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Journal of Applied Mechanics Discussion

A Conservation Theorem for Constrained Multibody Systems[1]

J. G. Papastavridis.[2] In Wang (1993), a laudable effort is made to remove the theoretical shortcomings of Kane and Levinson (1988). However, in the process Wang misrepresents the far more general exposition of Papastavridis (1991); he also introduces a number of fundamental errors which render his paper physically meaningless.

Let us begin with the latter. Wang states and proves (sic) that the power equation of *every* constrained mechanical system, under *arbitrary* forces, integrates to a corresponding first integral (constant) of motion of that system; i.e., that *every* constrained mechanical system has an energy integral! But, in general, the power of *nonpotential* forces does *not* equal the $(\cdots)^{\cdot}$ derivative of a scalar energetic function; i.e., *the corresponding elementary work in general, is not the exact (or total) differential of a potential (or work) function.* Calling a power-like quantity $\dot{Z}$, Eq. (53), and then integrating it to Z, Eq. (54), is mathematically impermissible and physically meaningless—in general, *no Z exists.* (Similar remarks apply to the generally *nonexistent Q*, Eqs. (57)–(59).) Contrary to Wang's implication, power equations, in general, do *not* lead to energy/conservation theorems. A first integral of motion is a (first-order) algebraic relation among time, positions, and velocities holding for any system motion. Of course, *after* the problem has been solved and a particular motion has been found, Wang's $\dot{Z}$, $\dot{Q}$, etc., become *known functions of time* (or of equivalent path parameters), and can be integrated to yield Z, Q, etc., *but that doesn't make them integrals of motion.* Therefore Wang's transition from Eqs. (57)–(58) to Eq. (59), the centerpiece of his paper, is, generally, invalid.

Incidentally: (i) Wang's "definition" (56) is well known; (ii) while his allegedly new definition of the Hamiltonian, Eqs. (98), (99), can be found in most advanced dynamics expositions (e.g., Gantmacher (1970), Greenwood (1976), and Papastavridis (1991), Eqs. (12), (13), (14), (55)); (iii) Wang's terminology Eq. (34), ". . .$\dot{Q}$ is the net power flowing into the system" confuses the first law of thermodynamics with the purely mechanical law of kinetic energy, e.g., Langhaar (1962), Fox (1967), according to which the rate of change of the sum $K+V=$ Total Energy equals the power of *nonpotential* external *and* internal forces; and the integral of $\dot{Q}$ is neither a constant, nor "the first integral of motion of the system."

Next, analytical dynamics is about *impressed* forces and forces of *constraint* (reactions); whereas "elementary" mechanics is about external and internal forces. Contrary to Wang's claims: (i) Papastavridis (1991), whose equations easily specialize to multibody systems, obtains power equations *with* constraint reactions in *holonomic* variables, Eqs. (11)–(15); and, *purposely*, *without* constraint reactions in *nonholonomic* variables, Eqs. (57)–(71); and indicates how to connect them and obtain reaction power equations in nonholonomic variables, if needed (Remark on Eq. (52)). The absence of constraint reactions in Eqs. (57), (58), (71) is an advantage; but if one wants, he/she can easily do that as shown there; or by applying the well-known theorem of kinetic energy Eqs. (11), (136–139.2). Hence Wang's statement". . . this useful information is not provided by the power equation given by Papastavridis (1991)" is a misrepresentation. And "this useful information" is not provided by Wang's Eqs. (52), (53) either, but by his Eqs. (67), (89); and there are general equivalents to both sets of equations in Papastavridis (1991).

In sum: The power equations of Papastavridis (1991) are based on the fundamental division of forces into *impressed* and *constraint reactions.* On the other hand, the law of kinetic energy involves *all* forces, external and internal. Hence, comparing power equations based on such different force classifications, as Wang does, leads, in general, to meaningless, or flawed, results. One cannot expect to obtain the power of constraint forces from equations of motion specifically formulated to exclude them (even though Papastavridis (1991) treats both cases). And if Papastavridis' speculation on the root causes of the Kane and Levinson (1988) limitations is invalid, then Wang should show the precise physical meaning of the absence of satisfaction of Kane and Levinson's conditions; and exactly how that affects Wang's $\dot{Z}$, Eq. (53).

References

Fox, E. A., 1967, *Mechanics*, Harper and Row, New York, pp. 92–97.

Gantmacher, F., 1970, *Lectures in Analytical Mechanics*, Mir, Moscow, pp. 48–51.

Greenwood, D. T., 1976, *Classical Dynamics*, Prentice-Hall, Englewood Cliffs, NJ pp. 66–78.

Kane, T. R., and Levinson, D. A., 1988, "A Method for Testing Numerical Integrations of Equations of Motion of Mechanical Systems," ASME Journal of Applied Mechanics, Vol. 55, pp. 711–715.

Langhaar, H. L., 1962, *Energy Methods in Applied Mechanics*, John Wiley and Sons, New York, pp. 10–13.

Papastavridis, J. G., 1991, "On Energy Rate Theorems for Linear First-Order Nonholonomic Systems," ASME Journal of Applied Mechanics, Vol. 58, pp. 536–544.

Author's Closure[3]

I find that Professor Papastavridis' discussion adds nothing but confusion to the paper under discussion. To set the record straight I am responding to his comments, including that I "misrepresented" his work, and that my paper has "a number of fundamental errors," below.

1 I could not agree with him that the paper under discussion "is made to remove the theoretical shortcomings of Kane and Levinson (1988)." The so-called "shortcomings" were reme-

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