ANFIS-based approach for the estimation of transverse mixing coefficient
Z. Ahmad, H. Md. Azamathulla and N. A. Zakaria

ABSTRACT
Understanding of the fate of pollutants, disposed of in streams, is a matter of concern in recent years for the effective control of pollution. Transverse mixing of the pollutants in open channels is arguably more important than the longitudinal mixing and near-field mixing. Several attempts have been made to establish the relationship between the transverse mixing coefficient and bulk channel and flow parameters such as width, depth, shear velocity, friction factor, curvature and sinuosity. This paper presents adaptive neuro fuzzy inference system (ANFIS) approach to predict the transverse mixing coefficient in open channel flows. Available laboratory and field data for the transverse mixing coefficients covering wide range of channel and flow conditions are used for the development and testing of the proposed method. The proposed ANFIS approach produces satisfactory results ($R^2 = 0.945$) compared to the artificial neural network (ANN) model and existing predictors for mixing coefficient.

Key words | ANFIS, ANN, mixing, open channel flow, pollutant, transverse

INTRODUCTION
Streams have been used for the disposal of various industrial and municipal wastes for a long time. Understanding of mixing of such pollutants in streams is a matter of concern in recent years for the effective control of pollution in the streams. Most of the natural streams are relatively shallow compared with their length and width. Thus when pollutants are disposed of at a point in a stream, it mixes quickly over the entire depth and then continues to spread in the longitudinal and transverse flow directions. Excluding the initial distance required to achieve mixing in the vertical direction, the problem can be efficiently modelled by a two-dimensional depth-averaged mixing equation, i.e., transverse mixing equation (Ahmad 2007, 2008).

The process of transverse mixing of a conservative and neutrally buoyant substance in a steady flow through a straight channel and for the constant injection of the pollutants is modelled by the principle of conservation of mass of the substance and written as (Lau & Krishnappan 1997; Seo et al. 2006):

$$\frac{\partial}{\partial s}(UDC) = \frac{\partial}{\partial z} \left[ DE_z \frac{\partial C}{\partial z} \right]$$

where $C =$ depth-averaged concentration; $D =$ depth of flow; $U =$ depth-averaged velocity in the longitudinal direction; $s$ and $z =$ longitudinal and transverse distances, respectively; $E_z =$ mixing coefficients in the transverse directions. Analytical and numerical solutions of Equation (1) for the different boundary conditions are available in the literature (Boxall & Guymer 2003a, b; Ahmad 2007).

Several attempts have been made to establish the relationship between the transverse mixing coefficient and bulk channel parameters such as width, depth, shear velocity $U_s$, friction factor, curvature and sinuosity (Sayre & Chang 1968;
Several studies and suggested that $\frac{E_z}{\text{DU}^*}$ does not vary with $\text{Krishnappan }$ (2003). Guymer and Ahmad (2007) reviewed several studies and found that $\frac{E_z}{\text{DU}^*}$ does not vary with width of the channel. Based on the experimental study in a rectangular channel, Chau (2000) proposed $\frac{E_z}{\text{DU}^*} = 0.15$ and Ahmad (2007) proposed $\frac{E_z}{\text{DU}^*} = 0.15$.

This paper deals with the estimation of transverse mixing coefficient using soft computing technique ANFIS. The accuracy of the proposed equation is checked with un-used experimental data.

### THE ANFIS NETWORKS

ANFIS, first introduced by Jang (1995), is a universal approximator and, as such, is capable of approximating any real continuous function on a compact set to any degree of accuracy. Thus, in parameter estimation, where the given data are such that the system associates measurable system variables with an internal system parameter, a functional mapping may be constructed by ANFIS that approximates the process of estimation of the internal system parameter.

The ANFIS is functionally equivalent to fuzzy inference systems. The hybrid learning algorithm, which combines gradient descent and the least-squares method, is introduced, and the issue of how the equivalent fuzzy inference system can be rapidly calibrated and adapted with this algorithm is discussed herein. Most of the previous works that address artificial neural networks (ANN) applications to water resources have included the feed forward type of the architecture, where there are no backward connections, which are trained using the error back propagation scheme or the feed forward back propagation (FFBP) configuration. Drawbacks of ANN include that it needs more training time and the difficulties in detecting hidden neurons in hidden layer for better predictions (Azamathulla & Ghani 2010). Therefore, the present study applies a new soft computing technique—ANFIS. The input in ANFIS is first converted into fuzzy membership functions, which are combined together. After following an averaging process to obtain the output membership functions, the desired output is finally achieved.

### DESCRIPTION OF COLLECTED DATA AND DIMENSIONAL ANALYSIS

Several experimental studies for the transverse mixing in straight rectangular laboratory channels have been conducted (Kalisinske & Pien 1944; Elder 1959; Sawyer & Chang 1968; Sullivan 1968; Engelund 1969; Okoye 1970; Prych 1970; Holley & Abraham 1973; Engmann 1974; Miller & Richardson 1974; Weibel & Schatzmann 1984; Nokes 1986; Lau & Krishnappan 1997). These provide estimates of the transverse mixing coefficient $E_z$ provided the flow does not depart significantly from the plane shear flow. Ahmad (2007) performed experiments at Hydraulics Laboratory of Civil Engineering Department, Indian Institute of Technology Roorkee, India and measured the concentration profiles of the tracer at downstream stations resulting due to steady state injection of the tracer, in the upstream of a flume. A brief description of his experimental work is described below:

The experiments were performed in a recirculating concrete flume of width 1.0 m, depth 0.30 m, length 19 m and bed slope 0.000652 (Figure 1). The water was supplied to the flume through an overhead tank. An orifice meter was fitted in the delivery pipe for the discharge measurement. At the inlet of the flume, flow straighteners and wave suppressor were provided to align the flow and to suppress the surface disturbances, respectively. The rhodamine WT was used as tracer due to its high detectability and conservative nature. Injection sampler, used to inject dye, consisted of two tubes of size 2 mm diameter placed at 25 mm spacing (Figure 2). The tubes had a number of vertical holes at the interval of 20 mm.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Empirical formulae for the transverse mixing coefficient</th>
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<tr>
<td>Straight channels</td>
<td>$0.15 &lt; \frac{E_z}{(\text{DU}^*)} &lt; 0.30$</td>
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<tr>
<td>Gentle meandering channels</td>
<td>$0.30 &lt; \frac{E_z}{(\text{DU}^*)} &lt; 0.90$</td>
</tr>
<tr>
<td>Curved channels</td>
<td>$1.0 &lt; \frac{E_z}{(\text{DU}^*)} &lt; 3.0$</td>
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and connected to a manifold. The tracer was supplied to the manifold through a 6 mm polythene pipe from a constant head tank containing dye. The sampler was designed to represent the plane source of width 25 mm. The experiments were performed for width of channel \( B = 100 \text{ cm}, 50 \text{ cm}, 40 \text{ cm}, 30 \text{ cm}, \) and 20 cm and for five to seven different discharges for each width of the channel. Tracer of concentration 9,815 ppm was continuously injected at a constant rate through the injection sampler located at 7 m downstream of the flume head and near left bank of the flume. Samples of water were collected in the glass test tubes at an instance at every 2 m distances in the downstream and at various transverse distances. The collected water samples were analysed in the Fluorometer for their concentration. The blank concentration was deducted from the observed concentration, and then the concentration profiles at different downstream stations across the width were normalised with the mass of the injected tracer. Observed concentration profiles across the width at various longitudinal distances clearly indicate that as tracer cloud moves in the downstream, it mixes transversely and its peak concentration decreases with distance.

The measured tracer concentration was used to estimate the transverse mixing coefficient using a numerical method developed by Ahmad (2007). The above data are used in this study to propose predictor for the transverse mixing coefficient.

Literature review reveals that the transverse mixing coefficient would be a function of width of channel \( B, D, U \) and \( U^* \). Thus, the functional relationship for dimensionless mixing coefficient through dimensional analysis may be written as

\[
\frac{E_z}{D U^*} = f \left( \frac{B}{D}, \frac{U}{U^*} \right)
\]

(2)

The values \( E_z/\alpha \), \( B/D \), and \( U/U^* \) in the collected data vary from 0.088 to 0.146; 2.86 to 17.80; and 4.57 to 13.20, respectively.
DEVELOPMENT OF ANFIS MODEL

The network of ANFIS as shown in Figure 3 works as follows: let \( x \) and \( y \) be the two typical input values fed at the two input nodes, which will then transform those values to the membership functions (say bell-shaped) and give the output as follows (note in general, \( w \) is the output from a node; \( m \) is the membership function, and \( x, y \) in Equation (3)

\[
\mu_{Mi}(x) = \frac{1}{1 + |(x - c_1)/a_1|^b_1} \quad (i = 1, 2)
\]

where \( a_1, b_1, \) and \( c_1 \) are changeable premise parameters. Similar computations are carried out for the input of \( y \) to obtain \( \mu_{Ni}(y) \). The membership functions are then multiplied in the second layer, e.g.

\[
w_i = \mu_{Mi}(x) \cdot \mu_{Ni}(y) \quad (i = 1, 2)
\]

Such products or firing strengths are then averaged:

\[
\overline{w_i} = \frac{w_i}{\sum w_i} \quad (i = 1, 2)
\]

Nodes of the fourth layer use the above ratio as a weighting factor. Furthermore, using fuzzy if-then rules produces the following output (an example of an if-then rule is: If \( x \) is \( M_1 \) and \( y \) is \( N_1 \), then \( f_1 = p_1x + q_1y + r_1 \)):

\[
\overline{w_i} f_i = \overline{w_i} (p_1x + q_1y + r_1)
\]

where \( p, q, \) and \( r \) are changeable consequent parameters. The final network output \( f \) was produced by the node of the fifth layer as a summation of all incoming signals, which is exemplified in Equation (6).

A two-step process is used for faster training and to adjust the network parameters to the above network. In the first step, the premise parameters are kept fixed, and the information is propagated forward in the network to layer 4. In layer 4, a least-squares estimator identifies the important parameters. In the second step, the backward pass, the chosen parameters are held fixed while the error is propagated. The premise parameters are then modified using gradient descent. Apart from the training patterns, the only user-specified information required is the number of membership functions for each input. The description of the learning algorithm is given in Jang & Sun (1995).

The scenarios considered in building the ANFIS model inputs and output are shown in the network (Figure 4). From the collected data sets (166) used in this study, around 75% of these patterns (125) were used for training (chosen randomly until the best training performance was obtained), while the remaining 41 patterns (25%) were used for testing, or validating, the ANFIS model. Software program code was developed to perform the analysis.

The computed \( Ez/DU^* \) is compared with the observed one for testing the model (Figure 5). The coefficient of determination \( (R^2) \) and root mean square error (RMSE) for training the model are 0.968 and 0.0051, respectively, while for testing 0.945 and 0.0062, respectively.

ANN MODEL

A FFBP neural network was trained with the same data sets used earlier for training the ANFIS modelling. It was found that the best model representing the satisfactory estimation of the

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**Figure 3** | ANFIS Network Architecture.

**Figure 4** | The ANFIS model.
mixing coefficient is in the form of the ANN with two inputs, 10 hidden neurons in the one hidden layer and one output. The R² and RMSE for training the model are 0.742 and 1.0235, respectively, while for testing 0.68 and 1.347, respectively.

RESULTS AND DISCUSSION

The results of the empirical equations in Table 1 are calculated using all of the experimental and collected data set and results of them are compared with measured data. Based on the results of these equations, none of these empirical equations have good results and they show considerable errors in comparison with measured data. The values of these statistical indexes show the poor performance of empirical equations for prediction of transverse mixing coefficients.

The testing results of the proposed new ANFIS model and ANN model are compared with the statistical parameters, i.e., R² and RMSE. Such comparison reveals that the proposed ANFIS model predicts fairly (R² = 0.945) accurately the transverse mixing coefficient compared to the ANN model and existing empirical equations for prediction of transverse mixing coefficients.

NOTATIONS

B = Width of channel
C = depth-averaged concentration
D = depth of flow
Ez = mixing coefficients in the transverse direction
Mᵢ and Nᵢ = fuzzy sets
U = depth-averaged velocity in the longitudinal direction
a₁, b₁, and c₁ = changeable basis parameters
p, q, and r = changeable consequent parameters
s = longitudinal distance
ωᵢ = firing strength
x, y = inputs
z = transverse distance
U* = Shear velocity
μ = the membership function

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