKEBE

Department of Physics, University of Tokyo

(Received October 27 1953)

The correction factors for the linear combinations of pseudoscalar and tensor interaction, and of pseudoscalar and axial vector interaction for the first forbidden $0 \rightarrow 0$ (yes) transition are deduced by taking the following into consideration; i) the effect of the finite size of nucleus, ii) contribution of a term which contains a derivative of lepton wave function in the case of pseudoscalar interaction. The beta-ray spectrum of RaE is examined by the linear combination of pseudoscalar and tensor interaction.

§ 1. Introduction

Of the Fermi theory of beta-decay the following seem to be currently accepted as having been established that the scalar ($S$) or the vector ($V$) interaction is necessary to explain the decay schemes of C$^{10}$ and O$^{14}$, from the necessity of Gamow-Teller selection rule to account for the short life transitions with a unit spin change the axial vector ($A$) or the tensor ($T$) interaction must be included, and the $T$ interaction is more favoured than the $A$ interaction by the experiments on the correlation between electron and recoil nucleus in He$^6$ decay. By these arguments and the so-called Fierz interference the combination $(S, T)$ or $(V, T)$ seems to fit in with the experimental data. On the other hand, as to the pseudoscalar ($P$) interaction, we have no evidence for its necessity except one case, namely, the explanation of beta-ray spectrum of RaE by Petschek and Marshak with $(T, P)$ combination.

To explain the beta-ray spectrum of RaE, Petschek and Marshak have at first calculated the correction factor, assuming that the nuclear charge is concentrated at the centre of nucleus, and then the effect of the finite size of nucleus, calculated by Rose and Holmes, taken into account. This procedure seems to be incomplete because of the following three reasons. First, although we must take 84 for the decay of RaE as the atomic number $Z$, the effect of the finite size of nucleus calculated by Rose and Holmes is that for $Z=83$. Secondly, Petschek and Marshak have taken the effect of the so-called finite deBroglie wave length into consideration in their calculation of the correction factor without the effect of the finite size of nucleus, but Rose and Holmes have not considered the former effect. So it may become meaningless to combine these two calculations. Thirdly, if the pseudoscalar nuclear matrix element $\langle P \rangle$ is small, the procedure to evaluate the radial part

*) The contents of this paper were read at the Kyoto meeting of the Physical Society of Japan on the 7th May, 1953.
of the lepton wave function at the boundary of the nucleus becomes incomplete. Then we must proceed to the next order of approximation, and should consider the term which contains a derivative of lepton wave function. Ahrens et al.\textsuperscript{10)} have pointed out this fact, too. According to Ruderman's\textsuperscript{12)} calculation, however, the matrix element \( \beta_{T_5} \) is not so small owing to the so-called pair term, if the nuclear force arises from pseudoscalar mesons with pseudoscalar coupling. But Brueckner et al.\textsuperscript{14)} have shown, although their calculations is incomplete as they have considered only the simplest case, that there is a possibility to reduce this term. Even if the nuclear matrix element \( \beta_{T_5} \) is not small, the term containing the derivatives of lepton wave function may contribute to the correction factor, for, in order to explain the decay of RaE by the linear combination \((T, P)\) the main terms should cancel each other.

By the above mentioned reasons it seems worth while to reexamine the pseudoscalar interaction and the beta-ray spectrum of RaE. Therefore, we have deduced the matrix element for pseudoscalar interaction in § 2, considered the deviation of the lepton wave function due to the effect of the finite size of nucleus in § 3, deduced the correction factors for the \( P, (T, P) \) and \( (A, P) \) interaction in § 4, and lastly in § 5 tried to explain the beta-ray spectrum of RaE using these results.

§ 2. Pseudoscalar interaction

The matrix element for the pseudoscalar interaction may be written as:

\[
H = \lambda_p \int d\tau (\mathbb{F}_J^* Q_k \beta_{T_5} \Phi_i) (\psi^* \beta_{T_5} \phi).
\]  

In this expression \( \lambda_p \) is the coupling constant of the nuclear particle with the lepton field, \( \Phi_i \) and \( \Phi_f \) are the wave functions of the initial and final nuclei, \( Q_k \) is an operator which replaces \( \Phi_i \) with a wave function describing a nucleus in which the \( k \)-th neutron is replaced by a proton. The quantities \( \psi \) and \( \phi \) are the quantized wave functions of the electron and anti-neutrino, respectively, normalized to one particle in a sphere of unit radius.

To use only the positive energy solutions of the Dirac equation\textsuperscript{14)}, we consider an operator

\[
C = \beta \gamma_y
\]

then

\[
(\psi^* (+) \beta_{T_5} \phi (-)) = (\psi (+) C \beta_{T_5} \phi (+))^*
\]

\[
= i (-\psi_{\beta_1} + \psi_{\beta_2} + \psi_{\beta_3} + \psi_{\beta_4})^*
\]

\[
= \sum_{k=1}^{4} R_K(\rho) \Theta_k(\theta, \varphi).
\]

(3)

\( \psi \) and \( \phi \) are the components of \( \psi (+) \) and \( \phi (+) \), respectively, where \( \phi (+) \) and \( \phi (-) \) represent the positive and negative energy state solutions of Dirac equation. The \( \psi_{\beta_i} \) for
Pseudoscalar Interaction in the Theory of Beta-decay and RaE

675

the case of pure Coulomb potential are calculated by Rose. For $\phi_i$, see reference 14).

$R_k(r)$ and $\theta_k(\theta, \varphi)$ are defined as follows:

$$
\begin{align*}
R_1(r) \theta_1(\theta, \varphi) &= -i(\phi_1 \phi_2)^*, \\
R_2(r) \theta_2(\theta, \varphi) &= i(\phi_2 \phi_3)^*, \\
R_3(r) \theta_3(\theta, \varphi) &= -i(\phi_3 \phi_4)^*, \\
R_4(r) \theta_4(\theta, \varphi) &= i(\phi_4 \phi_1)^*.
\end{align*}
$$

(4)

If we expand $R_k(r)$ at the nuclear radius $\rho$, we get for eq. (1):

$$
H = \lambda_p \sum_{k=1}^4 \left[ R_k(\rho) \int Q \beta \gamma_5 \theta_k(\theta, \varphi) + \sum_{m=0}^\infty \frac{R_k(n)(\rho)}{n!} \int Q \beta \gamma_5 (r-\rho)^n \theta_k(\theta, \varphi) \right]
$$

(6)

For the sake of simplicity, we have omitted $dr$, $\gamma_f$, and $W$ in the expression (6). $R_k^{(n)}(r)$ is the $n$-th derivative of $R_k(r)$ with respect to $r$. The first term is the customarily used transition matrix element for the $P$ interaction.\(^{10,17}\)

The effect of nuclear force affects the results only if the nuclear force arises from pseudoscalar mesons with pseudoscalar coupling. So we assume that nucleon satisfies the following Dirac equation:

$$
(E + \alpha \cdot p + \beta M - i \beta \gamma_5 U) \psi = 0.
$$

(7)

$E$, $p$, and $M$ stand for energy, momentum and mass of nucleon; $-i \beta \gamma_5 U$ is the potential for the nucleon.

If we consider only the first two terms in (6), then

$$
H = \lambda_p \sum_{k=1}^4 \left[ R_k(\rho) \int Q \beta \gamma_5 \theta_k(\theta, \varphi) + \frac{i}{2M} R_k'(\rho) \int Q \cdot (\sigma \cdot \gamma + 2U)(r-\rho) \theta_k(\theta, \varphi) \right].
$$

(8)

Hereafter, we confine ourselves to the transition $0 \rightarrow 0$ (yes). In this case

$$
\theta_k(\theta, \varphi) = \frac{1}{4\pi}, \ k = 1, 2, 3, 4,
$$

(9)

and if we put

$$
R(r) = \sum_{k=1}^4 R_k(r),
$$

(10)

eq(8) becomes as follows:

$$
H = \lambda_p \gamma \frac{i}{2M} \left[ \int R'(\rho) + R(\rho) \right] \int \sigma \cdot r,
$$

(11)

where
According to the calculation of Ahrens et al.\textsuperscript{9,10},

\[ \Gamma/\rho = A^3 (aZ/2\rho)^2, \quad A = 1 + (W_0 - 2.5) A^{1/3}/Z. \]  

This relation may be correct as an average over a range of \( A, Z \) values, but it seems somewhat inadequate to apply this to individual elements. Hence we regarded \( \Gamma/\rho \) as a parameter of which value should be determined from experimental data.

The second term in the numerator of \( \lambda' \) is of the next order to the first as to \( \rho \), and if we neglect that term, \( \lambda' \) becomes 1. But as we do not know to what extent \( U \) contributes, we will write \( \lambda' \) instead of 1. The fact that Ruderman could make \( \int \beta \gamma_s \) large is due to the effect of \( U \). We therefore may not be able to take \( \lambda' \) as 1, that is, if \( \int \beta \gamma_s \) is really large. And, as it may be seen in the following chapters, to take \( \lambda' \) as 1 or not is only equivalent to multiplying \( \lambda_p \) and \( \Gamma/\rho \) by constant factors or not.

\section*{§ 3. The effect of the finite size of the nucleus}

To reexamine the beta-ray spectrum of RaE, we must make eq. (13) include the effect of the finite size of the nucleus. This effect has been calculated by Rose and Holmes\textsuperscript{7} and Malcolm\textsuperscript{10}. As it has been mentioned in the introduction, Rose and Holmes have computed for \( Z=83 \), and as they have not considered the effect of the finite deBroglie wave length, we cannot use their result in our case. Malcolm has calculated the effect for \( Z=84 \), but his work came to our notice after the present investigation was finished and we could not use his results. The correction of the second term of eq. (13) by the finite nucleus has been calculated by neither Rose and Holmes nor Malcolm.

For these reasons, we had to calculate the effect of the finite size of nucleus for \( Z=84 \). We have done it according to the procedure given by Rose and Holmes. We use their notation in the following discussion.\textsuperscript{8)}

\textsuperscript{8)} \( x = \pm (j+1/2) \); \( \psi_n(r) = r f_n(r), \quad \omega_n(r) = r g_n(r) \), where \( f_n \) and \( g_n \) are regular radial wave functions in a Coulomb field. When we consider the effect of the finite nucleus, \( \psi_n(r) \) and \( \omega_n(r) \) are replaced by \( \Theta_n(r) \) and \( \Gamma_n(r) \).

\[ \mu_n' = \left( \frac{\psi / \Theta}{\psi^{(1)} / \Theta^{(1)} - \Theta / \psi} \right)_{x, at p}, \quad \mu_n = \left( \frac{\psi / \Theta}{\psi^{(1)} / \Theta^{(1)} - \Theta / \psi} \right)_{x, at p}. \]

\( \psi \) and \( \Theta \) are irregular solutions for pure Coulomb field. \( \psi_n^{(1)} \) and \( \Theta_n^{(1)} \) are regular solutions for the potential \( V_s(r) \) which is determined by the charge distribution inside the nucleus. The \( x \) which is outside the parentheses is common for all the quantities inside.
Rose and Holmes have calculated this effect for
\[ L_x = \left( \frac{1}{2} \rho^2 F \right) \cdot \left( \frac{\partial^2}{\partial \rho^2} - \frac{m^2}{\rho^2} \right) \] etc.,
but we have calculated the following quantities for the convenience of later calculations:
\[ A_x = \frac{\Phi_x(\rho)}{\partial^2 \Phi_x(\rho)} \right) / \partial \rho^2 \]
\[ B_x = \frac{\Phi_x(\rho)}{\partial^2 \Phi_x(\rho)} \right) / \partial \rho^2 \]
\[ C_x = \frac{\Phi_x'(\rho)}{\partial^2 \Phi_x'(\rho)} \right) / \partial \rho^2 \]
\[ D_x = \frac{\Phi_x''(\rho)}{\partial^2 \Phi_x''(\rho)} \right) / \partial \rho^2 \]
(14)
(15)
The primes attached to \( \Phi, \Phi' \) etc. denote the derivatives with respect to \( \rho \).
The results of numerical calculations are shown in Figs. 1, 2, 3 and 4 for the uniform nuclear charge distribution.

As it is unnecessary in the following discussions to consider the cases \( |x| \geq 3 \), we have not computed these cases. The greater the value of \( |x| \) the more the wave function is pushed away, and the effect of the finite nucleus becomes smaller and smaller. Then this effect may be neglected for these cases except the very special cases e.g. the case where the correction factors cancel each other almost entirely when some types of interaction are combined.

---

Fig. 1, \( A_x \)

Fig. 2, \( B_x \)
§ 4. Correction factors

The correction factor $C_p$ of pseudoscalar interaction for $0 \rightarrow 0$ (yes) transition may be written:

$$C_p = \frac{1}{4} \frac{\lambda^2}{F(Z, W) (2 M_p)^2} \left| \sum_{j_1, l_1, m_1} \sum_{j_2, l_2, m_2} \left| r_{j_1 l_1 m_1} \right|^2 \left| R'(\rho) + \Gamma R(\rho) \right|^2 \right| (18)$$

in which $W$ is the electron energy, $\rho = (W^2 - 1)^{1/2}$ is its momentum, $q$ is the neutrino momentum, and $j_1, l_1, m_1$ and $j_2, l_2, m_2$ are the total angular momentum quantum number, azimuthal quantum number and magnetic quantum number of electron and neutrino, respectively. And

$$F(Z, W) = 4 (2 \rho \gamma)^{2 \gamma - 2} \gamma^{\alpha Z W / \rho} | \Gamma (\gamma + i \alpha Z W / \rho) |^2 / \Gamma^3 (2 \gamma + 1), \quad (19)$$

where

$$\gamma = \sqrt{1 - u^2 Z^2Z^\gamma}.$$
In this case, we can consider the linear combinations $(T, P)$ and $(A, P)$. And we have calculated not only $C_p$, but also the cross correction factors for these two combinations. In such cases where $R'(\rho)$ does not exist, these are calculated by Smith \cite{20} and Pursey \cite{21}, and for neutron decay by Kotani et al. \cite{20}.

The results are:

$$C_p = \frac{\lambda'}{2M} \left| \int \sigma \cdot r \right|^2 \left[ l_{0p} + 2l_{1p} + 2l_{2p} + l'_{2p} + 2l_{3p} + l_{4p} + m_{0p} + 2m_{1p} + m_{2p} + 2n_{0p} + 2n_{1p} + 2n_{2p} + 2n_{3p} + n_{0p} \right]$$

$$+ \frac{2\rho^2 \rho^2}{(3!)^2} \frac{F_1(Z, W)}{F(Z, W)} l_{0p}'$$ \hspace{1cm} (20)

$$C_{p1} = \frac{\lambda'}{2M} \left| \int \sigma \cdot r \right|^2 \left[ l_{0p} + l_{1p} + l_{2p} - m_{0p} - m_{1p} - m_{2p} - n_{0p} - n_{1p} - n_{2p} - n_{3p} - n_{4p} \right]$$

$$- n_{0p} - n_{1p} - n_{2p} - n'_{3p} - n_{3p} - n'_{4p}$$ \hspace{1cm} (21)

$$C_{Ap} = \frac{\lambda'}{2M} \left| \int \sigma \cdot r \right|^2 \left[ l_{0Ap} + l_{1Ap} + l_{2Ap} + m_{0Ap} + m_{1Ap} + m_{2Ap} + n_{0Ap} + n_{1Ap} + n_{2Ap} + n_{3Ap} + n_{4Ap} \right]$$

$$+ n_{0Ap} + n_{1Ap} + n_{2Ap} + n_{3Ap} + n_{4Ap} + n_{5Ap} + n_{6Ap} + n_{7Ap}. \hspace{1cm} (22)

In this calculation, we have assumed

$$\int \beta \sigma \cdot r = - \int \sigma \cdot r. \hspace{1cm} (23)

Above are used the abbreviations:

$$F_1(Z, W) = (4!)^2 (2\rho^2)^{2\lambda - 1} e^{\rho Z W / \rho} | \Gamma(\gamma' + iuZW / \rho) |^2 I^2(2\gamma' + 1),$$

and

$$l_i = l_i^L \frac{x_{ik} + y_{ik}}{2} + l_i^R \frac{x_{ik} - y_{ik}}{2}$$

$$m_i = m_i^L \frac{x_{ik} + y_{ik}}{2} + m_i^R \frac{x_{ik} - y_{ik}}{2}$$

$$n_i = n_i^L \frac{x_{ik} + y_{ik}}{2} + n_i^R \frac{x_{ik} - y_{ik}}{2}$$

\hspace{1cm} (24)

\[ i = 0, 1, 2, \ldots, \lambda = P, TP, \text{ and } AP \]

$l_i^L, l_i^R, m_i, m_i^L, n_i^L$ and $n_i^R$ are given as follows:

$$l_i^L = (1 + \gamma) / 2,$$
\[ I_0^s = - \frac{1 + \gamma - \omega^2 Z^2}{2}, \]
\[ I_1^s = \frac{-1}{2r + 1} \left\{ \frac{r}{W} + (2r + 3) W \right\} \cdot \frac{uZ}{2\rho}, \]
\[ I_2^s = (r + 1) \cdot \frac{uZ}{2\rho}, \]
\[ I_3^s = \frac{1}{2(2r + 1)(2r + 2)} \left\{ - \frac{(\gamma + 1)^2 \rho^2 + 2u^2 Z^2 \{ \gamma + (r + 2) W \}^2}{W} \right\}, \]
\[ I_4^s = \frac{1}{2(2r + 1)(2r + 2)} \left\{ \gamma (\gamma + 1)^2 \frac{\rho^2}{W} - 2u^2 Z^2 (\gamma + 1)^2 W \right\}, \]
\[ I_5^s = \frac{2}{(2r + 1)^2} \left\{ - \frac{1 - \gamma}{\omega^2 Z^2} \rho^2 + 2 \{ 1 + 2r + 2 (r + 2) W \} W \right\} \cdot \left( \frac{uZ}{2\rho} \right)^2, \]
\[ I_7^s = - \frac{2}{(2r + 1)^2} \left\{ - \frac{1 - \gamma}{\omega^2 Z^2} \rho^2 + \left\{ \frac{1}{W} + (8r + 9) W - 4u^2 Z^2 W \right\} \right\} \cdot \left( \frac{uZ}{2\rho} \right)^2, \]
\[ I_8^s = \frac{1}{(2r + 1)^3 (2r + 2)} \left\{ \gamma (\gamma + 1) (\gamma + W) \frac{\rho^2}{W} + 2r (r + 3) \rho^2 W \right. \]
\[ \left. - 2u^2 Z^2 W \{ 3 (\gamma + W)^2 + 2 (r + 1) W \} \right\} \cdot \frac{uZ}{2\rho}, \]
\[ I_9^s = \frac{-1}{(2r + 1)^3 (2r + 2)} \left\{ \gamma (\gamma + 1)^2 + 2 \gamma (\gamma + 3) \right\} \rho^2 \]
\[ - 2u^2 Z^2 \{ (3\gamma + 4) W^2 - \gamma \} + 4u^2 Z^2 W^2 \right\} \cdot \frac{uZ}{2\rho}, \]
\[ I_4^t = \frac{1}{2(2r + 1)^3 (2r + 2)^2} \left\{ (\gamma + 1)^3 \rho^4 - 4u^2 Z^2 \rho^4 \{ \gamma (\gamma + 1) + (r^2 + 4r + 1) W \} \right. \]
\[ \left. + 4u^2 Z^3 W^2 \{ 2 (\gamma + 1) + (r + 3) W \} \right\}, \]
\[ I_5^t = \frac{-1}{2(2r + 1)^3 (2r + 2)^2} \left\{ (\gamma + 1)^3 \frac{\rho^4}{W} + \omega^2 Z^2 \rho^4 \left\{ \frac{(\gamma + 1)^2}{W} \right. \right. \]
\[ \left. \left. - (9 \gamma^2 + 22 \gamma + 5) W \right\} + 4u^2 Z^4 \{ - \gamma + (4\gamma + 5) W \} - 4u^2 Z^6 W^3 \right\}], \]
\[ I_0^{st} = 2 + \gamma, \]
\[ I_0^{st} = - (2 + \gamma - \omega^2 Z^3 / 2), \]
\[ m_0^L = 2 \cdot \frac{1 - \gamma}{\omega^2 Z^2} \cdot \left( \frac{uZ}{2\rho} \right)^2, \]
\[ m_0^S = 2 \cdot (1 - \gamma - \omega^2 Z^3) / \omega^2 Z^2 \cdot (uZ/2\rho)^2, \]
Pseudoscalar Interaction in the Theory of Beta-decay and RaE

\[ m_1^L = -\frac{1}{2\gamma + 1} \left\{ \frac{\gamma}{W} + (1 - 2\gamma W) \right\} \cdot \frac{aZ}{2 \rho}, \]

\[ m_1^S = (1 - \gamma) \cdot \frac{aZ}{2 \rho}, \]

\[ m_2^L = \frac{1}{2(2\gamma + 1)^3} \left\{ (1 + \gamma) \frac{p^3}{W} + 2a^2Z^2(1 - 2\gamma p^2) \right\}, \]

\[ m_2^S = \frac{1}{2(2\gamma + 1)^3} \left\{ (1 + \gamma) \frac{p^3}{W} + a^2Z^2 \left( \frac{1}{W} + W \right) - 4a^4Z^4W \right\}, \]

\[ n_0^L = (\gamma - 1) \frac{p^3}{4}, \]

\[ n_0^S = -\frac{\gamma}{W} \cdot \frac{aZ}{2 \rho}, \]

\[ n_1^L = \frac{-2}{2\gamma + 1} \left\{ \frac{1 - \gamma}{\rho^3} \frac{p^3}{W} + \frac{1}{W} + 2W \right\} \cdot \left( \frac{aZ}{2 \rho} \right)^3, \]

\[ n_1^S = 2 \left( \frac{aZ}{2 \rho} \right)^3, \]

\[ n_1^{L'} = \frac{1}{2(2\gamma + 1)^3} \left\{ (1 + \gamma) \frac{p^3}{W} + a^2Z^2 \left( \frac{1}{W} - 2W \right) \right\}, \]

\[ n_1^{S'} = a^2Z^2/2, \]

\[ n_2^L = \frac{-1}{(2\gamma + 1)(2\gamma + 2)} \left\{ (3\gamma - 1) p^3 - 2a^2Z^2(1 + W^3) \right\} \cdot \frac{aZ}{2 \rho}, \]

\[ n_2^S = \frac{-1}{2(2\gamma + 1)^3} \left\{ -\frac{\gamma}{W} + 2a^2Z^2W \right\}, \]

\[ n_2^{L'} = \frac{-1}{(2\gamma + 1)^3} \left\{ (1 + 4\gamma) p^3 - 4a^2Z^2W^3 \right\} \cdot \frac{aZ}{2 \rho}, \]

\[ n_2^{S'} = \frac{1}{(2\gamma + 1)^3} \left\{ \frac{\gamma}{W} - 4a^2Z^2(1 + \gamma) \right\} \cdot \frac{aZ}{2 \rho}, \]

\[ n_3^L = \frac{-1}{2(2\gamma + 1)^3(2\gamma + 2)} \left\{ (\gamma + 1)^3 \frac{p^3}{W} + a^2Z^2 \left( \frac{1 + \gamma}{W} - 2W(1 + 4\gamma) \right) \right\}, \]

\[ n_3^S = \frac{-a^2Z^2}{2(2\gamma + 1)^3(2\gamma + 2)} \left\{ (1 + 5\gamma + 2\gamma^2)p^3 - 2a^2Z^3(3 + 2\gamma)W^3 \right\}. \]

\[ x_{i\lambda} \text{ and } y_{i\lambda} \text{ are shown in Table I. In this table, the, following abbreviations are used:} \]
Table Ia.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_0$</td>
<td>$(\theta_0 + \theta_0')^2$</td>
<td>$(\beta_0 + \beta_0')^2$</td>
<td>$\beta_0 \sigma_1$</td>
<td>$\beta_0 \tau_{-1}$</td>
<td></td>
</tr>
<tr>
<td>$t_1$</td>
<td>$(\theta_1 + \theta_1')^2$</td>
<td>$(\beta_1 + \beta_1')^2$</td>
<td>$\beta_1 \sigma_1$</td>
<td>$\beta_1 \tau_{-1}$</td>
<td></td>
</tr>
<tr>
<td>$t_2$</td>
<td>$(\theta_2 + \theta_2')^2$</td>
<td>$(\beta_2 + \beta_2')^2$</td>
<td>$\beta_2 \sigma_1$</td>
<td>$\beta_2 \tau_{-1}$</td>
<td></td>
</tr>
<tr>
<td>$t_3$</td>
<td>$(\theta_3 + \theta_3')^2$</td>
<td>$(\beta_3 + \beta_3')^2$</td>
<td>$\beta_3 \sigma_1$</td>
<td>$\beta_3 \tau_{-1}$</td>
<td></td>
</tr>
<tr>
<td>$t_4$</td>
<td>$(\theta_4 + \theta_4')^2$</td>
<td>$(\beta_4 + \beta_4')^2$</td>
<td>$\beta_4 \sigma_1$</td>
<td>$\beta_4 \tau_{-1}$</td>
<td></td>
</tr>
<tr>
<td>$t_5$</td>
<td>$(\theta_5 + \theta_5')^2$</td>
<td>$(\beta_5 + \beta_5')^2$</td>
<td>$\beta_5 \sigma_1$</td>
<td>$\beta_5 \tau_{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

Table Ib. $I = -i\Gamma_1 = -i\gamma_5 \gamma_\sigma \gamma_\tau$. 

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_0$</td>
<td>$(q/3 + I)$</td>
<td>$(\theta_0 + \theta_0')^2$</td>
<td>$(q/3 + I) \beta_0 \sigma_1$</td>
<td>$q(\theta_0 + \theta_0') \tau_{-1} / 3$</td>
<td></td>
</tr>
<tr>
<td>$t_1$</td>
<td>$(q/3 + I)$</td>
<td>$(\theta_1 + \theta_1')^2$</td>
<td>$(q/3 + I) \beta_1 \sigma_1$</td>
<td>$q(\theta_1 + \theta_1') \tau_{-1}$</td>
<td></td>
</tr>
<tr>
<td>$t_2$</td>
<td>$(q/3 + I)$</td>
<td>$(\theta_2 + \theta_2')^2$</td>
<td>$(q/3 + I) \beta_2 \sigma_1$</td>
<td>$q(\theta_2 + \theta_2') \tau_{-1}$</td>
<td></td>
</tr>
<tr>
<td>$t_3$</td>
<td>$(q/3 + I)$</td>
<td>$(\theta_3 + \theta_3')^2$</td>
<td>$(q/3 + I) \beta_3 \sigma_1$</td>
<td>$q(\theta_3 + \theta_3') \tau_{-1}$</td>
<td></td>
</tr>
<tr>
<td>$t_4$</td>
<td>$(q/3 + I)$</td>
<td>$(\theta_4 + \theta_4')^2$</td>
<td>$(q/3 + I) \beta_4 \sigma_1$</td>
<td>$q(\theta_4 + \theta_4') \tau_{-1}$</td>
<td></td>
</tr>
<tr>
<td>$t_5$</td>
<td>$(q/3 + I)$</td>
<td>$(\theta_5 + \theta_5')^2$</td>
<td>$(q/3 + I) \beta_5 \sigma_1$</td>
<td>$q(\theta_5 + \theta_5') \tau_{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

$\beta_0 = (\gamma D_{-1} - B_{-1}) \rho^{-2} + B_{-1} \Gamma \rho^{-1}$, 

$\beta_0' = -(\gamma D_{-1} + B_{-1}) / 12$, 

$\beta_1 = (\gamma + 1) D_{-1} - B_{-1} + \rho \Gamma B_{-1}$, 

$\beta_2 = (\gamma + 2) D_{-1} - B_{-1} + \rho \Gamma B_{-1}$, 

$\theta_0 = (\gamma C_1 - A_1) \rho^{-2} + A_1 \Gamma \rho^{-1}$, 

$\theta_0' = - (\gamma C_1 + A_1) / 12$, 

$\theta_1 = (\gamma + 1) C_1 - A_1 + \sigma \Gamma A_1$, 

$\theta_2 = (\gamma + 2) C_1 - A_1 + \rho \Gamma A_1$.
Pseudoscalar Interaction in the Theory of Beta-decay and RaE

\[ a_0 = \frac{(\gamma C_{-1} + \rho \Gamma A_{-1})}{3}, \]
\[ a_1 = \frac{(\gamma + 1) C_{-1} + \rho \Gamma A_{-1}}{3}, \]
\[ \zeta_0 = \frac{(\gamma_1 C_2 + \rho \Gamma A_2)}{3}, \]
\[ \zeta_1 = \frac{(\gamma_1 + 1) D_1 + \rho \Gamma B_1}{3}, \]
\[ \xi_0 = \frac{(\gamma D_1 + \rho \Gamma B_1)}{3}, \]
\[ \xi_1 = \frac{(\gamma + 1) D_1 + \rho \Gamma B_1}{3}. \]

If we do not take the effect of the finite nucleus into account, the second terms in eq. (24) vanish. \( l_f, m_f^2 \), and \( n_f^2 \) are the constituents of \( L, M, \) and \( N \) given by Konopinski and Uhlenbeck. For reference, we re-write the Konopinski and Uhlenbeck’s correction factors for \( 0 \rightarrow 0 \) (yes) transition by our notation with the finite size correction:

\[ C_x = \int \sigma \cdot r \left[ l_{0T} + m_{0T} + 2 m_{1T} + m_{2T} + 2 m'_{2T} - n_{0T} - n_{1T} - n_{1T}' \right], \]
\[ C_A = \int \sigma \cdot r \left[ l_{0A} + m_{0A} + 2 m_{1A} + m_{2A} + 2 m'_{2A} + n_{0A} + n_{1A} + n_{1A}' \right]. \]

Table II

| \( x \) | \( T \) | \( y \) | \( A \)
|---|---|---|---|
| \( i_0 \) | \( \sigma \sigma_1 / 9 \) | \( \sigma \sigma_1 / 9 \) | \( l_{0A} = (1 + 3 \gamma / q) l_{0T} \)
| \( m_0 \) | \( \tau_1^2 \) | \( \sigma_1^2 \) | \( m_{0A} = m_{0T} \)
| \( m_1 \) | \( \tau_1^2 \) | \( \sigma_1^2 \) | \( m_{1A} = m_{1T} \)
| \( m_2 \) | \( \tau_1^2 \) | \( \sigma_1^2 \) | \( m_{2A} = m_{2T} \)
| \( m'_2 \) | \( \tau_1^2 \) | \( \sigma_1^2 \) | \( m_{2A}' = m_{2T}' \)
| \( n_0 \) | \( 2 \sigma \sigma_1 / 3 \) | \( 2 \sigma \sigma_1 / 3 \) | \( n_{0A} = (1 + 3 \gamma / q) n_{0T} \)
| \( n_1 \) | \( 2 \gamma \sigma \sigma_1 / 3 \) | \( 2 \gamma \sigma \sigma_1 / 3 \) | \( n_{1A} = (1 + 3 \gamma / q) n_{1T} \)
| \( n'_1 \) | \( 2 \gamma \sigma \sigma_1 / 3 \) | \( 2 \gamma \sigma \sigma_1 / 3 \) | \( n_{1A}' = (1 + 3 \gamma / q) n_{1T}' \)

§ 5. Beta-ray spectrum of RaE

The beta-ray spectrum of RaE was explained by Konopinski and Uhlenbeck with the \( T \) interaction, and by Smith with the linear combinations \( (V, T) \) and \( (S, A) \). All of them had regarded this transition as second forbidden. But from the prediction of the shell model and the \( f \)-value, this transition seems to be first forbidden. Nakamura, Umezawa and the author have shown that this spectrum can not be explained by the first forbidden transition of the tensor interaction. Radiative correction and mesonic correction may be unable to explain this singular spectrum.

Recently, Petschek and Marshak have succeeded in the explanation of this spectrum by the linear combination \( (T, P) \). But, as was mentioned in the introduction, it seems necessary to reexamine their calculation.

Using the above obtained correction factor, we have attempted to know whether the
RaE beta-spectrum can be explained by \((T, P)\) or not*. We have not considered the linear combination \((A, P)\), for the \(A\) interaction may not exist. The RaE beta-spectrum has been measured by many authors. We have used the data of Langer given in the Annual Review of Nuclear Science.

We have tried to explain the RaE beta-spectrum by adjusting the two parameters \(\lambda = \lambda_T / (\lambda_P M / 2M)\) and \(\Gamma / \rho\).

The condition that the three points on the Kurie plot for electron energy \(W_0\) (= maximum energy), \(W_1\) and \(W_2\) should lie on a straight line is represented by a quadratic for \(\lambda\) and \(\Gamma / \rho\). The graphs of these quadratics for various values of \(W_1\) and \(W_2\) are shown in Fig. 5. These are hyperbolas. They coincide practically with each other except near the apices, and we may be able to regard them as one hyperbola. In the explanation of the RaE beta-spectrum, \(\lambda\) and \(\Gamma / \rho\) must be based on this hyperbola.

From Fig. 5, we can conclude \(\lambda \cdot \Gamma / \rho < 0\). By the estimation (14) given by Ahrens et al., the value of \(\Gamma / \rho\) is about 200, and we cannot explain the spectrum by this value for any value of \(\lambda\).

But not all of the points on this hyperbola can explain the RaE beta-spectrum, for, although many hyperbolas seem to coincide with each other, they do not necessarily coincide at every point.

Therefore we have plotted the Kurie plots of RaE for various points on the hyperbola. And we found that when

\[*\) Very recently, Yamada has shown that this spectrum can be explained by \((S, T)\), assuming the transition to be first forbidden.\]
or putting $\lambda' = 1$, when

$$0.098 > \int \beta_T \cdot t \sigma \cdot t^\ast > -0.41,$$

Fig. 6a. The Kurie plots of $\text{RaE}$ for the points based on the part of the upper branch of the hyperbola and which is near the line (26). The values in parentheses indicate $3\beta_T/\lambda' \cdot t^\ast$ and $\lambda_T/\lambda'$. The topmost Kurie plot is the one which is obtained by neglecting the terms which contain the derivative of the lepton wave function. We have assumed $W_0 = 3.29$. 

\begin{tabular}{|c|c|c|}
\hline
$\Gamma/\rho$ & $\lambda$ \\
\hline
$20M(10)$ & 4920 ($-1.34$) \\
$27800(7.0)$ & $-2M(-1)$ \\
$2M(1)$ & $-468(-0.13)$ \\
$M(0.5)$ & $-220(-0.060)$ \\
$1000(0.27)$ & $-106(-0.029)$ \\
$800(0.22)$ & $-80(-0.022)$ \\
$300(0.14)$ & $-35(-0.0095)$ \\
$440(0.12)$ & $-25(-0.0068)$ \\
$400(0.11)$ & $-18(-0.0049)$ \\
$367(0.10)$ & $-10(-0.0027)$ \\
$365(0.099)$ & $-10(-0.0027)$ \\
$360(0.098)$ & $-8(-0.0022)$ \\
\hline
\end{tabular}
we can not explain the RaE beta-spectrum for any ratio of the coupling constants of the $T$ and $P$ interaction.

If the magnitude of $\beta_5$ or $\Gamma/\rho$ is so large as was shown by Ruderman$^{13}$), the RaE beta-spectrum can be undoubtedly explained. And even if it reduces to about one tenth of Ruderman's value by virtue of the damping effect calculated by Brueckner et al.$^{13}$, we can explain the RaE beta-spectrum by the linear combination $(T, P)$. But when we introduce more complicated processes into the calculation of Brueckner et al., if the magnitude of $\beta_5$ more reduced, we cannot explain the RaE beta-spectrum by the linear combination $(T, P)$.

As for $\lambda$, if the RaE decay is $0 \rightarrow 0$ (yes), the region

$$120 > \lambda > -10,$$

or assuming $\lambda' = 1$

---

Fig. 6b. The Kurie plots of RaE for the points based on the part of the upper branch of the hyperbola, and which is near the line (25). The topmost Kurie plot is the one which is obtained by neglecting the terms which contain the derivative of the lepton wave function.
is forbidden*).

The asymptotes of this hyperbola are:

\[ \frac{\Gamma}{\rho} = -19\lambda + 680, \quad (25) \]
\[ \frac{\Gamma}{\rho} = -7.4\lambda + 160. \quad (26) \]

These are the relations between \( \frac{\Gamma}{\rho} \) and \( \lambda \), when the term \( R'(\rho) \) does not appear in eq. (13).

For the large values of \( |\frac{\Gamma}{\rho}| \) the part of the hyperbola which is close by the line (25) seems to fit in better with the experimental data, and for the small values of \( |\frac{\Gamma}{\rho}| \), the part which is near the line (26) seem to be better.

The best Kurie plots for various values of \( \frac{\Gamma}{\rho} \) and for \( \lambda = \pm 2M \) are shown in Fig. 6. These seem to explain the RaE beta-spectrum fairly well.

If a cancellation occurs between the terms which do not contain the derivatives of lepton wave function, it occurs also between the terms which contain the derivatives, and

\[ \begin{align*}
\frac{\Gamma}{\rho} &= -20M(-10) \\
\lambda &= 4950(1.35) \\
\frac{\Gamma}{\rho} &= -27500(-7.5) \\
2M(1) &= 516(0.141)
\end{align*} \]

* Fujita and Yamada\textsuperscript{30} and Kofoed-Hansen and Winther\textsuperscript{31} have obtained the extent to which the ratio of \( \lambda_\rho \) and \( \lambda_\tau \) can exist from the analysis of the beta-ray spectra of He\textsuperscript{6} and B\textsuperscript{13}.
the term $R'(\rho)$ in eq. (13) hardly affects the results except near the apex of the hyperbola.

Fig. 6d. The Kurie plots of $\text{RaE}$ for the points based on the part of the lower branch of the hyperbola, and which is close by the line (25).

<table>
<thead>
<tr>
<th>$\Gamma/\rho$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-69000(-18.8)$</td>
<td>$2M(1)$</td>
</tr>
<tr>
<td>$-20M(-10)$</td>
<td>$1970(0.54)$</td>
</tr>
<tr>
<td>$-2M(-1)$</td>
<td>$229(0.062)$</td>
</tr>
<tr>
<td>$-1500(-0.41)$</td>
<td>$115(0.31)$</td>
</tr>
</tbody>
</table>

Table III. The ratios of the nuclear matrix elements for $\lambda_T/\lambda_P = \pm 1$

<table>
<thead>
<tr>
<th>$\lambda_T/\lambda_P$</th>
<th>$-1$</th>
<th>$+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Takebe</td>
<td>19.3 or 7.03</td>
<td>$-18.8$ or $-7.49$</td>
</tr>
<tr>
<td>Petschek &amp; Marshak</td>
<td>13</td>
<td>$-13$</td>
</tr>
</tbody>
</table>

Acknowledgments

The author wishes to express his cordial thanks to Professor R. E. Marshak for his valuable discussions at Kyoto. In addition, the author wishes to thank Professor T. Yamanouchi, Professor M. Taketani, Professor S. Nakamura, Dr. M. Umezawa, Dr. M. Morita, Dr. M. Yamada and Dr. S. Okubo for their kind interest in this work. Miss M. Hayashi, Miss T. Nakano, Mr. T. Hibino, Mr. Y. Nakai, Mr. T. Takebe and Mr. M. Watanabe were very kind and helpful in the numerical calculations. The author is indebted to the “Yukawa Yomiuri Fellowship” for the financial aid.
Pseudoscalar Interaction in the Theory of Beta-decay and RaE 689

References

5) M. Taketani, S. Nakamura, K. Ono and M. Umezawa, Soryushiron-kenkyu (Mimeographed circular in Japanese) 4 (1952), No. 6, 24; No. 8, 45; No. 12, 96.
7) M. E. Rose and D. K. Holmes, Oak Ridge National Laboratory Report 1022 (1951); Phys. Rev. 85 (1951), 190. The author wishes to thank Dr. Rose for correspondence concerning these calculations.
9) T. Ahrens and E. Feenberg, Phys. Rev. 86 (1952), 64.
12) M. Ruderman, Phys. Rev. 89 (1953), 1227.
18) I. Malcolm, Phil. Mag. 45 (1932), 1011.
19) A. M. Smith, Phys. Rev. 82 (1951), 955.
20) D. L. Pursey, Phil. Mag. 42 (1951), 1193.
22) A. M. Smith, Phil. Mag. 45 (1952), 915.